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APPLICATION OF GENERALIZED GAUSSIAN DISTRIBUTION IN THE PROCESSING OF WIRE SCANNER DATA

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Abstract

Wire scanners are widely used for measuring beam emittance in both electron and hadron accelerators. Gaussian fitting is the most commonly used method in processing the wire scanner data. But in hadron machines, beams are normally not Gaussian distribution due to the action of nonlinear forces such as space charge effect. Under these circumstances, there would be big deviations if the wire scanner data was still fitted with normal Gaussian distributions. This paper introduces the superiority of generalized Gaussian distribution in the processing of wire scanner data. The wire scanner data measured in the ADS injector-I will be taking as an example to the necessity of using generalized Gaussian distribution to fit non-Gaussian signal data.

INTRODUCTION

Wire scanner, is one of the most commonly used tools to measure beam emittances, it is widely used in both electron and hadron accelerators [1-6]. The measured signal from wire scanners is the distribution of particles along the moving direction of the wires, it needs to be processed to obtain the beam sizes that we are interested in. The most commonly used fitting method in processing the measured data of wire scanners is the Gaussian distribution fitting, which assumes that the distribution of particles along the measured direction is Gaussian. But in real machines, especially hadron accelerators, the particle distributions are usually not Gaussian due to the nonlinear effects from space charge effect etc [7, 8]. In this case, if the wire scanner measured data still be fitted with Gaussian distributions, big deviations may appear in the final fitted beam sizes from the real beam sizes.

Generalized Gaussian (or ggGaussian) distribution [9] is a statistical analysis method which is widely used in processing images and video signals. It can describe the Root Mean Square (RMS) value of distributions with different shapes. This paper will compare the fitting results with distribution function and calculation results with formula, and their different sensitivity to errors. The measured data with wire scanners in ADS injector-I [10, 11] will be used as example to show the differences of fitting results between normal Gaussian and ggGaussian when the signal is not Gaussian.

DIFFERENT FITTING METHODS

The definition of gg Gaussian distribution gg (x; μ, σ, p) is:

$$gg(x;\mu,\sigma,p) = \frac{1}{2\Gamma\left(1+{}^{1}/_{p}\right)A(p,\sigma)} e^{-\left|\frac{x-\mu}{A(p,\sigma)}\right|^{p}} . \qquad (1)$$

In Eq. (1), $x, \mu \in R$, $p, \sigma > 0$, $\Gamma(t) = \int_0^\infty e^{-u} u^{t-1} du$ is the Γ function, and $A(p,\sigma) = \left[\frac{\sigma^2 \Gamma(1/p)}{\Gamma(3/p)}\right]^{1/2}$. In this definition, μ is the center of the distribution, σ is the RMS value of the distribution, and p represents the shape of the distribution. For the same σ, a bigger p means the distribution is flatter, or the variation of the distribution is more gently, as shown in Fig. 1.

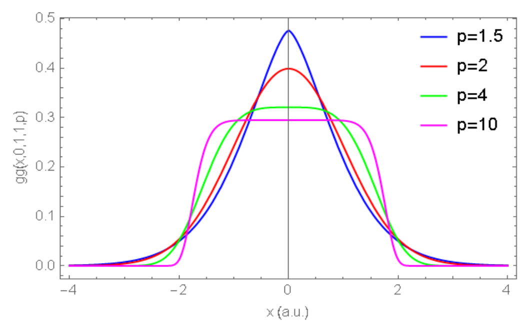

Figure 1: Comparison of ggGaussian distribution with different p value when the other parameters are the same.

It can be noted that when p=2, the ggGaussian distribution degenerated to the normal Gaussian distribution:

$$g(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left|\frac{x-\mu}{2\sigma}\right|^{2}} . \qquad (2)$$

The parameter σ in Eqs. (1) and (2) represents the RMS value of the distribution, it can be analyzed by the statistical formula, i.e.

$$\sigma = \sqrt{\langle x^2 \rangle} = \sqrt{\frac{\sum I_i (x_i - \mu)^2}{\sum I_i}} . \qquad (3)$$

Where I_i is the signal intensity at position x_i. Equation (3) is valid for any distributions, either Gaussian or non-Gaussian signals.

For an ideal standard Gaussian distribution, we could expect fitting with ggGaussian distribution, Gaussian distribution and calculation with Eq. (3), would give the same results. So we compare the results when dealing with signals whose distributions are not ideal Gaussian.

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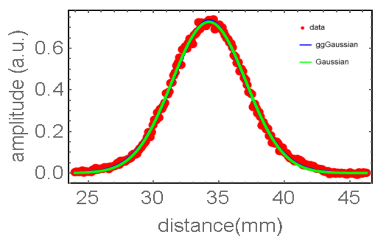
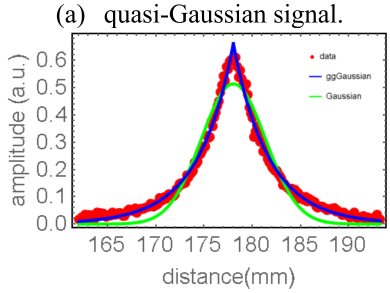
WIRE SCANNER DATA ANALYZE

Here we use the wire scanner measured signals in ADS injector-I as an example to compare the different results of fitting with ggGaussian, with normal Gaussian and the calculation with Eq. (3).

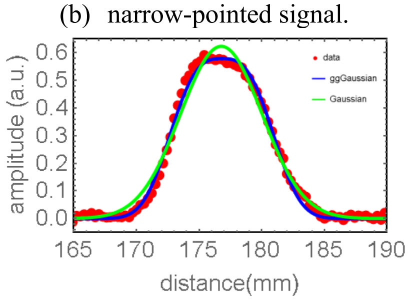
The shapes of wire scanner signals measured in ADS inject-I can be classified to three typical types – quasi-Gaussian, narrow-pointed and wide-plump. The measured signal and the fitting results with ggGaussian and Gaussian distribution functions are shown in Fig. 2.



(a) quasi-Gaussian signal.



(b) narrow-pointed signal.



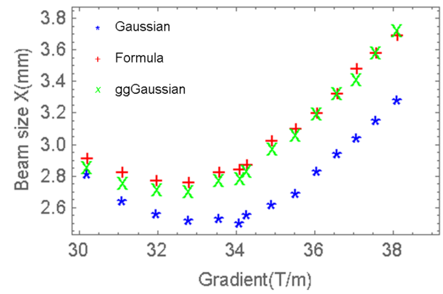
(c) wide-plump signal.

Figure 2: Fitting result for three typical signal distributions with ggGaussian and normal Gaussian distribution.

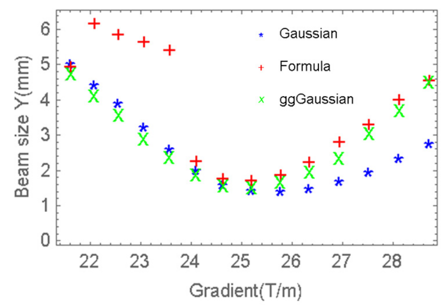
We can see from Fig. 2 that, for quasi-Gaussian signal, both the fitting with ggGaussian and normal Gaussian agree with the measured data very well; while for the narrow-pointed and wide-plump shaped signal, the fitting result of ggGaussian agrees much better than the one of normal Gaussian with the measured data.

The comparison of fitting results with ggGaussian and normal Gaussian and the calculation results with Eq. (2) of all the measured data in ADS Injector-I are shown in Fig. 3.

The corresponded p values of the fitting results of the ggGaussian distribution are shown in Fig. 4.

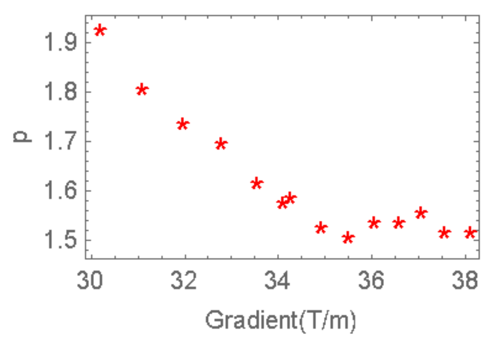


(a) Fitting and calculation results of beam sizes in horizontal plane.

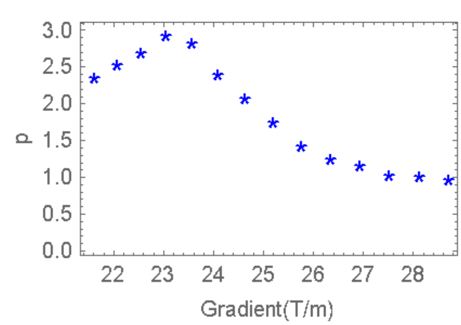


(b) Fitting and calculation results of beam sizes in vertical plane.

Figure 3: The comparison of fitting results with ggGaussian and normal Gaussian and the calculation results with Eq. (2) of all the measured data in ADS Injector-I.



(a) Fitted p value for horizontal plane.



(b) Fitted p value for vertical plane.

Figure 4: The corresponded p value of the ggGaussian fitting function for the measured data in ADS Injector-I.

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From Figs. 3 and Fig. 4 we can see that when p is about 2, the distribution of measured signal is quasi-Gaussian, so the fitting results of ggGaussian and normal Gaussian agrees with each other. As the p value gets away from 2, it represents the distribution of measured signal data deviate more and more from the Gaussian distribution, the difference of fitting results of ggGaussian and normal Gaussian also gets bigger. The fitting results using ggGaussian agree well with the calculation results using Eq. (3) for all signal distributions except for the four points in vertical distribution.

The big error of formula calculation in these four points at vertical distribution comes from the long tail of the measured data, which is shown in Fig. 5.

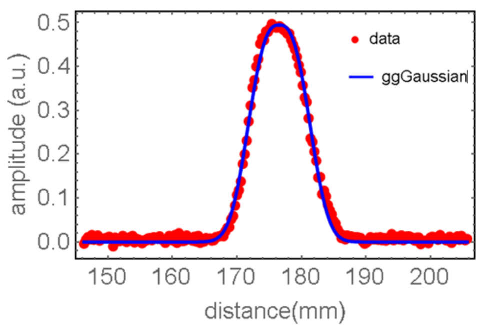

Figure 5: Signals with long tail measured in ADS injector-I.

We can see from Fig. 5 that, the measured data has very long tails. The amplitude of the tails is the background noises, which is not zero. When calculate the RMS sizes of the signal, all the noise signals will contribute to the results, which results in the big error of the final calculation results. When fitting the signal with ggGaussian distribution, the effect of background noise in the tail signal can be well inhibited as shown in Fig. 5.

SUMMARY

The ggGaussian distribution fit method can describe the RMS beam sizes very well when the wire scanner measured signals are not Gaussian distribution compared to the normal Gaussian distribution fit. It can well inhibit the effect of background noise compared to the formula calculation method. It is very beneficial to use ggGaussian distribution in fitting the wire scanner signals, especially in the lower high current hadron accelerators.

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