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APPLICATION OF THE FFA CONCEPT TO A MUON COLLIDER COMPLEX

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Abstract

A muon collider complex is one of the places where the concept of Fixed-Field Alternating Gradient (FFA) optics can be applied with great benefits. A vertical excursion FFA (vFFA) would provide isochronous acceleration of ultra-relativistic muon beams following pre-acceleration. This property, combined with fixed transverse tune, makes the vFFA an ideal accelerator for short-lived muon beams, given its lack of need for time variation of magnetic fields and RF frequency. Novel collider ring optics are designed based on skew quadrupole focusing inspired by a vFFA where the lowest order multipole is a skew quadrupole. This enables control of the momentum compaction factor. Neutrinos from the continuous decay of muons are spread out via orbit wiggling in the vertical direction as well as horizontal. This paper discusses the underlying principles of such a machine and describes some design examples.

VFFA FOR MUON ACCELERATION

A vertical excursion Fixed-Field Alternating Gradient Accelerator (vFFA) was first invented as an electron cyclotron [1]. Unlike a conventional cyclotron, an equilibrium orbit is not confined to a horizontal plane, and the strength of bending and focusing fields is scaled vertically. As a result, the shape of the trajectory is the same for different momenta, except at different vertical height. Since the dispersion function is finite in only the vertical direction, the momentum compaction factor is zero, like in a linac. For ultra-relativistic particles like high energy electrons, the slippage factor is closed to zero because $1/\gamma^2$ is approximately zero (where γ is the Lorentz factor). This leads to the possibility of isochronous and continuous acceleration of ultra-relativistic beams.

Muons after initial acceleration to a few 10s of GeV are already in the ultra-relativistic regime [2]. As a muon collider accelerator, the acceleration does not require continuous operation. On the other hand, a fixed magnetic field and fixed RF frequency give a big advantage.

Example of Muon Accelerator

Figure 1 shows one cell of a muon accelerator lattice example based on FODO optics. The orbit moves upward when accelerated. Figure 2 shows the magnetic field along the orbit for a FODO and FDF triplet lattice. It is clear that a large fraction of the normal bend at the focusing magnet is cancelled by the reverse bend at the defocusing magnet in these designs, which increases the circumference of the ring. The reverse bend should be minimised, and this is one of the targets for future design iteration.

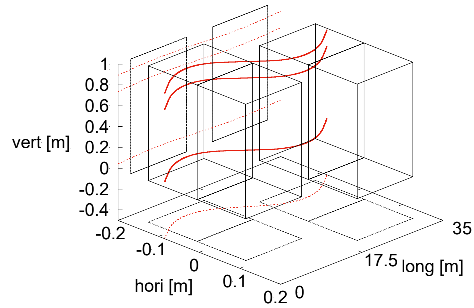
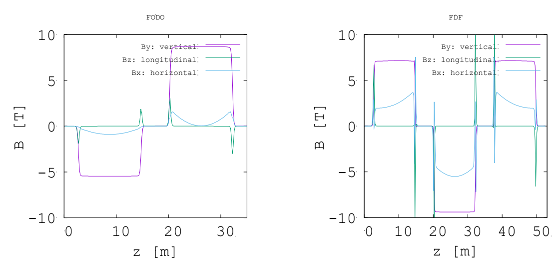


Figure 1: One cell of muon accelerator lattice based on FODO optics. The left brock is a defocusing magnet bending the beams in the reverse direction and the right one is a focusing magnet bending in the normal direction. The red curves are orbit at different momenta. The shape of the orbit is the same, but shifts upward when accelerated.



(a) (b)

Figure 2: (a) 3D magnetic field along the orbit for FODO lattice, (b) same for FDF triplet lattice. Main parameters are in Table 1. There is a finite longitudinal field even on mid-plane. All 3D fields increase in the vertical direction by $\exp(my)$ where y is the vertical coordinate and m is called effective field gradient.

Table 1: Main Parameters of vFFA Accelerators

	FODO	FDF
Energy	0.05 to 1.5 TeV	0.05 to 1.5 TeV
Cell len	35 m	35 m
# of cell	810	540
m-value	6.8	3.0
Tune	0.3957 / 0.0861	0.3510 / 0.1515
Orbit ex	0.50 m	1.13 m
Magnet	2 x 15 m	3 x 15 m
Field	8.7 T	10.6 T
Grad	240 T/m	240 T/m

Although the vertical orbit excursion is not large considering the momentum ratio from injection to extraction, it is preferable to reduce it further at the position of RF cavities. In the conventional FFA where the orbit moves in a horizontal place, a dispersion suppressor can be constructed by

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combining cells with different geometrical field index k [3]. It has not been successful so far, but a similar concept is being applied to the design of vFFA by combining cells with different effective field gradient m.

NOVEL OPTICS FOR MUON COLLIDER RING

Some constraints exist for muon collider ring optics. Firstly, a short bunch length should be preserved in order to keep high luminosity. In terms of optics, the momentum compaction factor should be as small as possible. Secondly, neutrino radiation tangential to the orbit (from the continuous muon decay) should not be focused in fixed directions. A collider ring orbit and optics have to take into account some mitigation measures.

We propose a collider arc comprised of skew quadrupoles with vertical displacement. As shown below, mitigation measures against these two constraints can be easily satisfied by this optics.

Design Principle

When focusing (NQF) and defocusing (NQD) normal quadrupoles are aligned on a circle, a periodic orbit and stable optics can be found as seen in Fig. 3 (a). The beam moves relatively outside at NQF and inside at NQD, though remains outside of the quadrupole axis in both. Even with the same gradient but opposite sign at NQF and NQD, stronger bending action at NQF because of the larger distance from the axis makes the net bending per unit focusing period finite and the orbit closes. A scaling FFA uses this same principle, although the magnet has a field gradient which is a nonlinear function of the distance from the machine centre - unlike a quadrupole magnet.

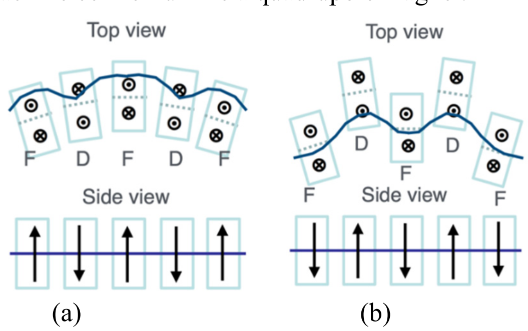

Figure 3: (a) without radial shift of NQD and NQF, (b) with radial shift. Arrow indicates the vertical magnetic field. Without radial shift, a closed orbit is found outside of the quadrupole axis. With radial shift, closed orbit exists inside the quadrupole axis and horizontal bend is flipped at NQD and NQF.

When a normal quadrupole is replaced by a skew quadrupole, a closed orbit can be still found but an oscillation is induced in the vertical plane instead of the horizontal. That is because a focusing skew quadrupole (SQF) acts as a normal bending above the magnet axis and a defocusing skew quadrupole (SQD) acts as a normal bending below the magnet axis, as shown in Fig. 4 (a).

Now let us discuss the momentum compaction factor. The momentum compaction factor α_p is defined as

$$dC/C = \alpha_p dp/p$$

where C is the orbit length, p is the beam momentum and α_p is the momentum compaction factor. A small momentum compaction factor makes the orbit length less dependent on the beam momentum.

One way to reduce the momentum compaction factor in a lattice with normal quadruples is to shift NQF and NQD differently in the radial direction, as shown in Fig. 3 (b). Without radial shift, a higher momentum particle finds its closed orbit at a larger radius. The orbit length becomes longer because the horizontal oscillation pattern remains similar but the average radius increases. With the radial shift, on the other hand, a higher momentum particle tends to go outward and finds a bending field decrease both at NQF and NQD. The increase of the orbit length by expanding outward is cancelled by flattening orbit oscillations. As a result, the momentum compaction factor is small or can even be zero. That is how a non-scaling FFA can minimise the orbit excursion. More detailed analysis of realising small momentum compaction factor based on the momentum dispersion function H can be found in [4].

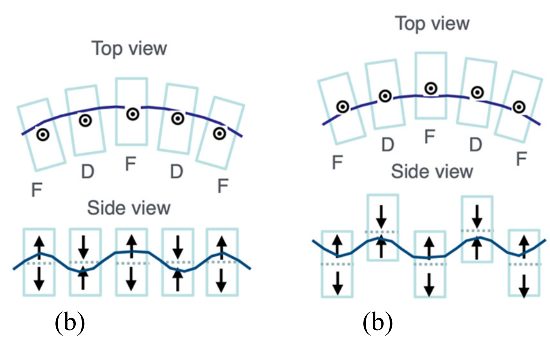

Figure 4: (a) without vertical shift of SQD and SQF, (b) with vertical shift. Arrow indicates the vertical magnetic field. Without vertical shift, closed orbit is found outside of the quadrupole axis. With vertical shift, closed orbit exists inside the quadrupole axis and vertical bend is flipped at SQD and SQF.

Control of the momentum compaction factor can be also applied in a skew quadrupole lattice. By shifting SQF and SQD in the vertical direction, the vertical bending directions at SQF and SQD are swapped as shown in Fig. 4 (b). As in the normal quadrupole lattice, a higher momentum particle expands the average radius but flattens the vertical oscillation because the particle sees less horizontal field when the average radius increases. As a result, the momentum compaction factor can be small or zero if necessary.

One noticeable difference between lattices with normal quadrupoles and skew quadrupoles is the reverse bending action. In a lattice with normal quadrupoles, small or zero momentum compaction factor lattice needs a reverse bend which increases the total circumference. In a lattice with skew quadrupoles, control of the momentum compaction

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factor is managed by bending in the vertical direction and hence there is no impact on the ring circumference.

Example of Muon Collider Arc

Figure 5 shows the muon collider arc cells with normal and skew quadrupoles when we set the momentum compaction factor to zero. In the normal FODO cell, NQD and NQF are displaced horizontally by +/- 40 mm. In the skew FODO cell, SQD and SQF are displaced vertically by +/- 40 mm.

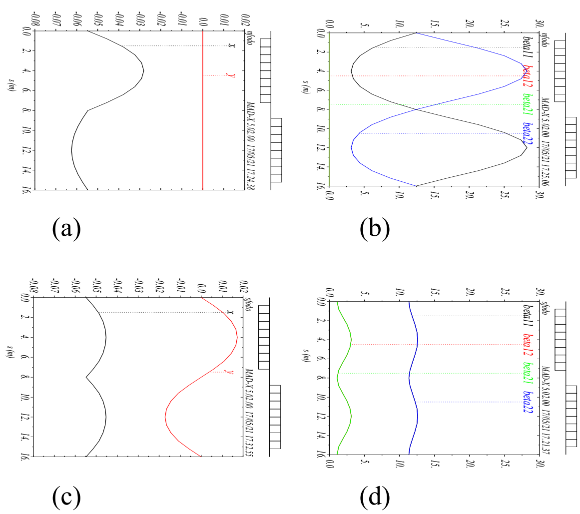

Figure 5: (a) orbit of normal FODO, (b) optics of normal FODO, (c) orbit of skew FODO, (d) optics of skew FODO. A kink of x (horizontal) orbit is due to the rotation of the coordinate system for QD and QF. Second half of x orbit in (a) shows the reverse bend whereas both magnets have normal bend in (c). Vertical orbit oscillation of around +/- 20 mm is induced in the skew FODO.

Correction by Sextupole

For the correction of higher order terms of the momentum compaction factor and chromaticity, skew sextupole components are included in the magnet (Fig. 6).

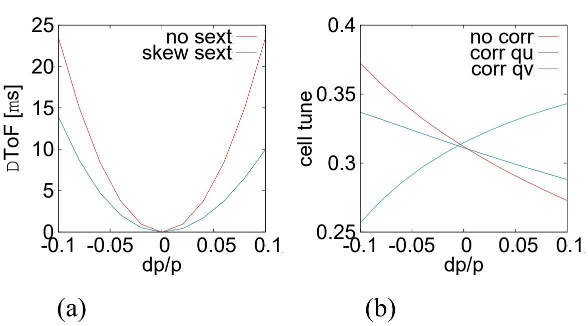

Figure 6: (a) time of flight per cell, (b) red is eigen tune without skew sextupole, green and blue are eigen tune after skew sextuple correction.

Wiggling Orbit

Muons are continuously decaying in the collider ring, meaning that neutrino radiation is a big issue. When the orbit is within a plane, the radiation stays in a plane as well, with a divergence of the beam less than 1 mrad. The wiggling orbit of a skew FODO arc spreads out the radiation in the vertical direction. The vertical angle in this particular example is around +/- 8 mrad, which is one order of magnitude larger than the beam divergence. The amplitude of vertical orbit oscillations is controlled by optics parameters (Table 2).

Table 2: Comparison of Normal and Skew FODO Lattice

	Normal FODO	Skew FODO
Circum.	6080 m	6080 m
Cell	16 m	16 m
# of cell	380	380
Tune	0.3131	0.3131
a_p	0	0
Magnet	2 x 6.4 m	2 x 6.4 m
Field	20 T	14 T
Grad.	240 T/m	240 T/m

CONCLUSION

vFFA for muon acceleration and a novel skew FODO lattice for a muon collider arc have been considered. The vFFA has big advantages in this application due to removing the need for ramping magnetic field and modulating RF frequency. On the other hand, reduction of reverse bending and squeezing the orbit excursion require future study to realise the vFFA's full potential.

A collider arc comprising skew quadrupole magnets with vertical shift can control the momentum compaction factor without introducing reverse bending. The wiggling orbit in the vertical direction helps to spread out neutrino radiation from the muon decay. Nonlinear elements such as a skew sextupole correct higher order momentum compaction factor and chromaticity. This renders the skew FODO lattice a promising candidate for future muon collider rings. Although low-beta insertion has not yet been designed, a simple 45-degree rotation of the existing design of low-beta insertion in the normal quadrupole scheme should be a starting point.

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