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MODELLING SEEDED SELF MODULATION OF LONG ELLIPTICAL BUNCHES IN PLASMA

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Abstract

The stability of particle bunches undergoing seeded self-modulation (SSM) over tens or hundreds of meters is crucial to the generation of GV/m wakefields that can accelerate electron beams as proposed for use in several ultra-high energy plasma-based linear colliders. Here, 3D particle-in-cell simulations using *QuickPIC* are compared to an analytical model of seeded self-modulation (SSM) of elliptical beam envelopes using linear wakefield theory. It is found that there is quantitative agreement between simulations and analytical predictions for the envelope in the early growth of the SSM. A scaling law is derived for the reduction of the maximum overall modulation growth rate with aspect ratio and is found to match well with simulation.

INTRODUCTION

Proton-driven plasma wakefield acceleration (PDPWFA), has been proposed to overcome the problem of energy depletion of drivers in previous experiments, with the view of application towards a new generation of plasma-wakefield-based colliders for high energy physics research. However, current high-energy-content bunches, such as those of the Super Proton Synchrotron (SPS) used in AWAKE are too long by two orders of magnitude to efficiently drive a wakefield in plasma of suitable density . Therefore, the concept relies on the self-modulation of the long proton bunch in plasma due to an initial weak 'seed' wakefield driven by the unmodulated bunch which causes the bunch to compress and diverge at periodic intervals along its length. The resulting train of shorter micro-bunches, if formed so that they are positioned correctly within the wakefield [1], can then resonantly excite much stronger accelerating gradients in the plasma to accelerate a witness beam [2].

Seeding the self-modulation process requires an initial wakefield with a sufficiently strong longitudinal component at the plasma wavelength, $\lambda_p = 2\pi c\sqrt{\frac{m_e\epsilon_0}{n_p e^2}}$, where n_p is the plasma density, ϵ_0 is the permittivity of free space, c is the speed of light, and m_e and e are the electron mass and charge, respectively. This may be achieved by a smaller preceding bunch as proposed in AWAKE Run 2 [3] or by ionising the plasma with a co-propagating laser pulse placed at the midpoint of the Gaussian proton beam to create a discontinuity in beam longitudinal profile as seen by the plasma [4]. Such seeding is required to control the initial phase of the modulation process along the longitudinal beam profile, to ensure an efficient resultant microbunch arrangement upon saturation of the SSM growth [5].

It has been shown by numerical investigations in previous works that the seeded self-modulation (SSM) process may be sensitive to beam parameters such as emittance and radial spot size [6]. However, such works have almost consistently considered only transversely round bunches. Previously, it was shown that even slightly unequal aspect ratio of the driving beam leads to strong asymmetric profiles of the resultant microbunches [7] which is reflected in the transverse profiles of the resultant wakefields [8]. Here we present preliminary comparisons between an analytic model and 3D particle-in-cell simulations of the effect of unequal aspect ratio on macroscopic aspects of the instability growth.

ANALYTICAL MODEL

Following the approach of [9], we begin with the observation that the azimuthal Fourier cosine components, Ψ_m, of the linear plasma wakefield potential, $\Psi(r,\phi,\xi) = \sum_{m=0}^{\infty}\Psi_m(r,\xi)\cos m\phi$, depend on the number density profile $n_b(r,\phi,\xi) = \sum_{m=0}^{\infty} n_{b,m}(r,\xi)\cos m\phi$ of an arbitrarily shaped (but y-symmetric) bunch that generates them, as

$$\left(\partial_\xi^2 + k_p^2\right)\left(\partial_r^2 + \frac{\partial_r}{r} - \frac{m^2}{r^2} - k_p^2\right)\Psi_m = \frac{n_m}{n_0}\,, \quad (1)$$

where $\xi = s - ct$ is a co-moving coordinate along the length of the beam, moving along s at c over time t. In the following, we consider a density profile of a ξ-dependent transverse factor scaled to a peak density n_{b0}, $n_b(r,\phi,\xi) = n_{b0} f(r,\phi,\xi)$. For simplicity, we consider a round transverse beam profile $f_0(r)$ that has been 'squashed' with an additive perturbation as $f = f_0(r) + f_2(r)\cos(2\phi)$. We choose f_2 judiciously such that the total slice charge is independent of $\iint_{\text{slice}} f_2(r)\cos(2\phi)\mathrm{d}^2r$ over each transverse slice. We then find that for a general profile $f_0(r)$, $f(r,\phi,\xi)$ may be written in terms of the sum of the instantaneous transverse mean square sizes of the slice in x and y, $R = \sigma_x^2 + \sigma_y^2$ (with initial ξ-independent value R_0), and their difference $S = \sigma_x^2 - \sigma_y^2$, as

$$f(r,\phi,\xi) = \frac{R_0}{R(\xi)} f_0(r) - \frac{S(\xi)}{2R(\xi)} r\frac{df_0}{dr}\cos(2\phi)\,. \quad (2)$$

To make calculations tractable, we choose $f_0(r) = \mathscr{H}(\sqrt{2R} - r)$, giving

$$f(r,\phi) = \frac{R_0}{R}\mathscr{H}(\sqrt{2R} - r) + \frac{S}{2R} r\delta(r - \sqrt{2R})\cos(2\phi)\,, \quad (3)$$

where $\mathscr{H}(r)$ and $\delta(r)$ are the Heaviside and Dirac delta functions, respectively.

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For an ultra-relativistic beam where space-charge effects may be neglected, a convenient choice for describing the evolution of a beam slice is the well-known space-charge-free Kapchinsky-Vladimirsky (KV) envelope equation in σ_x and σ_y,

$$\sigma_x'' + \frac{P_x(\sigma_x, \sigma_y)}{\sigma_x} - \frac{\epsilon_x^2}{\sigma_x^3} = 0 \quad (4)$$

$$\sigma_y'' + \frac{P_y(\sigma_x, \sigma_y)}{\sigma_y} - \frac{\epsilon_y^2}{\sigma_y^3} = 0, \quad (5)$$

where primes denote derivatives with respect to propagation direction, s. The numerators in the second (forcing) terms are given by

$$\gamma_b m_b N_0 \cdot \begin{pmatrix} P_x(\sigma_x, \sigma_y) \\ P_y(\sigma_x, \sigma_y) \end{pmatrix} = q \int_{\text{slice}} \begin{pmatrix} x\partial_x \\ y\partial_y \end{pmatrix} (\Psi) f(r, \phi) \mathrm{d}^2 r \quad (6)$$

where m_b, q and γ_b are the mass, charge ($= e$ for a proton beam) and Lorentz factor of the beam particles, respectively, and $N_0 = \iint_{\text{slice}} f(r, \phi) \mathrm{d}^2 r$ for a given transverse slice. The x and y rms emittances, ϵ_x and ϵ_y, are defined as $\epsilon_x = \sigma_x^2 \sigma_{x'}^2 - ((\sigma_x^2)'/2)^2$ in terms of the beam rms x-angular spread $\sigma_{x'}$, and equivalently for ϵ_y. Our derivation of Eq. (6), based on that in [10], further assumes that slice profile evolves self-similarly in x and y.

The ξ-dependence in the solution Ψ of Eq. (1) enters through the well-known linear plasma wakefield Green's function operator [11], $\hat{G}[\ ] = \int_{\infty}^{\xi} \sin k_p(\xi - \xi')[\ ] d\xi'$. This allows P_x to be linearized in $\sigma_x(\xi')$ and $\sigma_x(\xi)$ with perturbations $\sigma_{x1}(\xi')$ and $\sigma_{x1}(\xi)$ about an initial value, σ_{x0}, giving, to linear order,

$$\begin{aligned} P_x = & P_{x0}(\sigma_{x0}, \sigma_{y0}) \\ & + P_{x1}^{(\xi)}(\sigma_{x1}(\xi), \sigma_{y1}(\xi); \sigma_{x0}, \sigma_{y0}) \\ & + P_{x1}^{(\xi')}(\sigma_{x1}(\xi'), \sigma_{y1}(\xi'); \sigma_{x0}, \sigma_{y0}) \\ & + O(\sigma_{x1}^2, \sigma_{y1}^2), \end{aligned} \quad (7)$$

and an equivalent expression for P_y. Finally, choosing σ_{x0} and σ_{y0} to be the long-beam emittance-matched values as by Schroeder et al. [12], simplifying similarly (assuming a slowly varying component of the envelope, $\hat{\sigma} : \sigma_1 = \hat{\sigma} \exp(ik_p\xi)/2 + \text{c.c.}$, with growth timescale $\gg$ plasma period and $\ll$ betatron period), in normalised units of $\hat{\xi} = k_p \xi$ and $\hat{s} = k_\beta s$,

$$\left(\partial_{\hat{\xi}} \partial_{\hat{s}}^2 + \frac{i}{2k_p^2 k_\beta^2} \underline{Q_1} \right) \hat{\sigma} = 0, \quad (8)$$

where the matrix $\underline{Q_1}$ is such that

$$\underline{Q_1} \begin{pmatrix} \sigma_{x1}(\xi) \\ \sigma_{y1}(\xi) \end{pmatrix} = -\left(\partial_\xi^2 + k_p^2 \right) \begin{pmatrix} P_{x1}^{(\xi')}/\sigma_{x0} \\ P_{y1}^{(\xi')}/\sigma_{y0} \end{pmatrix}, \quad (9)$$

$k_p = 2\pi/\lambda_p$, and $k_\beta^2 = m_e n_{b0} k_p^2 / 2 m_b n_p \gamma_b$. Since this is an equivalent differential operator to that in [12], growth rates and directions (in σ_x-σ_y-space) implied in Eq. (8) can be found by the eigendecomposition of $\underline{Q_1}$ and using the solution obtained for the corresponding equation for symmetrical SSM in [12]. This leads to a perturbation to the Schroeder growth rate parameter for symmetrical SSM, $\nu \to \nu_0 + \nu_2$, such that

$$\nu_0 = 4 I_2(\hat{r}_0) K_2(\hat{r}_0) \quad [12] \quad (10)$$

and

$$\frac{\nu_2}{\nu_0} \approx -\frac{1}{2} \left(\frac{S_0}{R_0} \right)^2 = -\frac{1}{2} \left(\frac{1 - h^2}{1 + h^2} \right)^2 \quad \text{for } \hat{r}_0 \lesssim 1, \quad (11)$$

where $\hat{r}_0 = k_p \sqrt{2R_0}$ and $h = \sigma_y / \sigma_x$ is the beam aspect ratio.

SIMULATIONS

Simulation Setup

To test the model, simulations were carried out using the 3D quasi-static particle-in-cell (PIC) code *QuickPIC* [13]. We use a uniform plasma density, $n_p = 7 \times 10^{-14}\,\text{cm}^{-3}$, corresponding to a plasma skin-depth of $c/\omega_p = \lambda_p / 2\pi = 200\,\mu\text{m}$, on a grid of $512 \times 512 \times 4096$ cells, spanning a volume of $12 \times 12 \times 130\,(c/\omega_p)^3$ in x, y, and ξ with 4 particles per cell. The long proton bunches were initialized with parameters similar to the SPS bunches arriving at AWAKE, with Lorentz factor $\gamma_b = 427$ and equal x and y rms emittances of 3.5 mm mrad, but neglecting the 0.035% momentum spread. The self-modulation seed was achieved by using a sharp longitudinal density step upto the maximum density in the beam profile at $\xi = 0$, placed $2\,c/_p$ from the front edge of the simulation window.

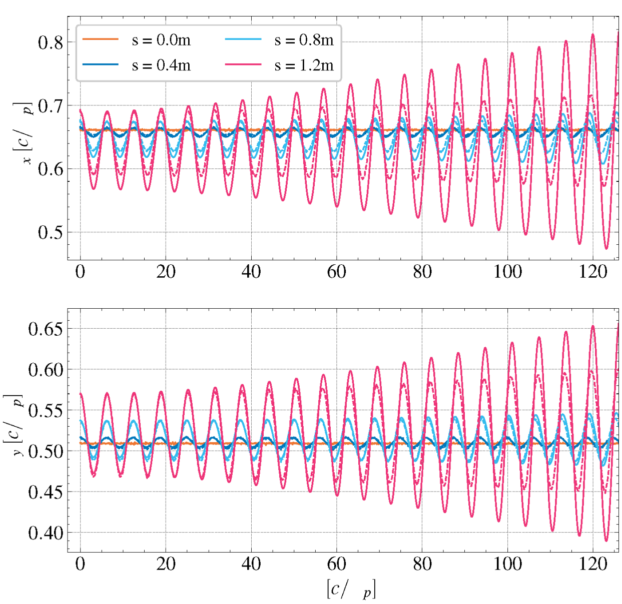

Figure 1: Comparison of analytic (solid lines) and simulation (dashed lines) envelope modulations for a beam with $S_0/R_0 = 0.2$ (aspect ratio $h = 0.8$) at four propagation distances of the beam.

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Envelope Modulation

The validity of the envelope model considered in the previous section was tested against a PIC simulation of a beam with aspect ratio $h = 0.8$ as in Fig. 1. Here, bunch density profiles were chosen to be flat-top in ξ and bi-gaussian in x and y with a peak density of $1 \times 10^{-2} n_p$ with $R_0 = 0.7$. The analytical envelope was obtained by using the numerical solution of Eqs. (4) and (5) with forcing terms P_x and P_y calculated using the Heaviside profile as given in Eq. (6), using a trapezium-rule integral over ξ to obtain $\hat{G}$ and a Runge-Kutta 4 scheme for time integration. It was found that while there is good agreement during the early stage of the modulation, the solutions at the tail of the beam overestimate the simulated envelopes as the SSM of the bunch in the simulated region approaches saturation. It is likely that this is primarily due to the hollowing-out of the defocusing parts of the beam, which is not captured in the Heaviside profile of the model.

Scaling Law For Growth Rate

As found in [12], the growth of pertubration to the ξ-envelope of the radius and the amplitude of the wakefield potential can be reasonably well approximated using a exponential-like functions of the growth rate ν:

$$\Psi \propto N^{-\frac{1}{2}} \exp N \tag{12}$$

where $N = A(\xi)\nu^{1/3} s^{2/3}$ where $A(\xi)$ is factor that depends on the beam and plasma parameters. Hence, in the limit of $s \to \infty$, the growth rate can be estimated upto a mutiplicative factor from the variation of the wakefield amplitude at position of fixed phase in ξ as the beam propagates, using $A(\xi_0(s))\nu^{1/3} \approx \partial_{s^{2/3}} \log \Psi_{0(s)}(s^{2/3})$. Figure 2 shows a comparison of the growth rate predicted for aspect ratios between $h = 1.0$ and $h = 0.5$ under the constraint $R_0 = 1.0$ by Eq. (11) vs. that of the PIC simulations. We find good agreement in the decrease in growth rate due to unequal aspect ratio.

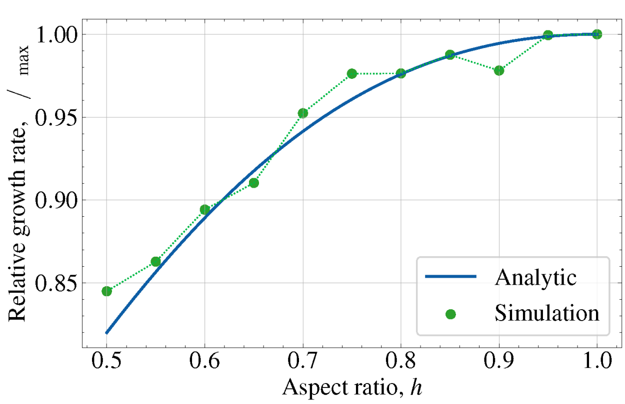

Figure 2: Comparison of variation of analytic and simulated values for the peak growth rates of the maximum wakefield potential with beam aspect ratio up to SSM saturation, relative to that for a symmetric beam ($h = 1.0$).

CONCLUSION

A comparison between predictions of an envelope-based theoretical model of the seeded-self-modulation of an elliptical beam and PIC simulations have been presented. While there is initial agreement between the predicted beam envelope and simulation, the model overestimates the growth as SSM approaches saturation. On the other hand, it is found that the model makes predictions that are in good agreement with simulation for the relative reduction in the maximum SSM growth rate due to variation in beam aspect ratio. Improvements to the model to reduce the divergence between predicted and simulated beam envelopes, as well as further investigations of the validity of the model for highly perturbed beam parameters will be addressed in an upcoming work.

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REFERENCES

[1] K. V. Lotov, "Physics of beam self-modulation in plasma wakefield accelerators", *Physics of Plasmas*, vol. 22, no. 10, p. 103110, 2015. doi:10.1063/1.4933129

[2] AWAKE Collaboration, E. Adli *et al.*, "Acceleration of electrons in the plasma wakefield of a proton bunch", *Nature*, vol. 561, pp. 363-367, 2018. doi:10.1038/s41586-018-0485-4

[3] AWAKE Collaboration, P. Muggli *et al.*, "Physics to plan AWAKE Run 2", *J.Phys.Conf.Ser.*, vol. 1596, p. 012008, 2020. doi:10.1088/1742-6596/1596/1/012008

[4] E. Gschwendtner *et al.*, "AWAKE, The Advanced Proton Driven Plasma Wakefield Acceleration Experiment at CERN", *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 829, pp. 76–82, 2016. doi:10.1016/j.nima.2016.02.026

[5] F. Batsch *et al.* (AWAKE Collaboration), "Transition between Instability and Seeded Self-Modulation of a Relativistic Particle Bunch in Plasma", *Phys. Rev. Lett.*, vol. 126, no. 16, p. 164802, 2021. doi:10.1103/PhysRevLett.126.164802

[6] K. V. Lotov *et al.*, "Parameter sensitivity of plasma wakefields driven by self-modulating proton beams", *Physics of Plasmas*, vol. 21, no. 8, p. 083107, 2014. doi:10.1063/1.4892183

[7] A. Perera *et al.*, "Stability of elliptical self-modulating long proton bunches in plasma wakefields", *J.Phys.Conf.Ser.*, vol. 1596, p. 012011, 2020. doi:10.1088/1742-6596/1596/1/012011

[8] A. Perera *et al.*, "Seeded Self-Modulation of Transversely Asymmetric Long Proton Beams in Plasma", in *Proc. 10th Int. Particle Accelerator Conf. (IPAC'19)*, Melbourne, Australia, May 2019, pp. 3757–3760. doi:10.18429/JACoW-IPAC2019-THPGW072

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[9] C. B. Schroeder *et al.*, "Coupled beam hose and self-modulation instabilities in overdense plasma", *Phys. Rev. E*, vol. 86, no. 2, p. 026402, 2012. doi:10.1103/PhysRevE.86.026402

[10] E. P. Lee and R. K. Cooper, "General Envelope Equation for Cylindrically Symmetric Charged-Particle Beams", *Particle Accelerators*, vol. 7, pp. 83-95, 1976.

[11] P. Chen, "A possible final focusing mechanism for linear colliders", Particle Accelerators, vol. 20, pp. 171–182, 1987.

[12] C. B. Schroeder *et al.*, "Growth and Phase Velocity of Self-Modulated Beam-Driven Plasma Waves", *Phys. Rev. Lett.*, vol. 107, no. 14, p. 145002, 2011. doi:10.1103/PhysRevLett.107.145002

[13] C. Huang *et al.*, "QUICKPIC: A highly efficient particle-in-cell code for modeling wakefield acceleration in plasmas", *Journal of Computational Physics*, vol. 217, no. 2, pp. 658–679, 2006. doi:10.1016/j.jcp.2006.01.039