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END-TO-END RMS ENVELOPE MODEL OF THE ISAC-I LINAC*

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Abstract

A full end-to-end simulation of the ISAC-I linear accelerator has been built in the first order envelope code TRANSOPTR. This enables the fast tracking of rms sizes and correlations for a 6-dimensional hyperellipsoidal beam distribution defined around a Frenet-Serret reference particle frame, for which the equations guiding envelope evolution are numerically solved through a model of the machine's electromagnetic potentials. Further, the adopted formalism enables the direct integration of energy gain via time-dependent accelerating potentials, without resorting to transit-time factors.

INTRODUCTION

Frequent re-tuning of the TRIUMF-ISAC linear accelerator [1] is necessary to service the various high energy experiments scheduled during machine operation. As a consequence of the separated function nature of the machine, tunes provided to operators find their origin in a variety of simulation codes, including PARMTEQ, PARMELA, Trace3D/TraceWin in addition to TRANSOPTR [2], which has been in continuous use and development at TRIUMF since 1984 [3]. In this proceeding, we present the full simulation of accelerated rms envelopes in the recently implemented ISAC-I linear accelerator in TRANSOPTR. This provides us with a single code simulation capability which includes native treatment of energy gain from accelerating potentials. First, an overview of the method of tracking root mean square (rms) beam envelopes using the Courant-Snyder hamiltonian for a relativistic, charged particle experiencing time dependent fields, is presented.

TRANSOPTR AND HAMILTONIAN BEAM DYNAMICS

We start with the s-dependent Courant-Snyder hamiltonian for relativistic, charged particles of mass m and charge q, travelling through electromagnetic fields themselves arising from potentials (ϕ,**A**), with the curvature parameter set to ∞:

$$H_s = -qA_s - \sqrt{\left(\frac{E-q\phi}{c}\right)^2 - m^2c^2 - (P_x - qA_x)^2 - (P_y - qA_y)^2}, \quad (1)$$

where the components of the vector potential are decomposed into its Cartesian components. Note that the scalar potential directly modifies the local definition of energy in the potential, to first order.

RMS Envelope Tracking

From the works of Brown [4] and Sacherer [5], we adapt to the σ-matrix formalism. Starting with a vector representation of the coordinates of the Frenet-Serret reference particle:

$$\mathbf{X} = (x, Px, y, Py, z, Pz)^T, \quad (2)$$

for an ensemble of N particles in an ellipsoidal beam distribution, the beam matrix provides the sizes and correlations across each canonical coordinate:

$$\boldsymbol{\sigma} = \frac{1}{N}\sum_{i=1}^{N} \mathbf{X}\mathbf{X}^T. \quad (3)$$

Sacherer's work demonstrated that the rms size of a distribution can also satisfy the Kapchinsky Vladimirsky equation and that these in fact depend on the linear components of the forces. Thus, the Hamiltonain of Eq. (1) is expanded to second order in each of the canonical coordinates:

$$H_s = H_0 + \sum_i \frac{\partial H}{\partial x_i}\Big|_0 x_i + \frac{1}{2}\sum_{i,j} \frac{\partial^2 H}{\partial x_i \partial x_j}\Big|_0 x_i x_j + \ldots \quad (4)$$

Since Hamilton's equations involve the evaluation of first order derivatives, Eq. (4) will produce solutions detailing the first order component of the motion needed to track the rms size of σ. Hamilton's equations may be expressed neatly as:

$$\frac{d\mathbf{X}}{ds} = \mathbf{F}(\mathbf{s})\mathbf{X}, \quad (5)$$

where the second order partial derivatives of H_s are stored in the **infinitesimal transfer matrix**:

$$\mathbf{F}(s) = \begin{pmatrix} \frac{\partial^2 H}{\partial P_x \partial x} & \frac{\partial^2 H}{\partial P_x^2} & \cdots & \frac{\partial^2 H}{\partial P_x \partial P_z} \\ -\frac{\partial^2 H}{\partial x^2} & -\frac{\partial^2 H}{\partial x \partial P_x} & \cdots & -\frac{\partial^2 H}{\partial x \partial P_z} \\ \frac{\partial^2 H}{\partial P_y \partial x} & \frac{\partial^2 H}{\partial P_y \partial P_x} & \cdots & \frac{\partial^2 H}{\partial P_y \partial P_z} \\ -\frac{\partial^2 H}{\partial y \partial x} & -\frac{\partial^2 H}{\partial y \partial P_x} & \cdots & -\frac{\partial^2 H}{\partial y \partial P_z} \\ \frac{\partial^2 H}{\partial P_z \partial x} & \frac{\partial^2 H}{\partial P_z \partial P_x} & \cdots & \frac{\partial^2 H}{\partial P_z^2} \\ -\frac{\partial^2 H}{\partial z \partial x} & -\frac{\partial^2 H}{\partial z \partial P_x} & \cdots & -\frac{\partial^2 H}{\partial z \partial P_z} \end{pmatrix}. \quad (6)$$

The matrix **F(s)** contains 36 terms, 21 of which are independent, due to the symmetry of mixed partial derivatives. Evaluating the s-derivative of the matrix σ, we obtain the **envelope equation**, relating its s-evolution to the potentials in the Hamiltonian of Eq. (1):

$$\frac{d\sigma}{ds} = \mathbf{F}(s)\sigma + \sigma\mathbf{F}(s)^T. \quad (7)$$

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TRANSOPTR solves the 21 independent coupled partial differential equations represented by Eq. (7), in addition to tracking energy and time:

$$\frac{dE_0}{ds} = \frac{\partial H}{\partial t} \tag{8}$$

$$\frac{dt_0}{ds} = -\frac{\partial H}{\partial E} = \frac{E_0}{P_0} = \frac{1}{\beta_0 c}. \tag{9}$$

For suitably defined potentials (ϕ,A) and a matrix **F(s)**, a beam optical element can be incorporated into the code.

Energy Gain From a Potential

To first order in TRANSOPTR, in the presence of a scalar potential Φ [6]:

$$c^2 P \Delta P = (E - q\Phi)\Delta E. \tag{10}$$

Energy gain in is directly integrated from the potential, in the case of a field $\mathscr{E}(s)$ scaled by a factor V_s:

$$E(s) = E_0 + qV_s \int_0^l \mathscr{E}(s)\cos(\omega t(s) + \phi_0)ds. \tag{11}$$

This allows for a straightforward treatment of acceleration, without resorting to transit time factors. All that is needed is information on the potential distribution itself. This avoids use of gap-field parametrizations: the field can simply be simulated in any Poisson solver, from the gap geometry. As shown below, it also enables the straightforward treatment of radiofrequency quadrupole (RFQ) envelopes, necessary for ISAC-I linac simulations.

RFQ F-Matrix

The two first terms of the Kapchinsky-Teplyakov potential for an RFQ are used to derive the **F**-matrix of an RF quadrupole arising from a specified vane geometry (s, A_{01}, A_{10}, k), shown at the top of Fig. 1 for the ISAC-RFQ [7]:

$$\mathbf{F} = \begin{pmatrix} 0 & \frac{1}{P} & 0 & 0 & 0 & 0 \\ -A_+ & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{P} & 0 & 0 \\ 0 & 0 & -A_- & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & B & \frac{1}{\gamma^2 P} \\ 0 & 0 & 0 & 0 & -C & -B \end{pmatrix} \tag{12}$$

$$A_\pm = \frac{qV_0 \sin(\omega t_0+\phi)(k^2 A_{10}\cos\psi \pm 4A_{01})}{4\beta c},$$

$$B = \frac{qV_0 A_{10}(k\sin\psi\sin(\omega t_0+\phi)+(\omega/(\beta c))\cos\psi\cos(\omega t_0+\phi))}{2\beta^2\gamma^3 mc^2},$$

$$C = \frac{qV_0(\omega/(\beta c))^2 A_{10}\cos\psi(qV_0A_{10})/(\beta^2\gamma^3 mc^2)}{4\beta c} \times \frac{(\cos\psi\cos^2(\omega t_0+\phi) - 2\sin(\omega t_0+\phi))}{4\beta c}.$$

and the shorthand $S = \sin(\omega t_0 + \theta)$ and $C = \cos(\omega t_0 + \theta)$ has been used. The spatial cosine term uses a function $\psi = \int k(s)ds$. The ISAC-RFQ is now in TRANSOPTR [7].

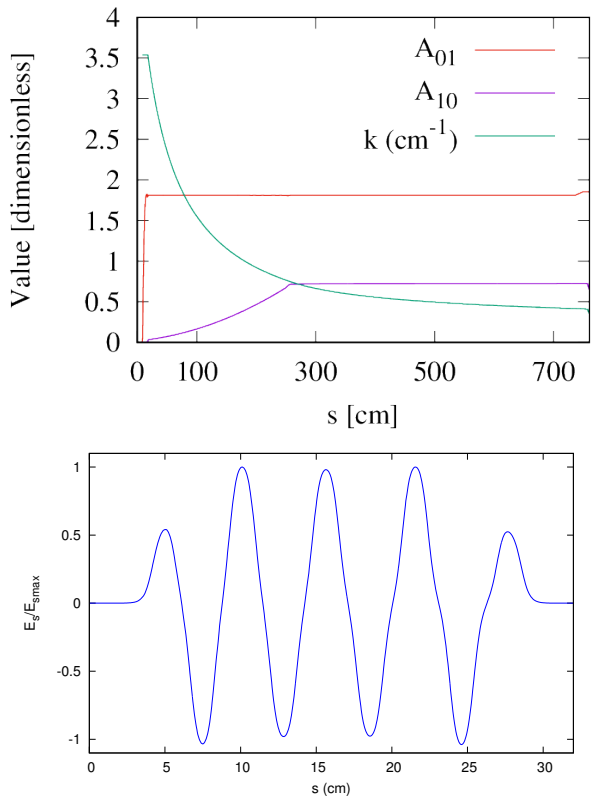

Figure 1: ISAC-RFQ vane modulation parameters (top) and example Opera-2D computed electric field $\mathscr{E}(s)$ for DTL Tank-1.

Axially Symmetric Linac F-Matrix

Originally implemented for use with the TRIUMF electron linear accelerator (elinac) [6], the following F-matrix relies upon a smooth function $\mathscr{E}(s)$:

$$\mathbf{F}(s) = \begin{pmatrix} 0 & \frac{1}{P_0} & 0 & 0 & 0 & 0 \\ \mathscr{A}(s) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{P_0} & 0 & 0 \\ 0 & 0 & \mathscr{A}(s) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\beta'}{\beta} & \frac{1}{\gamma^2 P_0} \\ 0 & 0 & 0 & 0 & \mathscr{B}(s) & -\frac{\beta'}{\beta} \end{pmatrix} \tag{13}$$

with:

$$\mathscr{A}(s) = -\frac{q}{2\beta c}\left(\mathscr{E}'(s)C - \mathscr{E}(s)S\frac{\omega\beta}{c}\right)$$

$$\mathscr{B}(s) = \frac{q\mathscr{E}(s)\omega S}{\beta^2 c^2}.$$

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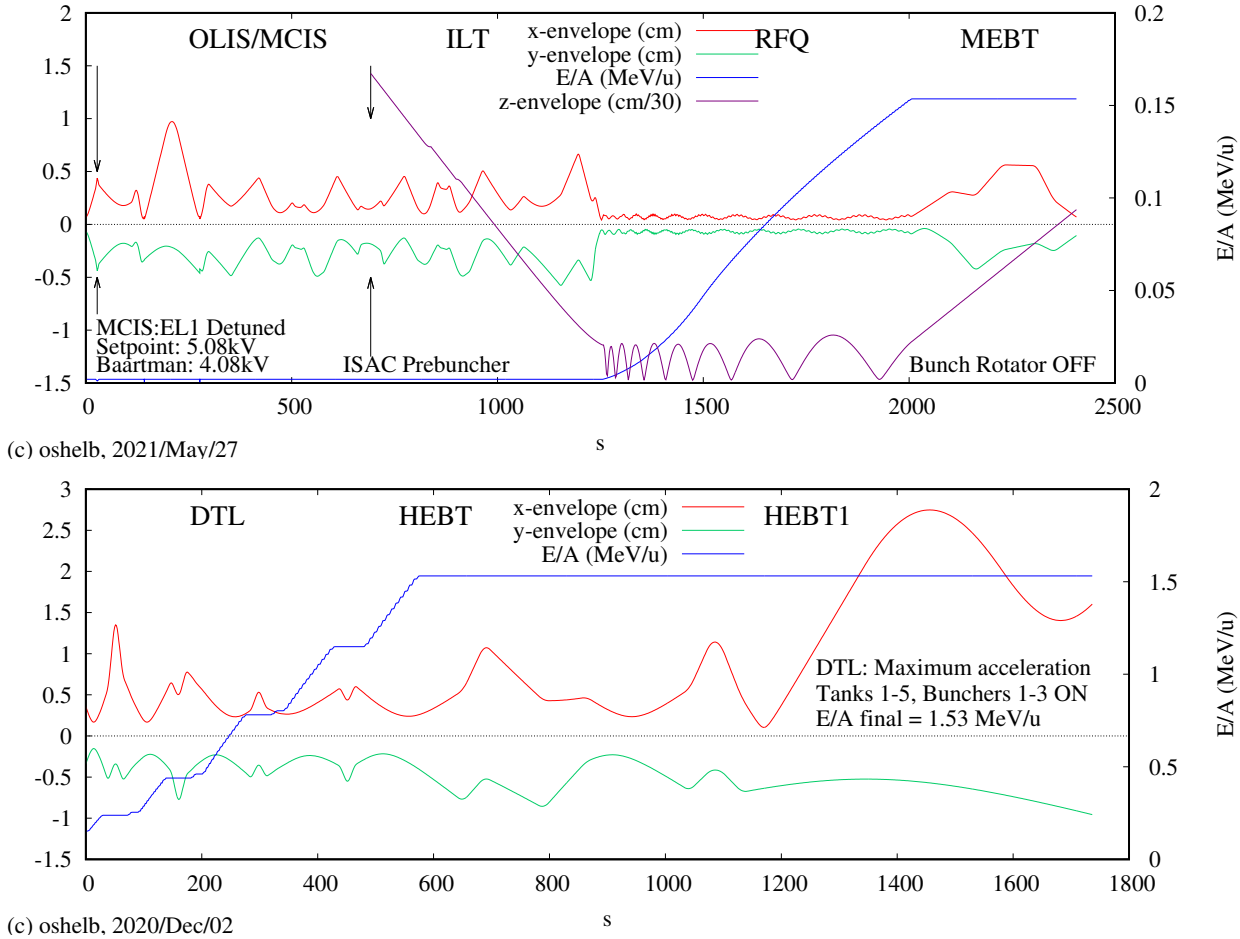

Figure 2: (**Top**) TRANSOPTR 2 rms envelopes for (x,y,z) dimensions of a 6D ellipsoidal beam centered around a reference particle undergoing acceleration in the ISAC-I linac. Simulation broken into two segments, starting from offline ion source (OLIS), through the ILT low energy section, ISAC-I RFQ and terminating at the medium energy section (MEBT). (**Bottom**) 2 rms envelopes for the same beam, undergoing full acceleration through the ISAC-DTL, after exiting the ISAC-RFQ.

The above parametrization is convenient as it calls upon $\mathscr{E}(s)$ & $\mathscr{E}'(s)$, which are the normalized magnitude of the on-axis electric field and its first derivative, with respect to the Frenet-Serret arclength coordinate s.

For the ISAC-DTL, engineering drawings were used to build a simplified model of the drift tubes in the Poisson solver Opera-2D, which allowed for the computation of $\mathscr{E}(s)$ for each cavity, with Tank-1 shown in Fig. 1, bottom. These maps were calibrated with beam to provide a relationship between the control system tank voltage parameter and the physical on-axis magnitude of the electric field in the cavities [8]. In addition, quadrupole effective lengths for the triplets located along the lattice of the machine were obtained from initial field surveys performed during machine commissioning. The full accelerator can be simulated in TRANSOPTR and is shown in Fig. 2, for a $^{20}Ne^{4+}$ beam.

CONCLUSION

End to end rms envelope tracking in the ISAC accelerator, using TRANSOPTR, provides a unified tool for machine and tune investigations. Since TRANSOPTR tracks a single particle, it typically enjoys subsecond execution time for full envelope computations. Ongoing development aims to use this model as the physics engine for model-coupled accelerator tuning, which has the potential to significantly reduce tuning time, by providing operators with realtime simulations and optimizations of the machine. This method further allows for a more elegant treatment of linac energy changes, doing away with transit time factors. We further note that, following the works of DeJong [9], based on that of Sacherer and Lapostolle [10], this envelope model is fully capable of a native first order space charge computation, as discussed in [6, 7].

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