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STUDY OF NONLINEAR PROPERTIES OF ESR VIA TUNE SCANS

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Abstract

The ESR storage ring at GSI is a key accelerator for the FAIR phase zero. This phase requires several highly specialized beam manipulations, which range from beam storage to deceleration of several ion species with the ultimate goal to provide intense highly charge ions to CRYRING. This plan will bring the ESR storage ring into a unique unexplored regime of accelerator operations where nonlinear dynamics, IBS, cooling, and high intensity will all become strongly interdependent. It is, therefore, necessary to acquire the best knowledge of the machine starting from its nonlinear dynamics properties. In this work, we present the development of a strategy to be used in the ESR, in which tune scans are used to explore the nonlinear properties of the accelerator. This approach is discussed with the help of simulations.

INTRODUCTION

The delivery of high beam quality in storage rings [1] and colliders [2] relies on the control of the accelerator optics. However, the presence in the magnets of uncontrolled nonlinear fields may affect significantly the dynamics and disturbs the beam operations: unwanted resonances and reduction of dynamic aperture are the consequences in synchrotrons, but in storage rings, nonlinear errors may affect the closed orbit deformation (COD) in particular for large off-momentum particles. This may disturb the mode of operations, which requires the accelerator optics to sit at the transition energy to reach an effective isochronous mode [3]. The optimization of a real lattice is pursued by making the best modeling of the linear and nonlinear accelerator optics. This is reached by making use of several approaches, which range from orbit response matrix methods [4], resonance driving terms approaches [5], nonlinear chromaticity [6], to nonlinear tune response [7]. Recently the use of machine learning techniques to reconstruct machine gradient errors is also explored [8]. In the following, we present the analysis of an alternative approach to evaluate the effect of nonlinear errors on the tunes in a storage ring of large momentum acceptance.

CLOSED ORBIT DEFORMATIONS

It is known from the early work of Ernest Courant what is the effect of linear errors in the linear dynamics of a particle. The major effect on a particle with small oscillation amplitude is a small deviation of the unperturbed tune, which is computed as

$$\Delta Q_x = \frac{1}{4\pi}\int_0^C \beta_x(s)k(s)ds$$

with $k(s)$ is a tiny gradient error at location s and $\beta_x(s)$ is the beta function at s. For large oscillations amplitudes, the presence of nonlinear magnets in the ring will create an amplitude-dependent detuning. The change of tune for a particle oscillating near the ideal reference closed orbit is negligible, however, if the closed orbit is not laying on the reference orbit, magnet feed-down will produce a local gradient, which effectively can be treated as a linear error. The same effect will appear if the source of nonlinear errors has the magnetic center displaced from the closed orbit. The concept of artificially deforming the closed orbit to induce a controlled feed-down is at the base of measuring tunes to retrieve the machine nonlinear errors [9]. This method produces a measurable detuning effect for sextupolar errors when creating a significant closed orbit distortion.

Suppose we now consider the situation of a machine with a large closed orbit deformation due to the combined effect of dispersion and particle off-momentum $\delta p/p_0$. The presence of nonlinear elements will create a deviation from the ideal tune, which, however, in this discussion can be considered as the "unperturbed tune", which we refer to as $Q_{x,0}$. We next perturb the closed orbit by slightly changing the dispersion as a result of artificially perturbing some quadrupoles. This procedure will produce a relevant effect on machines with a large dispersion: mainly there will be a contribution on the tune coming from the perturbation of the quadrupoles and a contribution originated from the change of the feed-down because of the change of the dispersion. With an analysis of the different contributions, it may be possible to disentangle the two effects.

The difference between the unperturbed closed orbit x_o and perturbed one x_k is described by the equation of $\delta x_o = x_k - x_o$, which reads

$$\begin{aligned}
&\delta x_o'' + \frac{k_x}{1+\frac{\delta p}{p_0}}(1+\delta k)\delta x_o + \frac{k_x}{1+\frac{\delta p}{p_0}}\delta k\, x_o = \\
&\mathcal{N}_{x,x}\delta x_o + \mathcal{N}_{x,y}\delta y_o + O_{\delta x_o,\delta y_o}(2), \qquad (1)\\
&\delta y_o'' + \frac{k_y}{1+\frac{\delta p}{p_0}}(1+\delta k)\delta y_o + \frac{k_y}{1+\frac{\delta p}{p_0}}\delta k\, y_o = \\
&\mathcal{N}_{y,x}\delta x_o + \mathcal{N}_{y,y}\delta y_o + O_{\delta x_o,\delta y_o}(2), \qquad (2)
\end{aligned}$$

where $\delta k(s)$ is the perturbation imposed on the quadrupole structure. The solution of this equation imposing the machine periodicity yields δx_o, δy_o. For convenience we defined

$$\begin{aligned}
\mathcal{N}_{x,x}(s) &= \frac{\partial}{\partial x}\mathcal{N}_x(x,y,s)|_{x_o(s),y_o(s)}\\
\mathcal{N}_{x,y}(s) &= \frac{\partial}{\partial y}\mathcal{N}_x(x,y,s)|_{x_o(s),y_o(s)}\\
\mathcal{N}_{y,x}(s) &= \frac{\partial}{\partial x}\mathcal{N}_y(x,y,s)|_{x_o(s),y_o(s)}\\
\mathcal{N}_{y,y}(s) &= \frac{\partial}{\partial y}\mathcal{N}_y(x,y,s)|_{x_o(s),y_o(s)}
\end{aligned} \qquad (3)$$

being $\mathcal{N}_y(x,y,s)$, $\mathcal{N}_y(x,y,s)$ the two functions which sum all the contributions of the magnets nonlinear components.

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Neglecting the high order terms, these equations become

$$\delta x_o'' + \frac{k_x}{1+\frac{\delta p}{p_0}}(1+\delta k)\delta x_o = -\frac{k_x}{1+\frac{\delta p}{p_0}}\delta k x_o + +O_{\delta x_o,\delta y_o}(2), \quad (4)$$

$$\delta y_o'' + \frac{k_y}{1+\frac{\delta p}{p_0}}(1+\delta k)\delta y_o = 0 + O_{\delta x_o,\delta y_o}(2). \quad (5)$$

Assuming the vertical closed orbit be corrected (i.e. going through the magnetic center of the accelerator elements) we have only the horizontal effect to discuss. As usual, we have $x_o(s) = D_{x,0}(s)\frac{\delta p}{p_0}$, and the perturbation of the quadrupoles will produce a new dispersion $D_{x,k}(s)$, which yields $x_k(s) = D_{x,k}(s)\delta p/p_0$. For convenience, we describe the new dispersion in terms of the initial one $D_{x,k}(s) = D_{x,0}(s) + \delta D_x(s)$, being $\delta D_x(s)$ the deviation with respect to the unperturbed dispersion. Then $\delta x_o(s) = \delta D_x(s)\frac{\delta p}{p_0}$ and therefore Eq. (4) reads

$$\delta D_x'' + \frac{k_x}{1+\frac{\delta p}{p_0}}(1+\delta k)\delta D_x = -\frac{k_x}{1+\frac{\delta p}{p_0}}\delta k D_{x,0}. \quad (6)$$

As $\delta k(s)$ is constant in each quadrupole the periodic solution of this equation is

$$\delta D_x(s) = \sum_{m=1}^{N_{quad}} \Lambda_{dk,m}(s)\Delta k_m \quad (7)$$

with N_{quad} the number of quadrupoles and

$$\Lambda_{dk,m}(s) = -\frac{\sqrt{\beta_{x,dp,dk}(s)}}{2\sin(\pi Q_{x,dp,dk})} \times$$
$$\times \int_s^{s+L} \frac{k_x}{1+\frac{\delta p}{p_0}}(t) D_{x,0}(t)\sqrt{\beta_{x,dp,dk}(t)}\Theta_m(t) \times$$
$$\times \cos(\psi_{x,dp,dk}(t) - \psi_{x,dp,dk}(s) - \pi Q_{x,dp,dk})dt. \quad (8)$$

being Δk_m the relative change of the m-th quadrupole strength and N_{quad} the number of quadrupoles and $\Theta_m(s)$ a function being 1 for s located within the m-th quadrupole and 0 otherwise. Here $\beta_{x,dp,dk}(s)$ is the beta function for the perturbed optics including the on-momentum, and $Q_{x,dp,dk}$ is the corresponding tune. The phase advance is defined as usual as $\psi_{x,dp,dk}(s) = \int_0^s 1/\beta_{x,dp,dk}(s')ds'$. Figure 1 shows the functions $\Lambda_m(s)$ for the case of a lattice without nonlinear terms. In each panel, there are two curves, one from the theory (black) and a green curve from the tracking. The quadrupoles which they refer to is indicated in each picture.

VARIATION OF TUNES

We consider here the tune of a particle oscillating with a small amplitude around the closed orbit. This tune is the result of the integration of all the feed down of all nonlinear fields. Let's suppose the total number of sextupolar errors is

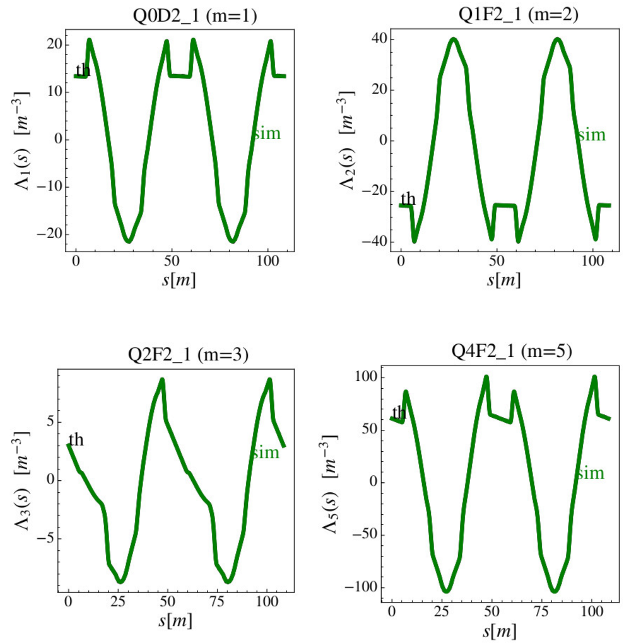

Figure 1: Function $\Lambda_m(s)$ at any longitudinal positions in the storage ring. We here compare the prediction of the analytic prediction with the prediction from the simulation.

N_{k2} and let us call $\tilde{Q}_{x,o}$ the tune of a particle on the perturbed closed orbit. Then the difference of the new tune with respect to the unperturbed one is

$$\tilde{Q}_{x,o} - Q_{x,o} = \sum_{m=1}^{N_{quad}} Q_m\Delta k_m +$$
$$+ \sum_{m=1}^{N_{quad}}\sum_{i=1}^{N_{k2}} Q_{i,m}\frac{\delta p}{p_0}K_{2i}\Delta k_m + O_{\delta p}(2)[1+\delta k],$$

with the coefficients Q_m, $Q_{i,m}$ defined as

$$Q_m = \frac{1}{4\pi}\int_0^L \beta_{x,dp}(s)k_x(s)\Theta_m(s)ds,$$
$$Q_{i,m} = \frac{1}{4\pi}\beta_{x,dp}(s_i)\tilde{\Lambda}_{dk,m}(s_i).$$

The function $\tilde{\Lambda}_{dk,m}$ is defined by

$$\beta_{x,dp,dk}(s)[x_o(s) + \delta x_o(s)] - \beta_{x,dp}(s)x_o(s) =$$
$$= \beta_{x,dp}(s)\sum_{m=1}^{N_{quad}}\tilde{\Lambda}_{dk,m}(s)\Delta k_m\frac{\delta p}{p_0}, \quad (9)$$

which is an extension of Eq. (8). Note that all these coefficients can be computed directly from the optical functions, i.e. by a computer model of the linear optics. The quantity $\tilde{Q}_{x,o} - Q_{x,o}$ is an experimental observable that directly depends on the nonlinear errors $K_{2,i}$, but that is also disturbed from the unwanted term Q_m.

NUMERICAL TESTS

Next, we apply the approach we have discussed in the previous section to explore the limits and strength of the method

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on a simplified model of the ESR storage ring lattice. This test is carried out assuming that the chromaticity is small and the results are shown in Fig. 2. The title of each picture indicates which sextupolar error (index i) is activated, and the x-axis indicates which quadrupole is perturbed (index m), to which relative strength Δk_m. We assigned on each dipole 3 sources of nonlinear errors; at entrance S_B1_##, in the body S_B_##, at the exit S_B2_##. Any of these components on the bends is activated for these numerical tests. The top pictures of Fig. 2 show a comparison of the detuning created by a single sextupolar error placed on the body of the bending magnets. In particular, the top pictures show the tune response for two different allocated errors ($i = 1, 4$) when two different quadrupoles are perturbed ($m = 1, 2$). In both cases, the integrated strength of the error is $K_2 = 0.005\ \mathrm{m}^{-2}$. The red curves are obtained from the theoretical model and the black curves are the result of the following procedure applied to a tracking code: the closed orbit for a particle with $\delta p/p_0 = 1.5 \times 10^{-2}$ is numerically found. Then by a small displacement of the test particle from the closed orbit, the tune $Q_{x,0}$ is computed with the turn-by-turn coordinates using an enhanced FFT analysis wihch applies special filters [10]. Next, the m-th quadrupole is perturbed of the amount Δk_m, and the same procedure is applied to compute the tune $\tilde{Q}_{x,0}$. Therefore we obtain the sequence $(\delta p/p_0, \Delta k_m, Q_{x,0}, \tilde{Q}_{x,0})$ for several Δk_m. By repeating this procedure inverting the sign of $\delta p/p_0$ we obtain another sequence $(-\delta p/p_0, \Delta k_m, Q_{x,0}, \tilde{Q}_{x,0})$ which subtracted from the previous allows omputing the contribution of the sextupolar error δQ shown in every picture of Fig. 2. The top two panels show a comparison of δQ as computed via tracking (black) and as through the theory (red). The results show an acceptable agreement. The pictures in the middle show the same type of comparison but for the same dipole magnet, and compare the ability of the method to be predictive for different strengths: on the left picture $K_2 = 1 \times 10^{-3}\ \mathrm{m}^{-2}$, and on the right picture it is $K_2 = 5 \times 10^{-3}\ \mathrm{m}^{-2}$. The two pictures at the bottom of Fig. 2 show instead the effect of the nonlinear error in the computation of the closed orbit. The perturbation of the dispersion is computed with the assumption that the nonlinear objects in the ring do not play a significant role in controlling COD, which then obeys Eqs. (8) and (9). If the COD is too deformed, the nonlinearities will play a significant role, and any subsequent COD deformation-induced via quadrupole perturbation will include a nonlinear effect and nonlinear deviations will appear in the prediction of the linearized theory. For the example shown here, this effect seems not dramatic.

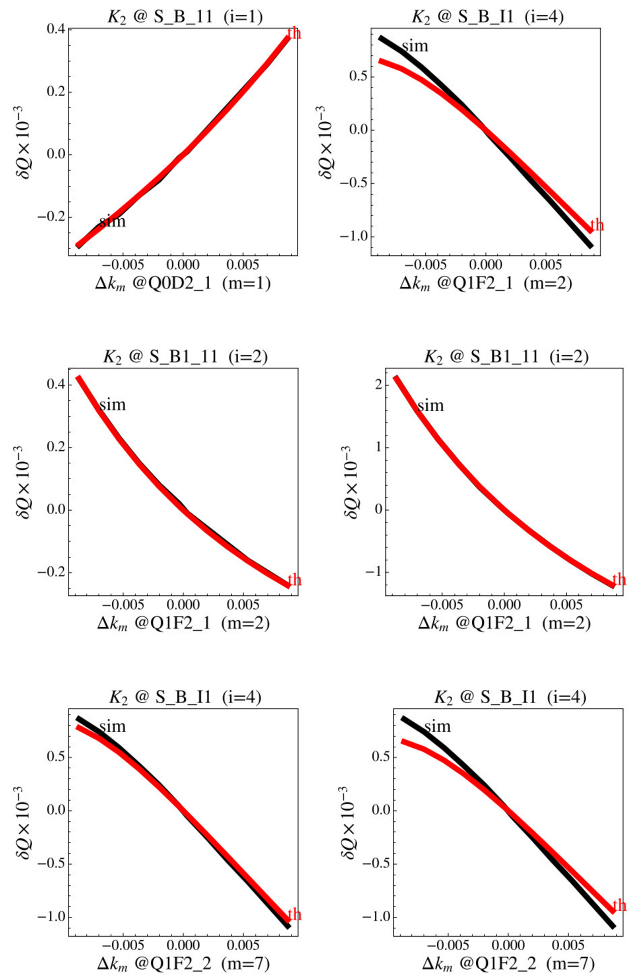

Figure 2: The top pictures show the comparison of the tune-shift produced by a sextupolar error as obtained from simulations (black) with those from the theory (red). Each of the two curves refers to a perturbation of a different quadrupole for a differently excited sextupolar error. The middle pictures test the predictivity of the method for a sextupolar error having two different strengths ($K_2 = 0.001\ \mathrm{m}^{-2}$ left, and $K_2 = 0.005\ \mathrm{m}^{-2}$ right). The bottom pictures assess the effect of the nonlinear error in determining the COD on the prediction of the tune-shift: the left panel include the effect of the nonlinear error, the right panel not.

OUTLOOK

The numerical tests carried out with small chromaticity indicate that the effect generated by sextupolar errors can be detected as a change of tune induced by the dispersion. Although the procedure of deforming COD through quadrupole perturbation is unusual, these results suggest that this method may be a useful tool for complement other types of optics investigation strategies. Further validation of this method under the influence of a significant chromaticity and the experimental testing is part of future work.

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