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EXAMINATION OF SEMI-ANALYTIC MODEL FOR MODE COUPLING INSTABILITIES*

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Abstract

A semianalytic model for studying beams at high SC tune shift is shown. It is a generalization of SWM [1] /ABS [2] for an arbitrary number of longitudinal phase space cycles, yielding more realistic longitudinal physics. The consequences of this generalization are explored; model is benchmarked against TRANFT [3] and analytical methods.

INTRODUCTION

At a strong enough space charge tune shift $\Delta Q_{sc} \gg Q_s$ we enter the space charge dominated regime. At this intensity regime, one cannot reliably treat space charge effects perturbatively. To analytically study this regime, one must find a way to directly include space charge forces. This can be accomplished by macroparticle tracking codes but this approach can become computationally intractable. Furthermore, tracking codes have a limited temporal resolution as well as artifacts that make some results of such simulations unclear or misleading [4].

SWM/ABS

The so called Square Well Model [1] or Airbag Square Well [2] is one of the few semi-analytic models for this system and is able to treat an arbitrary ΔQ_{sc} and wake functions. The model assumes that the beam is confined within a single square potential well, which provides longitudinal confinement for the beam. The beam consists of two lines of current, propagating from one edge of the square well to the other that comprise a single loop of current. At the edges of the square potential well the lines of current must satisfy continuity conditions. The governing equations for this model are

$$\frac{dx_+}{d\tau} = -\frac{i}{\nu_0}[x_+(\Delta Q_x - \xi\nu_0) + F + (x_+ - x_-)\Delta Q_{sc}/2] \quad (1)$$

$$\frac{dx_-}{d\tau} = \frac{i}{\nu_0}[x_-(\Delta Q_x - \xi\nu_0) + F + (x_- - x_+)\Delta Q_{sc}/2] \quad (2)$$

$$\frac{dF}{d\tau} = -\alpha F + (x_+ + x_-)\kappa W_0/2, \quad (3)$$

where x_+ and x_- are the slowly varying portion of the spatial positions (The complete spatial position $X_\pm = e^{-i\theta}x_\pm$, where θ is the position along the accelerator) for the lines of current along the bunch, while F is the wake force.

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Although physically interesting, this model has always been somewhat limited in application. A simple square well is an unrealistic representation of the longitudinal potential and creates a nonphysical phase space, resulting in imprecise predictions. A lack of Landau damping is identified as another source of error [2]. High frequency modes are comparatively easier to drive, which underestimate high frequency wake thresholds.

MULTI LOOP SQUARE WELL

It is possible to generalize upon SWM/ABS to a set of longitudinal step potential wells and their corresponding phase space cycles. We will refer to this method as the Multi Loop Square Well (MLSW). The cycles form 'loops' of current with similar continuity properties to single loop method, while square potential wells form subdomains with coupled linear ODEs characterizing transverse dynamics. A diagram of the phase space for this problem is given in Fig. 1. Modeling the longitudinal potential as a superposition of multiple square wells allows an arbitrarily complex longitudinal phase space to be formed. Distinct synchroton tunes can be selected for each current loop, but that is beyond the scope of this proceeding.

Multiloop Square Well Phase Space

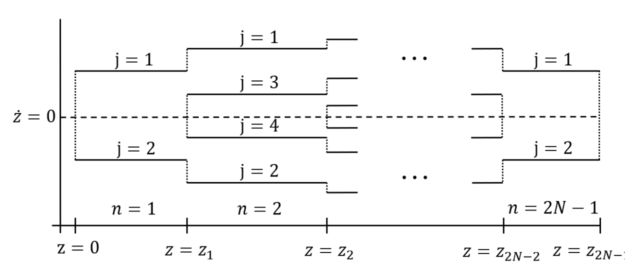

Figure 1: Diagram of arbitrary longitudinal phase space. n corresponds to subdomain and j to half loop of current.

For the generalization of this problem the equations of motion transverse dynamics are

$$g_1(z,\dot{z}) = -Q_x^2 + \xi(\dot{z}) + C_{sc}\Sigma_{j=1}^{2m}\rho_j^n \quad (4)$$

$$g_2(t,z) = -C_{sc}\Sigma_{j=1}^{2m}\rho_j^n\bar{\tilde{x}}_j^n + \Sigma_{k=1}^{\kappa}\hat{F}_k^n \quad (5)$$

$$\frac{d\bar{\tilde{x}}_j^n}{dz} = \frac{i\omega_0}{2Q_x\dot{z}_j^n}[(2Q_x\Delta Q_x\bar{\tilde{x}}_j^n + Q_x^2\bar{\tilde{x}}_j^n + g_1(z,\dot{z}))\bar{\tilde{x}}_j^n + g_2(t,z)] \quad (6)$$

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$$\frac{d\hat{F}_k^n}{dz} = \gamma_k \Sigma_{j=1}^{2m} \rho_j^n \bar{\tilde{x}}_j^n - \alpha_k \hat{F}_k^n, \quad (7)$$

where C_{sc} is the coherent space charge strength, and $C_{sc}\Sigma_{j=1}^{2m}\rho_j^n = 2\Delta Q_{sc}^n/Q_x$. In order to preserve linearity, only the coherent portion of the space charge is retained. Single bunch continuity conditions in between subdomains are satisfied by physical tune shift ΔQ_x values.

$$\bar{\tilde{x}}_j^n = \bar{\tilde{x}}_j^{n+1}, \hat{F}_k^n = \hat{F}_k^{n+1}. \quad (8)$$

The tune shift directly corresponds to the existence of an instability if $Im[\Delta Q_x] < 0$. It should be noted that due to longitudinal discretization, there is a discontinuity in the first derivative between any individual subdomains n and $n+1$.

The coupling between filaments of current is due to space charge and wakes effects. In the absence of these forces, the loops of current propagate independently.

Now as these are analytical models the MLSW and SWM/ABS do not employ macroparticles; each loop contains a continuum of particles traversing it. This feature allows the system to simulate an arbitrarily high space charge tune shift value. Additionally, while the model may have limitations to it, solutions are accurate to the numerical precision of the computer.

Wake and coherent modes are also modified with multiple loops. The solution for the single square well limit yields simple standing waves [2]. When this condition is broken by added complexity, higher order modes become more difficult to drive. This leads the drop of in instability strength at high frequency in Fig. 2.

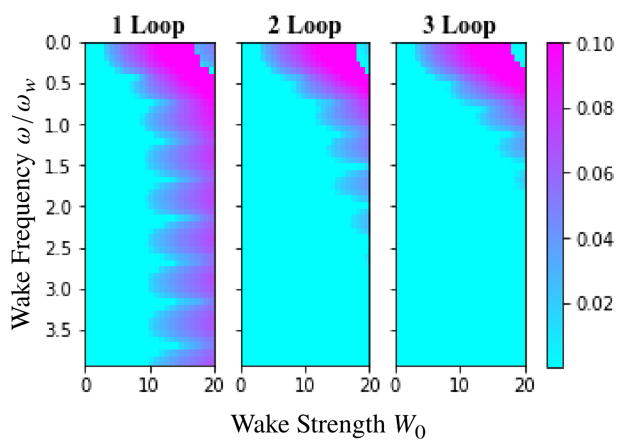

Figure 2: Instability strengths as a function of dimensionless wake strength and frequency. System is modelling an even distribution of longitudinal COM energies centered around $E_{z,COM} = 0$ and a z^2 potential. Higher order modes are more difficult to drive with multiple current loops, satisfying foundational assumptions.

Tune shifts become more complex in the multiloop case. When space charge and wakes are included, these loops of current couple between each other and each valid tune shift solution becomes separable into a set of N individual modes, where N is the number of unique loops of current. One can imagine the case in which the number of loops approach infinity, yielding a tune spread instead of a countable number of discrete tune solutions.

SELF CONSISTENCY WITH LIMITED PHYSICS SIMULATIONS

With the newly defined model it becomes important to ensure agreement with other similar models. A simulation derived from TRANFT [3] can by compared with two current loop case for MLSW generalization. This TRANFT based simulation is a macroparticle code and we expect it to have an equivalent result save for numerical effects from finite numbers of macroparticles. With sufficient macroparticles one obtains exceedingly similar solutions for equivalent inputs.

It is simple enough to jump from the two loop case to a larger number loops which can then be compared against the full TRANFT code using a Gaussian longitudinal distribution with linear RF. In this case we do not expect the result to be as similar as the two well case. The generalized MLSW has improved longitudinal resolution with the multiple loops of current, but there is still an inherent error associated with discretizing the Gaussian into discrete loops. In Fig. 3 we note that the 4 loop MLSW solution is very similar to the equivalent TRANFT solution. However, we do expect MLSW to converge to the TRANFT solution if sufficient number of wells and loops are used to approximate the Gaussian distribution.

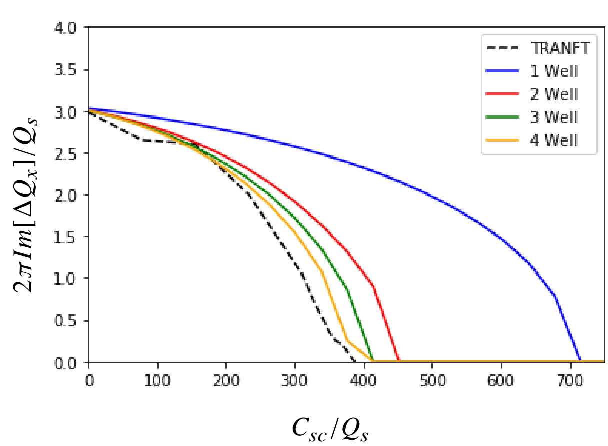

Figure 3: Instability strength of most unstable Eigenmode of a Gaussian distribution for TRANFT and MLSW under a step wake two times instability threshold. Space charge is strengthened to eliminate instability. Adding square wells causes the solution to approach TRANFT value indicating possible convergence. Note for the TRANFT case, C_{sc} is defined in terms of form factor $f = 3\sqrt{\pi/2}$.

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COMPARISON WITH RIGID-BEAM EQUATIONS

Rigid-Beam Equations [5] can be used to examine the efficacy of the MLSW. This model has been shown to agree with the single loop case [1], but is also solvable for specific cases such as a Gaussian beam profile. Since the MLSW relies on several of the same assumptions (like linear space charge) it can be compared to Rigid-Beam Equations and observe similar results. In practice it becomes difficult to compare the two; the N loop MLSW has N eigenvalues for every $+nQ_s$ mode present in the Rigid-Beam version.

However this is still of interest to us, Rigid Beam Equations can be brought to a much larger space charge than TRANFT, providing novel comparisons of the strong space charge regime. A 2 loop MLSW such as that found in Fig. 4 can be compared to the Guassian case for the Rigid-Beam Equations. With this it is possible to compare eigenvalues without too many extra modes.

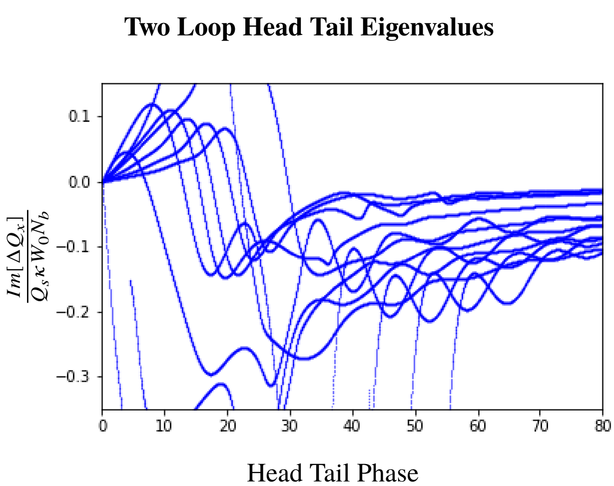

Figure 4: Rescaled imaginary components of two loop MLSW eigenvalues. These have a similar form [5] Fig. 3. $N > 1$ loop MLSW has more eigenvalues than Rigid-Beam making this sort of analysis more difficult in a many loop case.

CONCLUSION

This method acts as a tool to further studies into mode coupling instabilities. The generalizations allow the system to expand its purview to additional classes of problems, including higher cavity harmonics and systems with significant longitudinal tune spread.

There are several distinct directions for future work which can be more fully explored by this model. Examining the coherent modes of loops with differing synchrotron tunes has not yet been investigated, but should allow the modeling of nonlinear cavity focusing. Study of beam stability for beams with a long injection time such as the EIC [6] should also be of interest. Since the model is exactly solvable, it may also be of utility when evaluating tracking codes in the relevant limits. One application of particular interest to the authors is the study of of Burov's convective instability [2].

Some planned future work will need additional improvement of the model. These include propagating wakes in between bunches in a well filled storage ring. The current formalism can only solve the trivial zero mode. Applying approximate Landau damping [6] may be possible given sufficient computation power.

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