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TRANSIENT BEAM LOADING IN THE CBETA MULTI-TURN ERL

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Abstract

The Cornell-BNL ERL Test Accelerator (CBETA) is the first superconducting multi-turn ERL that has been commissioned at Cornell University in a low current mode. In this paper, we first discuss a new model of beam loading which is valid for the low injection energies used in CBETA. Using this model, we explore the effect of bunch patterns, beam turn-on, and turn-off transients on the fundamental mode of the 7-cell SRF cavities used in the main linac. In particular, we examine the operational constraints on the rf system at the design current of 40 mA.

INTRODUCTION

The Cornell-BNL ERL Test Accelerator (CBETA) [1] is a multi-turn Energy Recovery Linac (ERL) which uses Superconducting Radio Frequency (SRF) cavities for acceleration and a permanent magnet non-scaling Fixed Field Alternating (FFA) [2] return arc to recirculate the electron beam. The main linac used for energy recovery contains six 7-cell SRF cavities operating with a large loaded quality factor of 6×10^7 to reduce average power consumption. CBETA has been commissioned in a 1-turn configuration to an injection current of 70 µA and in a 4-turn configuration at 1 nA [3]. The commissioned beam current was constrained by the radiation dose absorbed by the permanent magnets arising from beam loss in the machine. During steady state operations, perfect energy recovery implies zero average fundamental mode beam loading in all rf cavities of the main linac which is realized by setting the cavity voltages, rf phases and re-circulation times to optimal values [4]. We achieved this in experiment and measured an energy recovery efficiency of $99.4 \pm 0.1\%$ on average in a 1-turn configuratio [5]. However, this doesn't account for transients which occur during machine turn-on, turn-off, beam loss or due to special bunch patterns especially at high currents. While a fixed shunt impedance description of beam loading is valid for an ultra-relativistic beam, 6 or 12 MeV bunches encountered in the first and last passes of CBETA can slip by ≈ 2.9 degrees with respect to the accelerating field and change the response of the fundamental mode to the passing bunches. We developed a model of beam loading which is a function of incident beam energy and phasing.

In the next section, we outline a new energy dependent model of beam loading which can be used to analyze the effect of transients in CBETA. Next we use the model to quantify power requirements and field stability during steady operations, startup and beam loss. Finally we summarize our results and discuss future work in the last section.

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BEAM LOADING THEORY

The field inside a rf cavity can be decomposed into a set of orthogonal eigenmodes which may be excited by rf waves guided in through an input coupler or by the passage of a particle beam through the cavity. The total vector potential inside the cavity can hence be written as $\vec{A}(\vec{r},t) = \sum_k a_k(t)\vec{\mathcal{A}}_k(\vec{r})$, where the resonant mode with frequency ω_k satisfies the *Helmholtz* equation $\vec{\nabla}^2\vec{\mathcal{A}}_k + \omega_k^2\vec{\mathcal{A}}_k = 0$ with the boundary conditions $\vec{\mathcal{A}}_k \times \mathrm{d}S = 0$. $a_k(t)$ represent the time-dependent eigenmode amplitudes which control the amount of energy and phase of the eigenmode k. We choose an arbitrary normalization where the eigenmode field $\vec{\mathcal{A}}_k$ contains 1 J of energy i.e. $0.5\epsilon_0\omega_k^2\int\vec{\mathcal{A}}_j\cdot\vec{\mathcal{A}}_k\,\mathrm{d}V = \delta_{jk}$. Assuming that the fundamental mode denoted by the subscript 0 is the dominant eigenmode and discounting the effect of space charge, the kinematics of the particles travelling along the axis of the cavity is governed by,

$$\frac{\mathrm{d}z_p}{\mathrm{d}t} = c\sqrt{1-\frac{1}{\gamma_p^2}}\,, \quad \frac{\mathrm{d}\gamma_p}{\mathrm{d}t} = -\frac{q}{mc^2}\dot{z}_p\vec{\mathcal{A}}_{z_0}(z_p)\dot{a}_0\,, \tag{1}$$

where $\gamma_p mc^2$ is the energy of particle p with position z_p and velocity $\dot{z}_p$.

We can arrive at a linear theory of beam loading assuming that the energy transferred to a single bunch is much smaller than the energy stored in the fundamental mode. In the absence of dissipation and any perturbation, we can define an eigenmode phasor which remains constant in time. The fundamental mode phasor may be defined as,

$$\tilde{\mathcal{F}}_0 \equiv (\dot{a}_0 + i\omega_0 a_0)\exp(-i\omega_0 t)\,, \tag{2}$$

so that $\dot{a}_0 = \mathcal{R}\{\tilde{\mathcal{F}}_0\exp(i\omega_0 t)\}$. Further, the absolute value of the phasor is related to the accelerating voltage V_c as $|\tilde{\mathcal{F}}_0| = V_c\sqrt{\omega_0/(2R/Q)}$. Now we consider a bunch of charge q_b entering the cavity at time t_0 with centroid position $z_b(t_0)$ and velocity $\dot{z}_b(t_0)$. Plugging in $\dot{a}_0(t) = \mathcal{R}\{\tilde{\mathcal{F}}_0(t_0)\exp(i\omega_0 t)\}$ into Eq. (1) gives the 0$^{\text{th}}$ order solution for the bunch position $z_b^{(0)}(t)$ and velocity $\dot{z}_b^{(0)}(t)$. The linear response of the fundamental mode to the passage of a single bunch in the absence of dissipation is given by $\tilde{\mathcal{F}}_0(t_0+T) = \tilde{\mathcal{F}}_0(t_0) + q_b\tilde{\mathcal{G}}_0(T)$ where T is the transit time of the bunch through the cavity and the induced phasor $q_b\tilde{\mathcal{G}}_0(T)$ is given by [6]

$$q_b\tilde{\mathcal{G}}_0(T) = \frac{q_b\omega_0^2}{2}\int_{t_0}^{t_0+T}\vec{\mathcal{A}}_{z_0}(z_b^{(0)}(\tau))\dot{z}_b^{(0)}(\tau)\mathrm{e}^{-i\omega_m\tau}\,\mathrm{d}\tau\,. \tag{3}$$

Hence each bunch passing through the cavity generates an induced phasor whose amplitude and phase depends on the energy and phasing of the incoming bunch.

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An active rf control system is used to stabilize the fundamental mode in response to dissipation, detuning, beam and other perturbations. In a system with $Q_L \gtrsim 10^7$, this is typically achieved using a proportional feedback loop with high gain K_p, which compensates for dissipation and effectively increases damping. In the presence of resonance detuning $\delta\omega << \omega_0$, the phasor evolution follows,

$$\begin{aligned}&\frac{\mathrm{d}\tilde{\mathscr{F}}_0}{\mathrm{d}t} + \left\{\frac{\omega_0}{2Q_L}(1+K_p) - i\delta\omega\right\}\tilde{\mathscr{F}}_0 \\ &= \frac{\omega_0 K_p}{2Q_L}\tilde{\mathscr{F}}_{sp} + q_b[\tilde{\mathscr{G}}_0(T)]_b\delta(t-t_b)\,,\end{aligned} \tag{4}$$

where each bunch induces a change in the fundamental mode phasor given by $q_b[\tilde{\mathscr{G}}_0(T)]_b$ and $\tilde{\mathscr{F}}_{sp}$ is the set point of the feedback loop. The forward power required by the rf system to stabilize the field is,

$$P_+ = \frac{K_p^2 Q_{ext}}{4\omega_0 Q_L^2}|\tilde{\mathscr{F}}_{sp} - \tilde{\mathscr{F}}_0(t)|^2\,. \tag{5}$$

Equations (1), (2), (3), (4) and (5) were solved numerically to analyze transient beam loading in CBETA.

RESULTS

We apply the linear beam loading model to the main linac cavities of CBETA. The relevant parameters of the rf system are shown in Table 1 Three out of the six 7-cell cavities are fitted with stiffening rings to reduce microphonics detuning. The field inside each cavity is controlled by separate digital Low Level Radio Frequency (LLRF) control systems driving solid state power amplifiers with maximum forward power capabilities of 5 kW and 10 kW for stiffened and unstiffened cavities respectively. Each cavity is also equipped with piezoelectric resonance tuners to compensate for transient detuning. While the rf system [7] and CBETA have already been commissioned to low currents (70 μA in 1-turn and 1 nA in 4-turn), future work will increase currents circulated in the machine which requires development of suitable procedures for current ramps, turn off, beam loss and putting limits on acceptable bunch patterns.

During steady state operation at the design current of CBETA, bunches from all 8 linac passes travel through the cavities in quick succession generating 8 induced phasors. Figure 1 shows phasors induced by the recirculating bunches (4 accelerating and 4 decelerating) in the first main linac cavity in two phasing configurations. The amplitudes are scaled to changes in accelerating voltage as $\chi_b \equiv q_b\sqrt{(\omega_0/(2R/Q))}[\tilde{\mathscr{G}}_0(T)]_b$ and the phases are referenced to the time averaged fundamental mode phasor. The 4 overlapping arrows in the first panel pointing to the right denote induced phasors from the decelerating bunches while only 3 out of the four arrows corresponding to the accelerating bunches overlap, with one significantly different due to a large phase slip arising from the lowest incident energy of 6 MeV. The second panel is phased according to the baseline design lattice of CBETA. The resultant phasor $\sum_b \chi_b$ exhibits significant imaginary components, 14.6 V and -17.2 V in the first and second configuration respectively. The average forward power required during steady state operations may be estimated as,

$$\langle P_+ \rangle = \frac{V_c^2 Q_{ext}}{8Q_L^2 R/Q}\left|1 - \frac{i\delta\omega}{\omega_{1/2}} - \frac{I_{inj}\sum_b \chi_b}{q_b V_c \omega_{1/2}}\right|^2, \tag{6}$$

where I_{inj} is the injection current, $\omega_{1/2} \equiv 0.5\omega_0/Q_L$ is the half bandwidth of the fundamental mode. Plugging in the design lattice values for the first main linac cavity and assuming $\delta\omega = 0$, we get an average power requirement of 36.5 kW which is predominantly used by the control loop to compensate for the large reactive beam loading. The amount of detuning required to minimize forward power is,

$$\delta\omega_{opt} = -\frac{I_{inj}}{q_b V_c}\mathscr{I}\left\{\sum_b \chi_b\right\}, \tag{7}$$

which yields a required detuning of 158 Hz. In practice, an automatic resonance control system will be used to detune the cavity in response to reactive beam loading.

Table 1: Operation Parameters of CBETA rf System

Parameter	Value	Unit
Injection Energy	6	MeV
Highest Energy	150	MeV
Linac Passes	8	
Main Linac Energy Gain	36	MeV
Fundamental frequency (f_0)	1300	MHz
Number of srf cavities	6	
Nominal gain per cavity	6	MeV
R/Q (fundamental mode)	387	Ω
Intrinsic quality factor (Q_0)	2×10^{10}	
Loaded quality factor (Q_L)	6×10^7	
Max. rf fwd. power (P_+)	5-10	kW
rf control loop frequency	12.5	MHz
Proportional gain (K_p)	300 (nominal)	
Harmonic number (h)	343	
Bunch charge (q_b)	123 (design)	pC
Laser frequency (f_{laser})	325	MHz

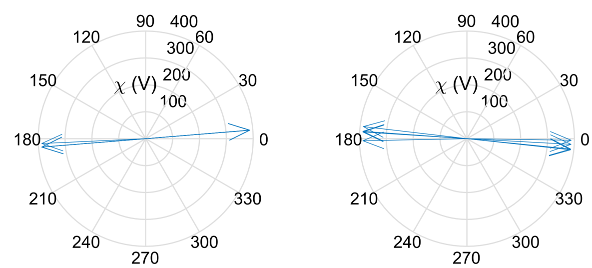

Figure 1: Beam induced fundamental mode phasors in the first main linac cavity with the phases referenced to the cavity set point phasor. While both plots have 8 arrows, multiple arrows overlap in the left plot.

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Steady state operations at CBETA may require special bunch patterns where not all linac buckets contain a bunch or the train may contain large gaps. Panel (a) of Fig. 2 shows a regular bunch pattern which can be used for operations. It is characterized by three timescales, the time between consecutive bunches T_b, the total length of a single continuous bunch train T_train, and the time period after which the patterns repeats T_rep. This bunch pattern recirculates around CBETA resulting in a complex train of bunches with different energies and complex time structure. We simulate this taking into account the length of each re-circulation pass and use the resulting bunch train as a source term in Eq. (4). Panel (b) shows peak forward power requirements (cavity suitably detuned according to Eq. (7)) and field stability of the first main linac cavity in the absence of microphonics detuning as a function of the length of the bunch train given $T_\mathrm{b} = 1/f_\mathrm{laser}$ and $T_\mathrm{rep} = 686T_\mathrm{b}$. The simulations predict low power consumption and best field stability either when few buckets are filled or when all buckets are filled which averages out the perturbations. In general, a bunch train with regularly spaced bunches result in lower peak power consumption and better field stability.

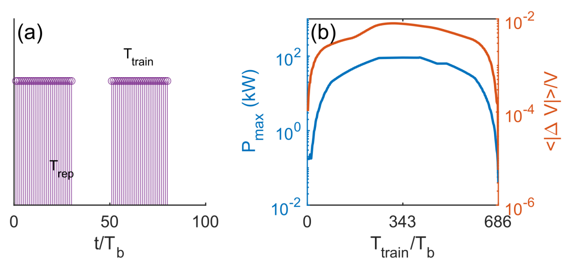

Figure 2: Influence of bunch pattern on field stability and power requirements of the first main linac cavity assuming no microphonics detuning. (a) Typical bunch pattern. (b) Peak power and field amplitude fluctuations as a function of train length.

Current transients also occur during machine startup and beam loss. During machine startup, the main linac has to provide energy to the beam before the high energy bunches can recirculate and deposit their energy back into the cavities establishing equilibrium. To reduce the amount of perturbations due to this energy transfer, the beam current must be ramped. This is achieved by gradually adding bunches to the train i.e increasing T_train while keeping q_b, T_b and T_rep constant. Figure 3 shows simulated fluctuations of the forward power consumed by the first main linac cavity in the absence of microphonics detuning during a current ramp from 0 mA to 40 mA in 10 ms during which the resonance tuner was also moved according to Eq. (7). The addition of each bunch perturbs the mode phasor and generates spikes in forward power which are still well within the maximum power capabilities (> 5 kW) of the solid state rf sources, leaving enough headroom to compensate for microphonics detuning. However detuning the cavity from 0 Hz to 158 Hz without exciting mechanical resonances would limit the maximum speed of the ramp to a few seconds. Conversely, in the event of beam loss, the bunches still recirculating in the machine can deposit all their energy into the SRF cavities and potentially cause a quench. Nevertheless this won't happen in CBETA due to the comparatively small stored beam energy and the full current can be lost safely as long as injection is turned off within a few micro-seconds of the loss event.

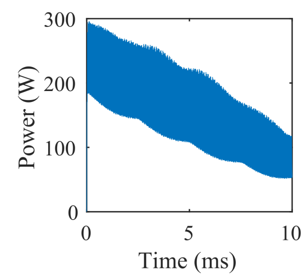

Figure 3: Time dependence of power in the first main linac cavity while ramping the injector current from 0 mA to 40 mA within 10 ms.

CONCLUSION

CBETA is a multi-turn SRF ERL which has been commissioned to 1 nA in a 4-turn configuration and 70 µA in a 1-turn configuration while achieving 99.4 ± 0.1% of energy recovery. We developed an energy dependent model of beam loading which accounts for incident beam energy and phasing and used it to analyze the effect of current transients during future high current operations. The new model predicts large reactive beam loading which needs to be compensated by detuning the cavities in order to minimize forward power requirements. The comparatively large detuning requirement of $\gtrsim$ 100 Hz at the design injection current of 40 mA will limit the speed of current ramps during startup to a few seconds. During steady state operations, a bunch train with regularly spaced bunches results in the best field stability and least power requirements.

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