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A FAST METHOD OF 2D CALCULATION OF COHERENT SYNCHROTRON RADIATION WAKEFIELD IN RELATIVISTIC BEAMS

Jingyi Tang*, Gennady Stupokov, Zhirong Huang
SLAC National Accelerator Laboratory, Menlo Park, CA, USA

Abstract

Coherent Synchrotron Radiation (CSR) is regarded as one of the most important reasons that limit beam brightness in modern accelerators. CSR wakefield is often computed in a 1D assuming a line charge, which can become invalid when beam has large transverse extension and small bunch length. On the other hand, the existing 2D or 3D codes are often computationally inefficient or incomplete. In our previous work[1] we developed a new model for fast computation of 2D CSR wakefields in relativistic beams with Gaussian distribution. Here we further generalize this model to achieve self-consistent computation compatible with arbitrary beam distribution and nonlinear magnetic lattice. These new features can enable us to perform realistic simulations and study the physics of CSR beyond 1D in electron beams with extreme short bunch length and high peak current.

INTRODUCTION

Coherent Synchrotron Radiation(CSR) is one of the main limits to improvements of the brightness of electron beam in storage rings, free electron laser (FEL) light sources and high-energy colliders. In a bending magnets, the beam radiates electromagnetic field and experiences a radiation reaction force. 1D models for this forces has been developed [2–5] and implemented in widely used simulation softwares like ELEGANT, which play an important role in lattice design and beam control for modern accelerators.

The 1D models are computationally efficient and work well in many scenarios. However since they neglect the transverse dimension of beam, 1D models are inaccurate when the beam has large transverse extension and short longitudinal length. An important part of the total force called compression force is neglected in relativistic beams moving in a curvilinear trajectory. The attention to this force was attracted by M. Dohlus [6] in 2002, who pointed out that if the beam is compressed (either longitudinally or transversely) the energy of its Coulomb field changes and this should result in a change of the kinetic energy of the beam particles. The departure from 1D theory has been explored both numerically and experimentally [7, 8]. On the other hand, existing 2D/3D CSR codes can be either slow in computation or missing some important part of effects.

In our previous work [1], we proposed a new model for beam with rigid Gaussian distribution, which enables fast computation of 2D CSR wake for arbitrary linear lattice. Here we generalize this model to work self-consistently with arbitrary beam distribution and nonlinear lattice. We first benchmark our model with CSRtrack and ELEGANT in the CSR chicane formulated in the 2002 DESY-Zeuthen workshop. Then we perform simulation for real beams on FACETII BC20 compressor. The effects of transverse and longitudinal CSR forces are also discussed.

* jytang@stanford.edu

2D/3D CSR MODEL FOR ARBITRARY BEAM DISTRIBUTION

Electron beam can be described by its time-dependent charge density $\rho(\boldsymbol{r},t)$ and velocity $\boldsymbol{v}(\boldsymbol{r},t)$, where $\boldsymbol{r}$ is the coordinate vector and t is the time. Energy loss per unit time and unit charge due to CSR along the beam can be given by

$$\mathscr{P} = q\,\boldsymbol{v}(\boldsymbol{r},t)\cdot\boldsymbol{E}(\boldsymbol{r},t), \tag{1}$$

where the electric field inside the beam can be derived using the scalar and vector potentials, which are the integrals over retarded space coordinate $\boldsymbol{r'}$ around the beam along the trajectory at preceding time $t' < t$

$$\begin{aligned}
\phi(\boldsymbol{r},t) &= \int d^3r'\,\frac{\rho(\boldsymbol{r'},t_{ret}(\boldsymbol{r},t))}{|\boldsymbol{r'}-\boldsymbol{r}|},\\
\boldsymbol{A}(\boldsymbol{r},t) &= \frac{1}{c}\int d^3r'\,\frac{\boldsymbol{v}(\boldsymbol{r'},t_{ret}(\boldsymbol{r},t))\,\rho(\boldsymbol{r'},t_{ret}(\boldsymbol{r},t))}{|\boldsymbol{r'}-\boldsymbol{r}|},\\
\boldsymbol{E}(\boldsymbol{r},t) &= -\nabla_r\phi(\boldsymbol{r},t) - \frac{1}{c}\frac{\partial\boldsymbol{A}(\boldsymbol{r},t)}{\partial t},
\end{aligned} \tag{2}$$

where $t_{ret} = t - |\boldsymbol{r}-\boldsymbol{r}'|/c$.

We will now make the following assumptions. A cold fluid approximation is applied that at point $\boldsymbol{r}$ the status of particles is determined by density $\rho(r,t)$ and velocity $\boldsymbol{v}(r,t)$. Second, we assume that the spread in velocity due to the angular and energy spread is negligible, which is valid for highly relativistic beams. Third, we assume that the size of the bunch is much smaller that the external scale of the problem under study. With further simplification, the energy loss due along the beam can be given by[9, 10]

$$W = W_1 + W_2 + W_3, \tag{3}$$

where

$$W_1(\vec{r},t) = -c\int\frac{d^3r'}{|\vec{r}'-\vec{r}|}\left[\vec{\beta} - (\vec{\beta}\cdot\vec{\beta}')\vec{\beta}'\right]\cdot\partial_{\vec{r}'}\rho(\vec{r}',t_{ret}), \tag{4a}$$

$$W_2(\vec{r},t) = c\int\frac{d^3r'}{|\vec{r}'-\vec{r}|}(\vec{\beta}\cdot\vec{\beta}')\rho(\vec{r}',t_{ret})\partial_{\vec{r}'}\cdot\vec{\beta}', \tag{4b}$$

$$W_3(\vec{r},t) = -\int\frac{d^3r'}{|\vec{r}'-\vec{r}|}\rho(\vec{r}',t_{ret})\vec{\beta}\cdot\partial_{t_{ret}}\vec{\beta}', \tag{4c}$$

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with $\vec{\beta} = \vec{v}/c$, $\vec{\beta} = \vec{\beta}(r,t)$, $\vec{\beta}' = \vec{\beta}(r, t_{ret})$.

To obtain the velocity and density distribution at retarded time for CSR calculation, we track macro-particles along the lattice. At each time step, particles are histogramed in $x-z$ plane. We apply 2D Savitzky-Golay filter to the distributions and calculate its derivatives. The distribution $\rho(x,z,t)$ and $v(x,z,t)$ and their derivatives are then recorded in a 3D matrix($t \times x \times z$). Note that for the calculation of CSR we only need to restore the time steps within a short distance(formation length) before the observation time. For time step t and arbitrary $\vec{r}$ inside the beam, we build interpolants based on the restored data and do linear interpolation to obtain ρ_{ret} and v_{ret}. The 2D CSR wake is then calculated given Eq. (3) and the reaction force is applied back to the beam in each step.

CSR WAKE IN BENCHMARK COMPRESSOR

As a benchmark, we calculated the CSR wake in a configuration studied at the CSR workshop at DESY-Zeuthen in 2002 [11]. The four magnets have the length $L = 0.5$ m with the bending radius $R = 10.35$ m resulting in the momentum compaction factor $R_{56} = 2.5$ cm. The electron beam is of Gaussian distribution in $x-z$ plane, with initial beam energy 5.0 GeV, charge 1 nC and slice energy spread is 10^{-4}. The initial rms bunch length of the beam is 200 µm with energy chirp $h = 36\,\mathrm{m}^{-1}$, and is compressed to 20 µm at the end of the chicane.

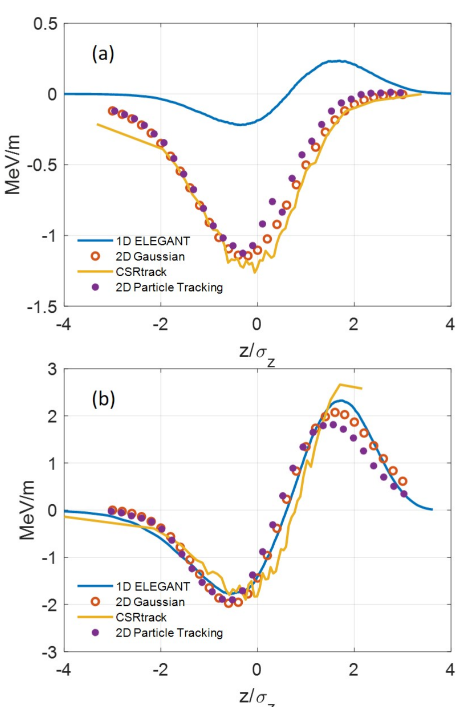

Figure 1: Longitudinal CSR wake along the central axis of the beam in the middle of the third (a) and fourth (b) bend of the benchmark chicane.

The longitudinal 2D CSR wake in term of energy loss rate is calculated along the central axis of the beam in the middle of the third and fourth dipole of the benchmark chicane. In both dipoles our model shows decent agreement with CSRtrack[12]. In the third dipole(Fig. 1)(a), where the transverse size of the beam is much larger than the bunch length due to the dispersion, 2D/3D models departure from the 1D results. As a comparison, in the fourth dipole(Fig. 1(b)) where the dispersion approaches zero, results from 2D/3D models and 1D model converge.

CSR WAKE IN FACET-II BC20 COMPRESSOR

Our second test case is the FACET-II upgraded BC20 compressor, the design of which can be found in [13]. It is composed of four dipoles with a series of quadrupoles and sextupoles in between. For simplicity in the case we tracked the beam from start-to-end simulation along the last dipole with CSRtrack and our model and compared the differences.

The final longitudinal and transverse phase space as well as the longitudinal CSR wake is shown in Figs. 2 and 3. Our model generates almost identical CSR wake compared with CSRtrack as well as very similar longitudinal phase space. If we compare Figs. 3(d) and 3(f), we can find that there are minor differences in the transverse phase space. We think the difference arise from the fact that our model only contains the longitudinal force for now, while CSRtrack computes both longitudinal and transverse forces. To verify this, we track the particles with longitudinal force from our model and transverse force from CSRtrack and get Fig. 3(b), which is in good agreement with the result from CSRtrack. It is also a good example showing the effects of transverse CSR force. In this case, it kicks the lower part of the beam in the transverse plane back to its center, and therefore partly cancels the effects of the longitudinal wake. In Fig. 2 we show the emittance calculated from the three models. With transverse wake included, our model agrees well with CSRtrack, while the emittance is slightly increased if only longitudinal force is considered.

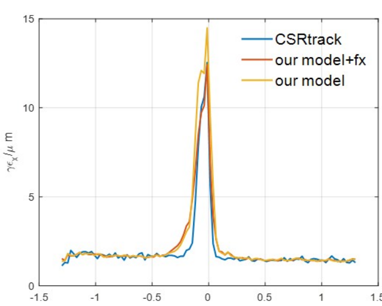

Figure 2: Slice emittance at the end of the last dipole of FACET-II upgraded BC20 compressor from CSRtrack(blue), our 2D model(yellow) and our 2D model with transverce force from CSRtrack(red).

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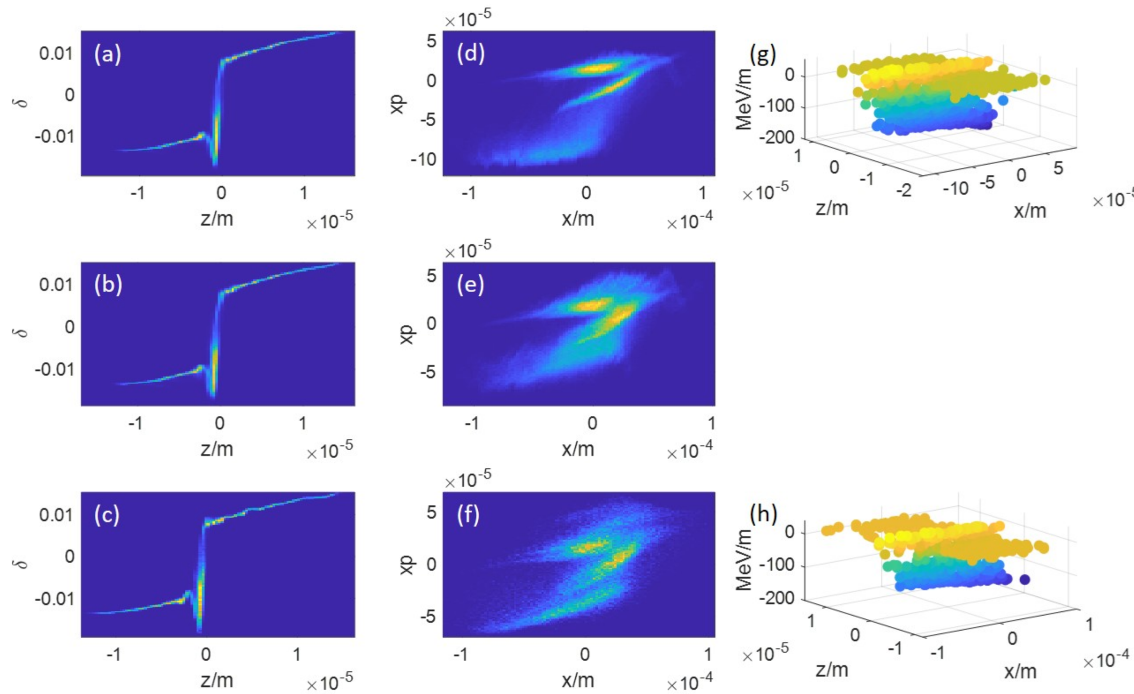

Figure 3: Longitudinal(a-c), transverse phase space(d-f) and longitudinal CSR wake(g,h) of the beam at the end of the last dipole of FACET-II upgraded BC20 chicane.(a), (d), (g) are results from our 2D model, (b), (e) from our 2D model with CSRtrack transverse wake, (c), (f), (h) from CSRtrack.

Our model can be fast in computation for several reasons. First it is based on histogramed distributions and therefore does not scale with the number of particles. Besides, the integrand in Eq. (3) is smooth and localized with W_1(which has the most contribution to the integration) only containing removable singularity. With these good properties the numerical integration can be performed fast and accurately. As a comparison, for the simulation of FACET-II dipole, CSRtrack takes about 97 cpu hours to finish, while our method only takes 20-30 minutes. Our model can be significantly faster compared with CSRtrack and produces identical results.

CONCLUSION

In this paper we introduce a new and fast method to achieve fast computation of CSR wake in 2D. We showed benchmarks with CSRtrack and obtained good agreement. The discrepancy between 1D and 2D model is observed and discussed. This model can be useful for research on relativistic beams under extreme compression.

REFERENCES

[1] G. Stupakov and J. Tang, "Calculation of the wake due to radiation and space charge forces in relativistic beams", *Phys. Rev. Accel. Beams*, vol. 24, p. 020701, Feb. 2021. doi:10.1103/PhysRevAccelBeams.24.020701

[2] J. B. Murphy, S. Krinsky, and R. L. Gluckstern, "Longitudinal Wakefield for Synchrotron Radiation", in *Proc. 16th Particle Accelerator Conf. (PAC'95)*, Dallas, TX, USA, May 1995, paper TPQ14, pp. 2980-2982.

[3] Ya. S. Derbenev, J. Rossbach, E. L. Saldin, and V. D. Shiltsev, "Microbunch radiative tail-head interaction", Deutsches Elektronen-Synchrotron, Hamburg, Germany, Rep. TESLA-FEL 95-05, Sep. 1995.

[4] E. L. Saldin, E. A. Schneidmiller, and M. V. Yurkov, "On the coherent radiation of an electron bunch moving in an arc of a circle", *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, vol. 398, pp. 373-394, 1997. doi:10.1016/S0168-9002(97)00822-X

[5] P. Emma and G. V. Stupakov, "CSR wake for a short magnet in ultrarelativistic limit", in *Proc. 8th European Particle Accelerator Conf. (EPAC'02)*, Paris, France, Jun. 2002, paper WEPRI029, pp. 1479–1481.

[6] M. Dohlus, "Field Calculations for Bunch Compressors", presented at the CSR Workshop, Zeuthen, Germany, Jan. 2002.

[7] A. D. Brynes *et al.*, "Beyond the limits of 1D coherent synchrotron radiation", *New Journal of Physics*, vol. 20, p. 073035, Jul. 2018. doi:10.1088/1367-2630/aad21d

[8] Y. Cai and Y. Ding, "Three-dimensional effects of coherent synchrotron radiation by electrons in a bunch compressor", *Phys. Rev. Accel. Beams*, vol. 23, p. 014402, Jan. 2020. doi:10.1103/PhysRevAccelBeams.23.014402

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-THXA05

[9] G. Stupakov, "New Method of Calculation of the Wake due to Radiation and Space Charge Forces in Relativistic Beams", in *Proc. 10th Int. Particle Accelerator Conf. (IPAC'19)*, Melbourne, Australia, May 2019, pp. 1223–1226. doi:10.18429/JACoW-IPAC2019-TUZZPLS3

[10] D. F. Ratner, *Much Ado About Microbunching: Coherent Bunching in High Brightness Electron Beams*, CA, USA: Stanford University, 2011.

[11] P. Emma, "CSR Benchmark Test-Case Results", presented at CSR Workshop, Berlin, 2002.

[12] M. D. Dohlus and T. L. Limberg, "CSRtrack: Faster Calculation of 3-D CSR Effects", in *Proc. 26th Int. Free Electron Laser Conf. & 11th FEL Users Workshop (FEL'04)*, Trieste, Italy, Aug.-Sep. 2004, paper MOCOS05, pp. 18–21.

[13] G. R. White, "A Design for 10 GeV, High Peak-Current, Tightly Focused Electron Beams at FACET-II", in *Proc. 8th Int. Particle Accelerator Conf. (IPAC'17)*, Copenhagen, Denmark, May 2017, pp. 807–809. doi:10.18429/JACoW-IPAC2017-MOPIK115