12th Int. Particle Acc. Conf. IPAC2021, Campinas, SP, Brazil JACoW Publishing
ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB006

THE IMPACT OF BEAM POSITION MONITOR TILTS ON COUPLING MEASUREMENTS

L. van Riesen-Haupt*, R. Tomás, CERN, Geneva, Switzerland

Abstract

The measurement and correction of coupling resonance driving terms is a key tool for improving the performance of synchrotrons. These terms are measured by exciting the beam and observing the subsequent motion in the horizontal and vertical planes through beam position monitors. This paper outlines the impact of tilt errors in these monitors to the distortion of the amount of coupling measured between the planes and how the computation of the resonance driving terms is affected by these tilts. It also attempts to use these results for mimicking tilt errors in simulations and discusses how discrepancies in measured resonance driving terms could be used to estimate the tilt errors that cause them.

BACKGROUND

To first order, particle accelerators are often designed using quadrupoles that cause particles to oscillate around a reference orbit as they traverse the machine. In an ideal approximation, the quadrupoles focus and de-focus the motion of the particles in two orthogonal planes leading to this motion being independent in the vertical and horizontal planes.

In this ideal case, at any given location, s, in the machine, the horizontal position, x, of a particle as it circles the accelerator n times is given as

$$x(n) = \sqrt{I_x \beta_x(s)} \cos(2\pi Q_x n + \phi_{x,0}), \tag{1}$$

where $\beta_x(s)$ is the local magnetic beta function and Q_x is the horizontal betatron tune, characterising the oscillations in the horizontal plane. I_x and $\phi_{x,0}$ depend on the initial conditions and location of the particle. A similar expression can be obtained for the vertical plane by substituting y for x.

The position can be divided by $\sqrt{\beta_x(s)}$ to give the normalised position, $\hat{x}$, which can be combined with its conjugate momentum, $\hat{p}_x$, to give the complex Courant-Snyder coordinate h_{x-}, which describes the particle's full position in phase space and is given by

$$h_{x-}(n) = \hat{x}(n) - i\hat{p}_x(n) = \sqrt{I_x} e^{i(2\pi Q_x n + \phi_{x,0})}. \tag{2}$$

In reality, however, manufacturing and alignment errors cause the motion in the two planes to be coupled, leading to the motion in the vertical plane to affect the horizontal position of a particle and vice-versa. As a result of this, the equation of the horizontal motion is modified to

$$\begin{aligned} h_{x-}(n) &= \sqrt{I_x} e^{i(2\pi Q_x n + \phi_{x,0})} \\ &-2if_{1001}(s)\sqrt{I_y} e^{i(2\pi Q_y n + \phi_{y,0})} \\ &-2if_{1010}(s)\sqrt{I_y} e^{-i(2\pi Q_y n + \phi_{y,0})}, \end{aligned} \tag{3}$$

where f_{1001} and f_{1010} quantify the local magnitude of the coupling effect and are referred to as the coupling resonance driving terms (RDT) [1]. The coupling RDTs are multiplied by terms that correspond to what would be the uncoupled motion in the orthogonal plane. In the vertical plane, the complex Courant-Snyder co-ordinate with coupling can be expressed as

$$\begin{aligned} h_{y-}(n) = \hat{y}(n) - i\hat{p}_y(n) &= \sqrt{I_y} e^{i(2\pi Q_y n + \phi_{y,0})} \\ &-2if^*_{1001}(s)\sqrt{I_x} e^{i(2\pi Q_x n + \phi_{x,0})} \\ &-2if_{1010}(s)\sqrt{I_x} e^{-i(2\pi Q_x n + \phi_{x,0})}. \end{aligned} \tag{4}$$

MEASUREMENT

In order to ensure that the accelerator optics are as close as possible to the design optics and to improve the stability of the beam, it is useful to measure and correct the coupling RDTs. The RDTs can be measured by computing h_- in both planes over many turns and taking a Fourier transform of this. The magnitude of the spectral lines at the frequencies corresponding to the tune of the orthogonal plane are directly proportional to the resonance driving terms.

Since a beam position monitor (BPM) can only measure the physical position, x, the normalised position, $\hat{x}$, and momentum, $\hat{p}_x$, have to be computed in order to obtain h_{x-}. $\hat{x}$ can be worked out by simply dividing x by $\sqrt{\beta_x(s)}$. On the other hand, $\hat{p}_x$ has to be computed using information from a downstream BPM, by using the fact that for a given BPM pair, identified as 1 and 2,

$$\hat{x}_2(n) = \hat{x}_1(n)\cos(\Delta\phi_x) + \hat{p}_{x1}(n)\sin(\Delta\phi_x), \tag{5}$$

where $\Delta\phi_x$ is the phase advance between the two BPMs. This phase advance can be be directly measured. One should also note that this equation assumes that there are no coupling sources between the BPMs. From this, one can get

$$\hat{p}_{x1}(n) = \frac{\hat{x}_2(n) - \hat{x}_1(n)\cos(\Delta\phi_x)}{\sin(\Delta\phi_x)}. \tag{6}$$

ROLL ERROR

In a real machine, the BPMs will not be perfectly aligned and will have small roll errors around the s-axis. This will result in some of the vertical motion being detected as horizontal motion and vice versa. This parasitic motion will

* leon.van.riesen-haupt@cern.ch

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB006

contribute to the spectral lines that are used to determine the coupling RDTs as described in [2]. Understanding exactly how these roll errors can affect the measured RDTs is important when modelling a realistic machine with alignment errors. Moreover, it helps understanding potential measurement uncertainties in real machines and under certain circumstances might make it possible to detect roll errors from RDT measurements.

The measured horizontal and vertical positions, $X(n)$ and $Y(n)$ respectively, for a BPM i with a roll error of θ can be computed using a rotation matrix as

$$\begin{pmatrix} X_i(n) \\ Y_i(n) \end{pmatrix} = \begin{pmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{pmatrix} \begin{pmatrix} x_i(n) \\ y_i(n) \end{pmatrix}, \tag{7}$$

where θ_i is the roll error and $x_i(n)$ and $y_i(n)$ are the actual horizontal and vertical position obtained by taking the real parts of Eqs. (3) and (4) and multiplying them by the square root of the corresponding β-function. The rotation from Eq. (7) can be applied to two BPMs, 1 and 2, before normalising the co-ordinates by dividing by the respective β-function. Next, the measured momenta $\hat{P}_x(n)$ and $\hat{P}_y(n)$ can be obtained using Eq. (6). These momenta are combined with the measured positions to give the measured complex Courant-Snyder co-ordinates $H_{x-}(n)$ and $H_{y-}(n)$.

SOLUTION

The terms for $H_{x-}(n)$ and $H_{y-}(n)$ can be factored by their complex exponentials giving

$$\begin{aligned} H_{x-}(n) = {} & A_x\sqrt{I_x}e^{i(2\pi Q_x n+\phi_{x,0})} \\ & -2iF_{1001,x}(s)\sqrt{I_y}e^{i(2\pi Q_y n+\phi_{y,0})} \\ & -2iF_{1010,x}(s)\sqrt{I_y}e^{-i(2\pi Q_y n+\phi_{y,0})} \\ & +B_x\sqrt{I_x}e^{-i(2\pi Q_x n+\phi_{x,0})} \end{aligned} \tag{8}$$

and

$$\begin{aligned} H_{y-}(n) = {} & A_y\sqrt{I_y}e^{i(2\pi Q_y n+\phi_{y,0})} \\ & -2iF^*_{1001,y}(s)\sqrt{I_x}e^{i(2\pi Q_x n+\phi_{x,0})} \\ & -2iF_{1010,y}(s)\sqrt{I_x}e^{-i(2\pi Q_x n+\phi_{x,0})} \\ & +B_y\sqrt{I_y}e^{-i(2\pi Q_y n+\phi_{y,0})}, \end{aligned} \tag{9}$$

where A_x, B_x, A_y and B_y are constants that depend on the roll errors, actual RDTs and machine optics and $F_{1001,x}$, $F_{1010,x}$, $F_{1001,y}$ and $F_{1010,y}$ are the coupling RDTs as measured using data from the horizontal and vertical plane respectively.

The full expression for A_x, after linearising for θ_1 and θ_2 is

$$\begin{aligned} A_x = 1 + \csc\Delta\phi_x \Bigg(& (f^*_{1001} - f^*_{1010})\sqrt{\frac{\beta_{y1}}{\beta_{x1}}}e^{-i\Delta\phi_x}\theta_1 \\ & -(e^{-2i\Delta\phi_y}f^*_{1001} - f^*_{1010})\sqrt{\frac{\beta_{y2}}{\beta_{x2}}}e^{-i\Delta\phi_y}\theta_2\Bigg), \end{aligned} \tag{10}$$

which would reduce to unity for $\theta_1 = \theta_2 = 0$, as expected. Similarly, B_x can be linearised and expressed as

$$\begin{aligned} B_x = -\csc\Delta\phi_x \Bigg(& (f_{1001} - f_{1010})\sqrt{\frac{\beta_{y1}}{\beta_{x1}}}e^{-i\Delta\phi_x}\theta_1 \\ & -(f_{1001} - e^{2i\Delta\phi_y}f_{1010})\sqrt{\frac{\beta_{y2}}{\beta_{x2}}}e^{-i\Delta\phi_y}\theta_2\Bigg), \end{aligned} \tag{11}$$

which reduces to zero in the absence of roll errors.

The linear expression for the measured RDTs are

$$\begin{aligned} F_{1001,x} = f_{1001} - \frac{\csc\Delta\phi_x}{4}\Bigg(& \sqrt{\frac{\beta_{y1}}{\beta_{x1}}}e^{-i\Delta\phi_x}\theta_1 \\ & -\sqrt{\frac{\beta_{y2}}{\beta_{x2}}}e^{i\Delta\phi_y}\theta_2\Bigg), \end{aligned} \tag{12}$$

$$\begin{aligned} F_{1010,x} = f_{1010} - \frac{\csc\Delta\phi_x}{4}\Bigg(& \sqrt{\frac{\beta_{y1}}{\beta_{x1}}}e^{-i\Delta\phi_x}\theta_1 \\ & -\sqrt{\frac{\beta_{y2}}{\beta_{x2}}}e^{-i\Delta\phi_y}\theta_2\Bigg), \end{aligned} \tag{13}$$

in both cases giving the correct value when there are no roll errors.

Taking into account similar expressions for the constants in the vertical plane, specifically, the measured RDTs would be

$$\begin{aligned} F_{1001,y} = f_{1001} + \frac{\csc\Delta\phi_y}{4}\Bigg(& \sqrt{\frac{\beta_{x1}}{\beta_{y1}}}e^{-i\Delta\phi_y}\theta_1 \\ & -\sqrt{\frac{\beta_{x2}}{\beta_{y2}}}e^{i\Delta\phi_x}\theta_2\Bigg) \end{aligned} \tag{14}$$

and

$$\begin{aligned} F_{1010,y} = f_{1010} - \frac{\csc\Delta\phi_y}{4}\Bigg(& \sqrt{\frac{\beta_{x1}}{\beta_{y1}}}e^{-i\Delta\phi_y}\theta_1 \\ & -\sqrt{\frac{\beta_{x2}}{\beta_{y2}}}e^{-i\Delta\phi_x}\theta_2\Bigg). \end{aligned} \tag{15}$$

$\pi/2$ FODO CELL

A special case arises when the two BPMs used for the measurement are located at the same position with a periodic FODO lattice with $\pi/2$ phase between them. Accelerators often use this kind of optics in their arcs. In this case $\Delta\phi_x = \Delta\phi_y = \frac{\pi}{2}$, $\beta_{x1} = \beta_{x2} = \beta_x$ and $\beta_{y1} = \beta_{y2} = \beta_y$ so that the expressions for the measured RDTs become

$$\begin{aligned} F_{1001,x} &= f_{1001} + \frac{i}{4R}(\theta_1 + \theta_2) \\ F_{1010,x} &= f_{1010} + \frac{i}{4R}(\theta_1 - \theta_2) \\ F_{1001,y} &= f_{1001} - \frac{iR}{4}(\theta_1 + \theta_2) \\ F_{1010,y} &= f_{1010} + \frac{iR}{4}(\theta_1 + \theta_2), \end{aligned} \tag{16}$$

where $R = \sqrt{\frac{\beta_x}{\beta_y}}$.

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB006

If one wanted to measure the roll angles of the two BPMs used for the measurement, one could eliminate the unknown actual RDT by computing the difference between the RDTs using data from the horizontal and vertical planes to give

$$\begin{aligned} \Delta F_{1001} &= \frac{i}{4}\left(\frac{1}{R}+R\right)(\theta_1-\theta_2) \\ \Delta F_{1010} &= \frac{i}{4}\left(\frac{1}{R}-R\right)(\theta_1+\theta_2). \end{aligned} \tag{17}$$

These expressions can be combined to solve for θ_1 and θ_2 giving

$$\begin{aligned} \theta_1 &= -2Ri\left(\frac{\Delta F_{1001}}{1-R^2}+\frac{\Delta F_{1010}}{1+R^2}\right) \\ \theta_2 &= -2Ri\left(\frac{\Delta F_{1001}}{1-R^2}-\frac{\Delta F_{1010}}{1+R^2}\right). \end{aligned} \tag{18}$$

The measured roll angles could be used for alignment or to compute the actual RDTs from the erroneous measured RDTs by solving Eq. (16).

IMPLEMENTATION IN SIMULATIONS

To test the accuracy of Eq. (18), a beam measurement in the Large Hadron Collider (LHC) was simulated using the MAD-X accelerator code [3]. For the simulation, the LHC run II injection optics were used for which the phase advance in the FODO cells is almost exactly $\pi/2$ [4]. The tracking was performed over 6600 turns using the MAD-X PTC implementation. After the tracking was completed a python script was used to manipulate the orbit data recorded at the two BPMs located next to quadrupoles 29 and 31 left of interaction point 3. The script read the horizontal and vertical orbit data and performed a rotation of −0.318 rad and 0.159 rad on the BPMs located next to quadrupoles 29 and 31 respectively, simulating roll errors of these magnitudes.

The resulting manipulated tracking data was analysed with a modified version of the BetaBeat python library, which is regularly used to analyse LHC optics measurements [5]. The library was modified so that the BPM pairs used for RDT measurements in the arcs would skip a BPM so that the phase advance between them would be $\pi/2$. Moreover, the code was also modified to compute an estimate of the roll angle for each BPM pair using the difference between measured RDTs from horizontal and vertical tracking data and Eq. (18). The results of this measurement are shown in Table 1.

Table 1: BPM Rolls Measured from Simulating an Ideal Machine. BPM Locations Indicate Adjacent Quadrupole Number Left of Interaction Point 3

BPM Pair	Measured / rad		Error / %	
	θ_1	θ_2	θ_1	θ_2
33 , 31	8.59×10^{-4}	0.160	-	0.73
31 , 29	0.162	−0.329	1.5	3.2
29 , 27	−0.329	8.63×10^{-4}	3.3	-

From Table 1 one can see that in the case of the LHC, Eq. (18) is accurate to a few percent. The inaccuracies may arise from the linearisation of the results, errors in the measured vertical and horizontal actions, I_x and I_y, which are described by constant A and B but ignored in the derivation of Eq. (18) or the fact that the LHC lattice does not have a phase advance of exactly $\pi/2$ by design. To test how well Eq. (18) holds up in a more realistic measurement, the tracking was repeated but with a lattice where typical LHC magnet errors based on magnetic measurements [6] and corrections were applied in MAD-X. A python script was used to simulate roll errors on the same two BPMs but this time the errors were reduced to −0.2 rad and 0.1 rad. The results are shown in Table 2.

Table 2: BPM Rolls Measured from Simulation Data with Errors. BPM Locations Indicate Adjacent Quadrupole Number Left of Interaction Point 3

BPM Pair	Measured / rad		Error / %	
	θ_1	θ_2	θ_1	θ_2
33 , 31	−0.0128	0.0824	-	17
31 , 29	0.0862	−0.195	11	2.6
29 , 27	−0.189	-1.55×10^{-5}	5.4	-

From Table 2, one can see that in a more realistic simulation, the relative error of the measured roll error significantly increases. This may be due to the fact that there are actual coupling sources between the BPM pairs used for the measurement, which were ignored in Eq. (6). These sources would result in additional errors in the measured RDTs, as described in [7] and have an impact on Eq. (18). Even in the absence of BPM roll errors, like in BPM 33, this results in erroneous roll angle estimates of several mrad. These effects become more dominant when the roll angle is decreased to several 100 µrad, at which point the method failed to give even indicative values of the BPM roll errors. A method using a single BPM, as described in [2] could be less vulnerable to error sources between the BPMs and could be explored in future studies.

CONCLUSIONS AND OUTLOOK

This paper has introduced a set of equations that describe how BPM roll angles affect the measurement of resonance driving terms. It also outlined how this effect could theoretically be used to estimate roll errors in BPMs and demonstrated this with ideal tracking data from the LHC and relatively large roll angles. The method was less successful when the tracking data came from a machine with magnet errors that introduced coupling sources. The effectiveness of this method method for machines with very low inherent coupling such as light sources or lepton colliders has to be further explored. Moreover, the equations for the measured RDTs can be used to apply roll errors in simulations that aim to predict the behaviour of a machine with realistic errors and corrections.

AKNOWLEDGEMENTS

We thank Andrea Franchi for useful discussions and Michael Hofer for help with the BetaBeat library.

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB006

REFERENCES

[1] A. Franchi, R. Tomás, and F. Schmidt, "Magnet strength measurement in circular accelerators from beam position monitor data", *Phys. Rev. Spec. Top. Accel. Beams*, vol. 10, no. 7, Jul. 2007. doi:10.1103/PhysRevSTAB.10.074001

[2] Y. Hidaka *et al.*, "Linear Optics Characterization and Correction Method Using Turn-by-turn BPM Data Based on Resonance Driving Terms With Simultaneous BPM Calibration Capability", in *Proc. North American Particle Accelerator Conf. (NAPAC'16)*, Chicago, IL, USA, Oct. 2016, pp. 605-608. doi:10.18429/JACoW-NAPAC2016-TUPOB52

[3] Methodical Accelerator Design - X, http://cern.ch/madx

[4] O. Brüning *et al.*, "LHC Design Report", CERN, Geneva, Switzerland, CERN-2004-003-V-1, 2004.

[5] pyLHC/OMC-Team, https://github.com/pylhc/Beta-Beat.src

[6] P. Hagen *et al.*, "WISE: A Simulation of the LHC Optics Including Magnet Geometrical Data", in *Proc. 11th European Particle Accelerator Conf. (EPAC'08)*, Genoa, Italy, Jun. 2008, paper TUPP091, pp. 1744-1746.

[7] R. Tomás *et al.*, "Measurement of global and local resonance terms", *Phys. Rev. Spec. Top. Accel. Beams*, vol. 8, no. 2, Feb. 2005. doi:10.1103/PhysRevSTAB.8.024001