12th Int. Particle Acc. Conf. IPAC2021, Campinas, SP, Brazil JACoW Publishing
ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB213

IMPORTANT DRIFT SPACE CONTRIBUTIONS TO NON-LINEAR BEAM DYNAMICS

J. Frank, M. Arlandoo, P. Goslawski, J. Li, T. Mertens, M. Ries
Helmholtz-Zentrum Berlin fuer Materialien und Energie GmbH (HZB), Berlin

Abstract

This paper presents an in-depth analysis of the non-linear contributions of drift spaces in beam dynamics for the creation of Transverse Resonance Island Buckets (TRIBs). TRIBs have been successfully generated in BESSY II and MLS at the Helmholtz-Zentrum Berlin fuer Materialien und Energie GmbH (HZB). They offer the possibility of generating a second stable orbit and, by populating the orbit with a different electron bunch pattern, allow to effectively have two distinct radiation sources in the same machine individually tailored to different user needs. We demonstrate the generation of TRIBs by order of non-linearity on simple lattice configurations by only treating the drift space as the non-linear element. Moreover, we also insert other non-linear magnets to show how they modify the already generated TRIBs from the drift spaces. We conclude by giving a qualitative analysis of the occurring effects, which provides a guideline as to when the linear approximation is insufficient and the non-linear contribution has to be taken into account.

INTRODUCTION AND MOTIVATION

Helmholtz-Zentrum Berlin fuer Materialien und Energie GmbH (HZB) has an ongoing effort to understand and leverage the highly non-linear effect of Transverse Resonance Island Buckets (TRIBs) [1–4]. The ongoing effort recently led to a successful two-orbit operation mode [5]. By filling the two distinct orbits with different electron bunches this operational mode expands on the range and diversity the storage ring based synchrotron light source can provide to its user community. For example it could in principle simultaneously allow to have:

1. a high temporal resolution, which drives the BESSY VSR [6] upgrade project,
2. high average brightness and photon flux-hungry experiments.

The realization of both types of experiments in the same storage ring is challenging since for coherent transverse radiation low emittance multi-bend-achromat (MBA) lattices are used, which allow only very long bunches of a few hundred pico-seconds length and thus hamper the time-resolved experiments.

The above problem may be resolved by a two-orbit operational mode where each orbit is then tailored to the specific user needs. On the other hand, such an operational mode can also provide a novel tool to study element-selective static and dynamic magnetism on magnetic recording materials, quantum materials, clusters, as well as molecular materials relevant for spintronics and catalysis by fast flipping the helicity of the circularly polarized X-rays in a synchrotron storage ring [7].

In order to address the above needs and provide multiple stable beams in the accelerator a further investigation of TRIBs is essential. In particular, the number, position, and size of the islands warrants a better understanding. For this reason we reduce the complex problem to the simple "toy model" of the focusing defocusing optical lattice (FODO cell) that is a standard building block in any storage ring and analyze the non-linear dynamics leading to TRIBs in a controlled setting. We show that the origin of TRIBs can also be traced to the non-linear contribution of drift spaces which are unavoidable in any optical lattice. In addition, the higher order magnets like sextupoles may generate islands and hence, a vital part of the analysis is which element presents the dominant contribution.

ACCELERATOR COMPONENTS

The central Hamiltonian of this investigation can be derived from relativistic Hamiltonian mechanics. In a particular gauge choice and switching from the independent variable time to the path along the central orbit the Hamiltonian for a charged particle passing through an electromagnetic region in the accelerator reads (Eq. (3.1) in [8])

$$H = \frac{\delta}{\beta_0} - \sqrt{\left(\delta + \frac{1}{\beta_0}\right)^2 - p_x^2 - p_y^2 - \frac{1}{\beta_0^2\gamma_0^2}} - a_s, \qquad (1)$$

where δ is the energy deviation relative to the energy of the reference particle, $\gamma_0 = (1-\beta_0^2)^{-\frac{1}{2}}$, the momenta are normalized w.r.t. the reference momentum $p_i = \frac{p_i}{p_0}$, and $a_s = \frac{q a_z}{p_0}$ is the normalized z-component of the vector potential for the electromagnetic field with charge q.

In this paper we will focus on the following three elements:

- drift space: $a_s = 0$
- quadrupole: $a_s = -\frac{k_1}{2}(x^2 - y^2)$
- sextupole: $a_s = -\frac{k_2}{6}(x^3 - 3xy^2)$,

where $k_n = \frac{q}{p_0}\frac{\partial^n B_y}{\partial x^n}$ relate to the magnetic fields of the magnets. We restrict the discussion to the transversal components x, p_x and set $\beta_0 = 1$.

Hamilton's equations can be solved exactly for drift spaces since only cyclic coordinates appear in the Hamiltonian. For the purpose of this paper we expand the drift space

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB213

Hamiltonian

$$H_D = \delta - \sqrt{(\delta+1)^2 - p_x^2} \tag{2}$$

$$\approx \frac{p_x^2}{2(\delta+1)} + H_D^{(4)} + H_D^{(6)} + H_D^{(8)} + \ldots, \tag{3}$$

where $H_D^{(n)}$ represents $O(n)$ terms. We note the $O(2)$ expansion is known as the paraxial approximation that leads to a linear transfer map. The paraxial approximation is used to represent quadrupoles as linear transfer maps and the sextupoles are treated as thin lens elements to have an analytic solution. The exact form of the transfer maps and its derivation can be found in [8], chapters 3 and 8.

It is clear that the drifts naturally couple the $x - p_x$, $y - p_y$, and $z - \delta$ sub-spaces and the solution is highly nonlinear. Moreover, the drift contribution appears as the kinetic term for all magnets encountered in the accelerator. By simplifying the Hamiltonians by either paraxial or thin lens approximations we loose this connection.

FODO LATTICE

The usual argument for approximating drift spaces as linear elements is based on the observation (for small values that are encountered in the accelerator ring), that expanding the square root beyond the leading order does not significantly improve the approximation for the next couple of terms. However, accounting for higher-order contributions introduces drastic topological changes and given the right setting provides qualitatively completely different phase space diagrams opposed to the expected ellipses for linear tracking.

We will demonstrate this in the optical lattice of a simple FODO cell (Fig. 1A). With the correct parameter settings this "toy model" creates particle oscillations that are depicted by closed phase space curves and is simple enough to allow for element by element tracking. In addition, we will modify the FODO cell (Fig. 1B) by inserting sextupole magnets.

RESULTS

For the following results we note that the transfer maps of all machine elements (w.r.t. the approximations made) have an analytic solution. We implemented the transfer maps as functions in Python and perform element-by-element tracking.

We first track through the FODO cell (Fig. 1A). We tuned the individual parameters to achieve a 1:4 resonance (4 islands). Figure 2 depicts the plots in the $x - p_x$ phase space, where we track by using the exact solution for the drift space H_D (Fig. 2A), only to order $H_D^{(2)}$ (Fig. 2B), to order $H_D^{(4)}$ (Fig. 2C), and to order $H_D^{(8)}$ (Fig. 2D). With the paraxial approximation (Fig. 2B) we can see how linear tracking misses the existence of TRIBs. Taking into account terms of order $H_D^{(4)}$ (Fig. 2C) the qualitative picture is recovered. Additional orders $H_D^{(6)}, H_D^{(8)}$ improve the approximation (Fig. 2D). Next, we track through the FODO cell (Fig. 1B), where we turn the sextupole magnets off. Effectively the FODO cell we are tracking through is akin to (Fig. 1A) with changed element lengths (and magnet strengths to stay on the 1:4 resonance line). Figure 3 depicts the plots in the $x - p_x$ phase space, where we again notice that the paraxial approximation (Fig. 3B) fails to reveal the existing islands, that are capture by higher order contributions of the drift spaces (Fig. 3C) and (Fig. 3D). Ultimately, we track through the FODO cell (Fig. 1B), where we turn the sextupole magnets on. Here we only changed the parameters $k_{31} = -k_{32} \neq 0$ and kept the remaining values fixed to illustrate the effect of

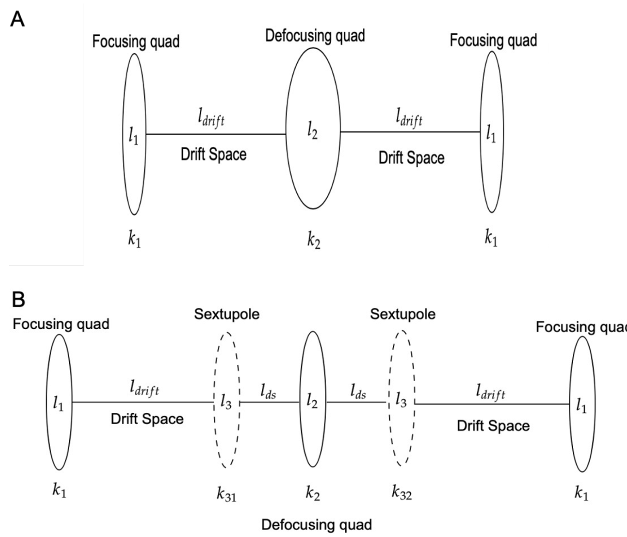

Figure 1: We assume the cells are periodically repeated and perform particle tracking through this optical lattice. The lengths of the elements are denoted by the $l's$ and the magnet strengths are denoted by the $k's$. **A** Schematic of the symmetric layout of the FODO cell. **B** Schematic of the symmetric layout of the FODO cell with inserted sextupole magnets. We begin by setting $k_3 = 0$ to track through the FODO cell where the only non-linear contributions are due to drift spaces.

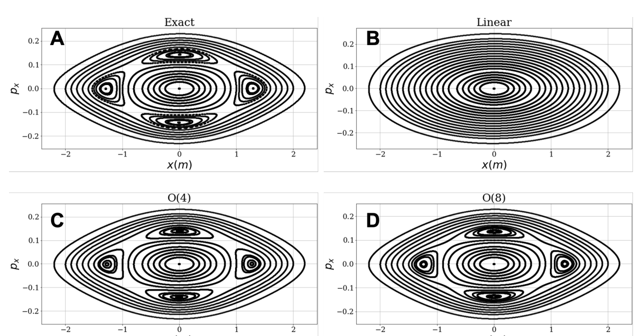

Figure 2: Phase space portraits in the $x - p_x$ subspace for tracking 10000 turns through the FODO cell (Fig. 1A). **A** Using exact drift space H_D. **B** The expansion cutoff at $H_D^{(2)}$ is a linear map. **C** Including contributions from $H_D^{(4)}$ reveals the islands. **D** The expansion including $H_D^{(8)}$ adds correction terms, but does not change qualitatively the phase diagram.

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB213

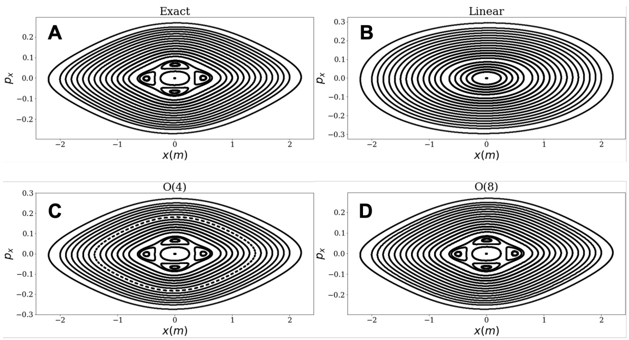

Figure 3: Phase space portraits in the $x - p_x$ subspace for tracking 10000 turns through the FODO cell (Fig. 1B). The sextupoles have been turned off ($k_{31} = k_{32} = 0$) so it is akin to tracking through the FODO cell (Fig. 1A) with adjusted parameters. **A** Using exact drift space H_D. **B** The expansion cutoff at $H_D^{(2)}$ is a linear map. **C** Including contributions from $H_D^{(4)}$ reveals the islands. **D** The expansion including $H_D^{(8)}$ adds correction terms, but does not change qualitatively the phase diagram.

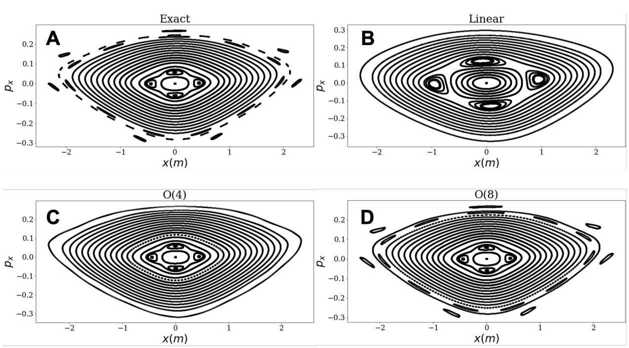

Figure 4: Phase space portraits in the $x - p_x$ subspace for tracking 10000 turns through the FODO cell (Fig. 1B). The sextupoles have been turned on ($k_{31} = -k_{32}$). **A** Using the exact square root expression for drift spaces. **B** The paraxial approximated linear map for the drift space gets islands through sextupole contributions. **C** The expansion up to $H_D^{(4)}$ neglects the higher order resonance for the edge particles. **D** The expansion to $H_D^{(8)}$ reveals further higher order resonance islands.

sextupole magnets. Figure 4 depicts the plots in the $x - p_x$ phase space, where we notice that the paraxial approximation (Fig. 4B) has been given a non-linear contribution from the sextupoles and thus shows TRIBs. We see that for the exact drift (Fig. 4A) the sextupoles lead to a higher resonance order of the outer particles - an effect captured by the $H_D^{(8)}$ contribution (Fig. 4D), but not by considering contributions up to $H_D^{(4)}$ (Fig. 4C).

CONCLUSION

We hope to have portrayed here the importance of non-linear contributions due to drift spaces in an accelerator. We have shown, that depending on the resonance in the optical lattice, one might need to go beyond terms of order $O(4)$ to recover the qualitative phase space plot. For that an analysis of the higher order terms leading to a resonance condition is crucial in order to establish which approximation is to crude for tracking purposes. Furthermore, by employing non-linear magnets we show their contribution for TRIBs.

The authors suspect that for the generation of stable TRIBs in a storage ring leading to two-orbit operational modes, the machine needs to be drift dominated with other non-linear magnets placed appropriately to stabilize this phenomenon. This suggestion is being investigate with various advanced mathematical tools and intermediate results from the authors can be found in [9, 10].

REFERENCES

[1] M. Ries *et al.*, "Transverse Resonance Island Buckets at the MLS and BESSY II", in *Proc. 6th Int. Particle Accelerator Conf. (IPAC'15)*, Richmond, VA, USA, May 2015, pp. 138–140. doi:10.18429/JACoW-IPAC2015-MOPWA021

[2] P. Goslawski *et al.*, "Resonance Island Experiments at BESSY II for User Applications", in *Proc. 7th Int. Particle Accelerator Conf. (IPAC'16)*, Busan, Korea, May 2016, pp. 3427–3430. doi:10.18429/JACoW-IPAC2016-THPMR017

[3] P. Goslawski, A. Jankowiak, F. Kramer, M. Ries, M. Ruprecht, and G. Wuestefeld, "Transverse Resonance Island Buckets as Bunch Separation Scheme", in *Proc. 8th Int. Particle Accelerator Conf. (IPAC'17)*, Copenhagen, Denmark, May 2017, pp. 3059–3062. doi:10.18429/JACoW-IPAC2017-WEPIK057

[4] F. Kramer, P. Goslawski, A. Jankowiak, M. Ries, M. Ruprecht, and A. Schaelicke, "Characterisation of the Second Stable Orbit Generated by Transverse Resonance Island Buckets (TRIBs)", in *Proc. 9th Int. Particle Accelerator Conf. (IPAC'18)*, Vancouver, Canada, Apr.-May 2018, pp. 1656–1659. doi:10.18429/JACoW-IPAC2018-TUPML052

[5] P. Goslawski *et al.*, "Two Orbit Operation at Bessy II - During a User Test Week", in *Proc. 10th Int. Particle Accelerator Conf. (IPAC'19)*, Melbourne, Australia, May 2019, pp. 3419–3422. doi:10.18429/JACoW-IPAC2019-THYYPLM2

[6] P. Schnizer *et al.*, "Status of the BESSY VSR Project", in *Proc. 9th Int. Particle Accelerator Conf. (IPAC'18)*, Vancouver, Canada, Apr.-May 2018, pp. 4138–4141. doi:10.18429/JACoW-IPAC2018-THPMF038

[7] K. Holldack *et al.*, "Flipping the helicity of X-rays from an undulator at unprecedented speed", *Commun. Phys.*, vol. 3, no. 61, 2020. doi:10.1038/s42005-020-0331-5

[8] A. Wolski, *Beam dynamics in high energy particle accelerators*, London, United Kingdom: Imperial College Press, 2014

[9] J. Frank *et al.*, "Novel Non-Linear Particle Tracking Approach Employing Lie Algebraic Theory in the TensorFlow Environment", presented at the 12th Int. Particle Accelerator Conf. (IPAC'21), Campinas, Brazil, May 2021, paper TUPAB215, this conference.

[10] J. Frank, M. Arlandoo, P. Goslawski, T. Mertens, and M. Ries, "Alpha Buckets in Longitudinal Phase Space: A Bifurcation Analysis", presented at the 12th Int. Particle Accelerator Conf. (IPAC'21), Campinas, Brazil, May 2021, paper TUPAB214, this conference.