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# LONGITUDINAL DYNAMICS WITH HARMONIC CAVITIES UNDER THE OVER-STRETCHING CONDITIONS

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*Abstract*

Harmonic cavities (HCs) are widely used to lengthen the bunches, mainly for increasing the Touschek lifetime or for suppressing the coupled-bunch instabilities in electron rings. There have been quite many studies on the beam dynamics with the consideration of HCs. We revisited the basic longitudinal beam dynamics with HCs. We successfully separated two peaks of longitudinal beam density of a over-stretching bunch from 267 ps ~ 1.77 ns.

## INTRODUCTION

Harmonic cavities(HCs) are widely used in new generation of light sources and diffraction limited storage rings allowing to perform complex operations on the beam bunches.

In 1967, Roger Gram and Phil Morton [1] manipulated the bucket shape and gave the “quasi-flat-top” condition. In 1980, Hofmann and Myers [2] discussed optimum stretched bunch meeting three conditions: first and second derivatives of potential at synchronous phase vanish; voltage gain per turn of synchronous particle equals to the total loss per turn. It should be mentioned that active HCs can satiesfy optimum conditions for bunch lengthening, but passive HCs can only satiesfy optimum conditions approximately [3].

Under optimum conditions, HCs can lengthen the bunch, increase Touschek lifetime, provide strong Landau damping, reduce the synchronous frequency, decrease the energy spread induced by coupled bunch oscillations, provide larger bunching factor and capture efficiency [4–11].

For some purposes, such as avoiding reduction of beam brightness or compress bunches for subsequent injection to FEL, HCs can also work in the shorten regime [12].

However, in the presence of HCs, the transients can be very large [13]. The high impedance of cavities can cause strong wakefields for a long time, which perturbates fields from generators and cause distortion of harmonic voltage, especially with nonuniform filling pattern.

In the uniform filling case, the Touschek lifetime increase is maximum [14]. However, because of clearing trapped ions, limitations of injector, gaps for beam kicker rise times or user requirement, the buckets are always working with a gap. This will cause nonuniform beam loading effects, phase shift, synchrotron tune spread, reduction of efficiency of bunch lengthening subsequently [12–16], and even reduce the stability with an increasing gap in the uniform filling pattern [3].

Negative effects, such as multi-bunch instability driven by higher-order modes (HOMs) and other effects mentioned above, can be partially suppressed by an additional compensation system, and even increase the beam current threshold [17–19].

In this paper, we tried to separate a bunch longitudinally by HC under overstretching conditions. We can obtain some sub-buckets with smaller time structure and manipulate their time gap. We review the equation of motion of longitudinal dynamics with HHCs firstly and derivate the longitudinal distribution for HHCs. The default assumption here is it can only discript longitudinal distribution within a bucket. Then we analyze the conditions for over-stretched bunches. The gap between two SFPs can range from 0 to 1.85 rad theoretically. But when the gap vanished, the sub-bucket height will vanish too. If we ignore results where half sub-bucket height is smaller than 0.1%, the gap between two SFPs will be 0.28 rad ~ 1.85 rad.

## LONGITUDINAL DYNAMICS

At first we review the longitudinal beam dynamics with active HC. Starting from S. Y. Lee's book [20], we have:

$$\begin{aligned} \dot{\phi} &= h_1 \omega_0 \eta \delta, \\ \dot{\delta} &= \frac{e\omega_0 V_1}{2\pi\beta^2 E}\mathscr{V}(\phi), \end{aligned} \tag{1}$$

where

$$\mathscr{V}(\phi) = \sin\phi - \sin\phi_s + r\sin[\phi_{2s} + h(\phi-\phi_s)] - r\sin\phi_{2s},$$

where $\phi_s$ and $\phi_{2s}$ are phases of synchronous particle in primary cavity and HC, and $r = V_2/V_1$.

Then we can get the Hamiltonian

$$\begin{aligned} H(\delta,\phi) = \frac{1}{2}h_1\omega_0\eta\delta^2 + \frac{e\omega_0 V_1}{2\pi\beta^2 E}\cdot\Big(\cos\phi + \phi\sin\phi_s \\ + \frac{r}{h}\cos\big[\phi_{2s} + h(\phi-\phi_{2s})\big] + r\phi\sin\phi_{2s}\Big). \end{aligned} \tag{2}$$

We consider the equilibrium condition where $\frac{\partial\rho}{\partial t} = 0$. Supposed that density distribution in phase space $\rho(\phi,\delta)$ has the Maxwell-Boltzmann form:

$$\rho(\phi,\delta) = \frac{1}{\sqrt{2\pi}\sigma_\delta}\exp\Big[-\frac{1}{2}\Big(\frac{\delta}{\sigma_\delta}\Big)^2\Big]\rho(\phi).$$

By Vlasov eqution, we can solve the longitudinal density distribution $\rho(\phi)$

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$$\rho(\phi) = C \cdot \exp\left[\frac{e}{2\pi h_1 \eta \beta^2 E \sigma_\delta^2} \int_{\phi_0}^{\phi} \mathscr{V}(\phi) d\phi\right],$$

here $C$ and $\phi_0$ were constants. This result is the same as Hofmann's [2].

At last, we can normalize the density distribution:

$$\bar{\rho}(\phi) = \frac{\rho(\phi)}{\int_{\phi_l}^{\phi_r} \rho(\phi) d\phi}, \tag{3}$$

where $\phi_l, \phi_r$ is the minimum and maximum phase in the bunch.

## EFFECTS OF OVERSTRETCHING

At over-stretching conditions, a bucket has two stable fixed points (SFPs), and $\phi_s$ and $\phi_{2s}$ are not synchronous phases in primary and harmonic cavity anymore. $\phi_s$ becomes a new unstable fixed point (UFP). But following equation still holds

$$\begin{aligned} \frac{U_0}{eV_1} &= \sin\phi_{s,1} + r \cdot \sin\phi_{2s,1} \\ &= \sin\phi_{s,2} + r \cdot \sin\phi_{2s,2} \\ &= \sin\phi_s + r \cdot \sin\phi_{2s}. \end{aligned} \tag{4}$$

By assuming sub-buckets, small buckets split from the original one, be of same bunch height, we can solve the longitudinal distance between these two SFPs numerically for different $r$. The relevant parameters were shown in Table 1 and results were plotted in Fig. 1.

Table 1: Parameters in Simulation

| Parameters | Symbols | Value and Units |
|---|---|---|
| Circumference | $C$ | 1360.4m |
| Beam Energy | $E_0$ | 6 GeV |
| Harmonic Number | $h_1$ | 756 |
| Higher Harmonic Number | $h_2$ | 2268 |
| Momentum Compaction Factor | $\alpha_c$ | 1.561e-5 |
| Radiation Energy Loss Per Turn | $U_0$ | 2.887 MeV |
| RMS of Momentum Deviation Rate | $\sigma_\delta$ | 1.061e-3 |

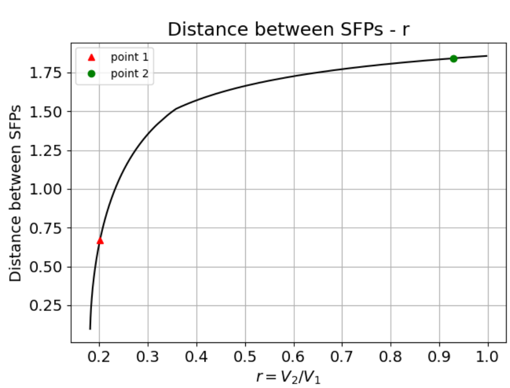


Figure 1: Sub-bucket distance for different $r$ when h = 3.

We choose two cases at point 1($V_1$: 3.644 MV, $V_2$: 0.733 MV, $\phi_{1s}$: 0.632) and point 2($V_1$: 3.644 MV, $V_2$: 3.385 MV, $\phi_{1s}$: 0.221) in Fig. 1 and plot their Hamiltonian torus (see Fig. 2) and longitudinal density distributions (see Fig. 3).

We need to mention that, in Fig. 2, when the distance between two SFPs is large enough, two sub-buckets will separate completely. So we should separately inject charges into these sub-buckets at similar conditions, adjusting two peak densities. In Fig. 2, we assumed two sub-buckets have equivalent charges. The over-stretching beam pulse can range from 0.28 rad ~ 1.85 rad. When distance between SFPs less than 2.78 rad, the half sub-bucket height will less than 0.1% that we ignoring here. Using parameters in Table 1, we can calculate the time distance between two SFPs is 267 ps ~ 1.77 ns.

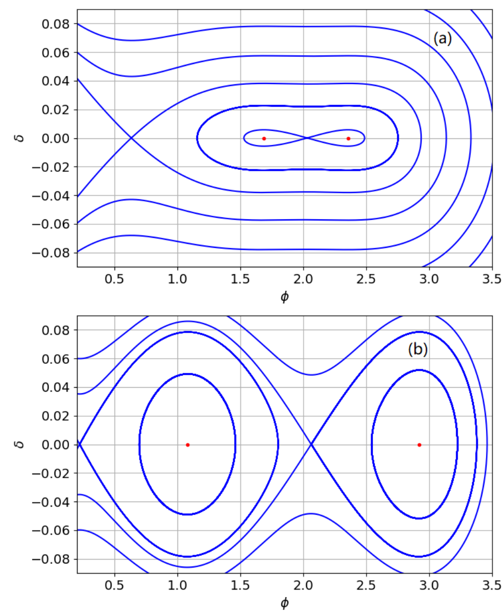


Figure 2: The plot shows Hamiltonian torus for point 1 and point 2 respectively. Figure (a) shows the two sub-buckets are in one bucket. Figure (b) shows these two sub-buckets are not in the same bucket because a large distance between them.

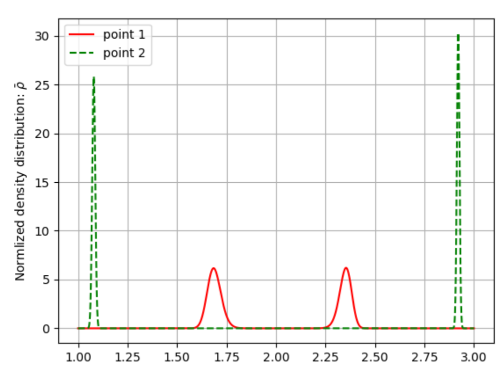


Figure 3: (1) Normlized density distribution for point 1 assuming a continuous density distribution. (2) Normlized density distribution for point 2 assuming two sub-buckets have equal charges, for discontinuous distribution at this condition. Two peak density can be adjusted by injecting different charges into two sub-buckets.

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## CONCLUSION

In this paper, we reviewed longitudinal beam dynamics with active harmonic cavities. Then we tried to seperate a bunch into two small bunched theoretically. The distance between two longitudinal density peaks of a over-stretching bunch can range from 0.28 ~ 1.85 rad (267 ps ~ 1.77 ns).

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