12th Int. Particle Acc. Conf. IPAC2021, Campinas, SP, Brazil JACoW Publishing
ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB254

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2021). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI

LIMITING COHERENT LONGITUDINAL BEAM OSCILLATIONS IN THE EIC ELECTRON STORAGE RING*

B. Podobedov†, M. Blaskiewicz, Brookhaven National Laboratory, Upton, NY, USA

Abstract

We study coherent longitudinal beam oscillations in the Electron Ion Collider (EIC) electron storage ring (ESR). We show that to avoid unacceptable hadron emittance growth due to finite crossing angle, the amplitude of these oscillations needs to be limited to a fraction of a millimeter. We estimate the amplitude of these oscillations under the two scenarios: 1) the beam is passively stable and the oscillations are driven by RF phase noise only; 2) a coupled-bunch instability, presently expected in the ESR, is damped by a longitudinal feedback system. We show that, for the 2nd scenario, comfortable specifications for RF phase noise and feedback sensor noise will be sufficient to maintain the oscillations within the required limits.

INTRODUCTION

The present ESR design is expected to have longitudinal coupled bunch instability, driven primarily by a narrow-band impedance due to the RF cavity HOM absorbers [1]. A strategy needs to be developed to cure this instability, either by passive damping with re-designed HOM absorbers, or with a longitudinal feedback system. Separately, to avoid unacceptable hadron emittance growth, the electron beam arrival time jitter in the crab cavities must be maintained below 1.1 ps rms, which imposes 0.33 mm rms limit on the amplitude of coherent longitudinal oscillations in the ESR, derived below. These oscillations are expected to be primarily driven by the RF phase noise. However, in the case that a feedback system is used to cure the instability, they can also come from the feedback system itself, which, in a certain frequency range, could amplify its own sensor noise. In this paper we analyse the expected magnitude of these oscillations for both approaches. For more in-depth treatment of this problem see [2].

LIMIT FOR LONGITUDINAL POSITION OSCILLATION AMPLITUDE IN ESR

Longitudinal motion of the electron bunches creates dipole beam-beam kicks on the hadrons because of the crossing angle. We begin by deriving the transverse kick from the electrons and then address emittance growth of the hadrons.

Let z denote the longitudinal position of an electron with respect to the zero crossing of the crab voltage with $z > 0$ at the head of the bunch. As this particle moves through the interaction region its orbit offset is

$$\Delta x(z) = \theta z - \theta \sin(kz)/k, \tag{1}$$

where θ is the half crossing angle, 12.5 mrad, and k is the wavenumber of the crab cavity (corresponding to 394 MHz frequency [1]). When the electron bunch centroid is oscillating as $z_0(t)$, the average offset of the bunch is

$$\langle \Delta x(t) \rangle = \int ds \Delta x(s) f(s - z_0(t)), \tag{2}$$

where $f(s)$ is the longitudinal distribution function of the electron bunch, which we model as a Gaussian of rms width σ_l. Expanding the sine to cubic order one gets

$$\langle \Delta x(t) \rangle \cong -\theta k^2 \sigma_l^2 z_0(t)/2, \tag{3}$$

where we assume the average oscillation amplitude is zero and that the oscillation is small compared with the bunch length. This transverse offset creates a beam-beam dipole kick which can drive hadron emittance growth. This was recently studied in the context of beam-ion instability [3]. (Other effects that may lead to proton emittance growth include RF noise in the main cavities with non-zero dispersion [4], and crab cavity RF noise [5]).

The hadron beam at the interaction point grows as

$$\frac{d}{dn}\sigma_x^2 = 8(\pi \Delta Q_{bb} \sigma_{xe})^2 \sum_{m=-\infty}^{\infty} \rho(m) \cos(2m\pi Q_x), \tag{4}$$

where n is turn number, σ_x is the horizontal beam size, ΔQ_{bb} is the beam-beam tune shift, σ_{xe} is the rms of $\langle \Delta x \rangle$, $\rho(m)$ is correlation function in turns, and Q_x is the tune.

Depending on the correlation function the sum can be very large, but since the synchrotron tune is smaller than the betatron tune, this is unlikely. As an initial estimate we take the sum to be 1. We take the initial beam size to be σ_{x0} = 0.1 mm, ΔQ_{bb} = 0.015, and n = 10 hours/ 12.8 microseconds, for the beam emittance to double. This yields σ_{xe}=14 nm. Using Eq. (3) one gets a rms value of z_0 of 0.08 mm for σ_l = 20 mm bunch length. The electron bunch length of about 1 cm results in the rms value of z_0 of 0.33 mm, or, equivalently, the electron beam arrival time jitter in the crab cavities must be below 1.1 ps rms.

MODEL FOR BEAM OSCILLATIONS

Noise-Driven Oscillator with Feedback

Relevant physics effects can be modelled by a noise-driven harmonic oscillator with feedback. For an oscillator with white-noise excitation and with derivative feedback,

$$\ddot{x} + 2\Gamma \dot{x} + \omega_0^2 x = \sigma\, \eta(t) - g \times (\dot{x} + \dot{\xi}(t)), \tag{5}$$

where x is a generic coordinate (not necessarily the horizontal beam position from the previous section), ω_0 is the natural frequency, $\Gamma > 0$ is the damping decrement, $\sigma\, \eta(t)$ is a stochastic force with zero mean and δ-function autocorrelation, $R(\tau) = \sigma^2 \delta(\tau)$, g and $\xi(t)$ are the feedback

* Work supported by Brookhaven Science Associates, LLC under Contract No. DE-SC0012704 with the U.S. Department of Energy.
† boris@bnl.gov

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB254

gain and sensor noise. If the drive noise, $\eta(t)$, and the sensor noise, $\xi(t)$, are uncorrelated, the expected power spectral density (PSD) of x is (see e.g. [6]),

$$S_x(\omega) = \frac{\sigma^2}{(\omega_0^2-\omega^2)^2+(2\Gamma+g)^2\omega^2} + \frac{g^2\omega^2}{(\omega_0^2-\omega^2)^2+(2\Gamma+g)^2\omega^2}\sigma_\xi^2. \quad (6)$$

Here the sensor noise PSD is assumed constant in the frequency range of interest, $S_\xi(\omega) = S_\xi = \sigma_\xi^2$. Denoting the sensor noise bandwidth (in rad/s) by B_ξ, the total integrated sensor noise power (in m^2) is

$$\sigma_s^2 = \frac{1}{2\pi}\int S_\xi(\omega)d\omega = \frac{B_\xi}{\pi}\sigma_\xi^2. \quad (7)$$

Integrating Eq. (6) over frequency, we get for the expected rms of the residual oscillation σ_x,

$$\sigma_x^2 = \frac{1}{2\pi}\int S_x(\omega)d\omega = \frac{{\sigma_{x0}}^2}{1+\frac{1}{2}g/\Gamma} + \frac{g^2\sigma_\xi^2}{2(2\Gamma+g)}, \quad (8)$$

where σ_{x0} denotes the expected rms of the residual oscillation without the feedback (i.e. at $g = 0$),

$$\sigma_{x0} = \sqrt{\langle x^2\rangle} = \frac{\sigma}{2\omega_0\sqrt{\Gamma}} = \frac{\omega_0}{2}\sqrt{\frac{S_x(0)}{\Gamma}}. \quad (9)$$

From Eqs. (7), (8) and (9), we obtain the fractional change in the power of the residual oscillations due to the feedback,

$$\frac{\sigma_x^2}{{\sigma_{x0}}^2} = \frac{1+\frac{\pi}{4}\frac{g^2}{\Gamma^2}\Gamma\sigma_s^2/B_\xi}{1+\frac{1}{2}g/\Gamma}. \quad (10)$$

As expected, increasing the feedback gain from zero results in the initial reduction of the residual oscillation power. At larger gains, the residual oscillations increase, eventually exceeding the level without the feedback.

To minimize σ_x for a given sensor noise, the feedback must be set to the optimum gain,

$$g_{opt} = 2\Gamma\left(\sqrt{1+\alpha^2}/\alpha - 1\right), \quad (11)$$

where we introduced parameter α (typically $\alpha \ll 1$) to express the normalized sensor noise amplitude,

$$\alpha = \sqrt{\frac{\pi\Gamma}{B_\xi}}\frac{\sigma_s}{\sigma_{x0}} = \frac{2\Gamma}{\omega_0}\sqrt{\frac{S_\xi}{S_x(0)}}. \quad (12)$$

The resulting minimum level of the oscillations is

$$\frac{{\sigma_{x,min}}^2}{{\sigma_{x0}}^2} = 2\alpha\left(\sqrt{1+\alpha^2} - \alpha\right), \quad (13)$$

therefore, for small sensor noise, the residual oscillations will be substantially reduced by the feedback. Figure 1 illustrates this point in the frequency domain.

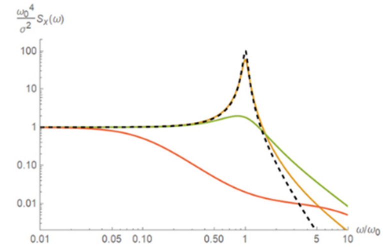

Figure 1: PSD of residual oscillations at the optimum gain for several feedback sensor noise levels at Γ/ω_0=0.05.

Feedback-Stabilized Beam Instability

So far we considered a stable situation, Γ>0, and showed that the optimized derivative feedback with a low-noise sensor can substantially reduce the residual noise due to a stochastic driving force.

In the context of storage ring longitudinal beam dynamics (well described by a harmonic oscillator model, see e.g. [7]) this applies to the case of longitudinally stable beam, driven by RF noise (primarily m = 0 mode). A bunch-by-bunch feedback system, which essentially acts on all coupled-bunch modes, can effectively reduce the residual beam noise in this case. In practice, feedback controllers utilize a band-limited differentiator implemented as a digital FIR filter [8], so they are more complicated than the one assumed here. Nevertheless, we believe that our model is appropriate for the preliminary design estimates.

Now we assume that, in addition to the white noise considered so far, there is an instability the feedback needs to damp. To proceed, we: 1) replace $\Gamma \to \Gamma_d - \Gamma_i$, in Eq. (5), where Γ_d is the radiation damping rate and Γ_i>0 is the instability growth rate; 2) require that the beam is stable with feedback, $\Gamma_d - \Gamma_i + g > 0$; 3) normalize the residual beam motion to the case without the feedback and instability, i.e. replace, $\Gamma \to \Gamma_d$, in Eqs. (9) and (12). Our final result for the minimum level of the residual oscillations at the optimum feedback gain is

$$\frac{{\sigma_{x,min}}^2}{{\sigma_{x0}}^2} = 2\alpha\left(\sqrt{1+\alpha^2(\Gamma_i/\Gamma_d-1)^2} + \alpha(\Gamma_i/\Gamma_d-1)\right), \quad (14)$$

which agrees with Eq. (13) for $\Gamma_i = 0$.

The reduction of the residual beam oscillation power due to the feedback, tuned to optimum gain, in the presence of instability is illustrated in Fig. 2. It shows that: 1) the feedback is ineffective when the sensor noise is large; 2) for any instability growth rate, a sensor with low enough noise, $\alpha < 2\Gamma_d/\Gamma_i$, allows for substantial reduction of the residual oscillations. For α≪1 the reduction factor is ~2α.

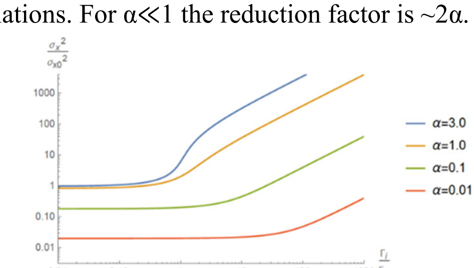

Figure 2: Residual beam oscillation power vs. instability growth rate for several feedback sensor noise levels.

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2021). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI

12th Int. Particle Acc. Conf. IPAC2021, Campinas, SP, Brazil JACoW Publishing
ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB254

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2021). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI

In Fig. 2 we normalize the oscillation power to that without the instability and feedback, Eq. (9) (with $\Gamma \rightarrow \Gamma_d$), which depends on the magnitude of the drive noise. For a storage ring application, the latter is primarily given by the RF phase noise. For modern electron rings, 0.1 degree rms RF phase noise is considered easily achievable, and the state-of-the-art is about an order of magnitude lower (see [2] for more discussion and measurements from NSLS-II).

ARRIVAL JITTER ESTIMATES FOR ESR

We take relevant parameters from [1], specifically the longitudinal radiation damping time, $\tau_d = 1/\Gamma_d$ =2000 turns (at 10 GeV), the instability growth rate $\Gamma_i = 2\pi/1000$ turns^{-1}, the synchrotron tune $\nu_s = 0.05$, and the RF frequency f_{RF} = 591 MHz.

First consider the case without the instability, so that the beam oscillations are driven by the RF phase noise only. From Eq. (6), the beam response is resonantly enhanced around the synchrotron frequency ω_s and then is sharply falling off at higher frequencies, see Fig. 1. Due to higher frequency noise being largely irrelevant, we limit it around ω_s with a bandwidth, $B_{RF}\sim\omega_s$.

If the phase noise of rms σ_ϕ (in degrees) is distributed uniformly in this bandwidth then its PSD, PSD_{RF}, is

$$\frac{1}{\pi}\int_{\omega_s-\frac{1}{2}B_{RF}}^{\omega_s+\frac{1}{2}B_{RF}} PSD_{RF}\,d\omega = \frac{B_{RF}}{\pi}PSD_{RF} = \sigma_\phi{}^2. \qquad (15)$$

At low frequency the beam follows the RF, so, from Eq. (6), we can express the beam motion PSD as

$$S_z(0) = PSD_{RF}\left(\frac{c}{f_{RF}\,360}\right)^2 = \frac{\pi}{B_{RF}}\left(\frac{c}{f_{RF}\,360}\right)^2\sigma_\phi{}^2. \quad (16)$$

Note that we use subscript z instead of x to indicate that the model is applied to longitudinal beam oscillations.

Substituting Eq. (16) into Eq. (9) (with $\Gamma \rightarrow \Gamma_d$) we obtain the final expression for the residual beam oscillation rms,

$$\sigma_{z0} = \sigma_\phi \frac{c}{f_{RF}\,360}\frac{1}{4}\sqrt{\frac{\pi B_{RF}}{\Gamma_d}}. \qquad (17)$$

Plugging in the CDR values, and taking, for instance, $\sigma_\phi = 0.1$ degree and $B_{RF} = \omega_s$, we obtain $\sigma_{z0} = 1.6$ mm, which significantly exceeds the 0.33 mm rms crab cavity limit worked out in the previous section.

For the beam oscillation amplitude to end up right at this limit, we must reduce the RF phase noise proportionally, to ~0.02 degrees rms, which could be somewhat challenging in practice. Due to many approximations made, it would also be prudent to assume a safety factor of 2-5 at this stage of the design. This, however, could bring the required RF phase noise spec beyond the state-of-the-art.

We now turn to the case of longitudinal instability being cured by the feedback. As discussed above, on top of curing the instability, the feedback can damp the residual beam oscillations by a large factor, as long as the feedback sensor has a good signal-to-noise ratio, see Eq. (14). To illustrate the required sensor performance in real units, we express Eq. (14) in terms of the integrated sensor noise, σ_s, by using Eq. (12) (with $\Gamma \rightarrow \Gamma_d$), to get

$$\sigma_{z,min}{}^2 = 2\sigma_{z0}{}^2\sqrt{\frac{\pi\Gamma_d}{B_\xi}}\frac{\sigma_s}{\sigma_{z0}}\left(\sqrt{1+\frac{\pi\Gamma_d}{B_\xi}\left(\frac{\sigma_s}{\sigma_{z0}}\right)^2(\Gamma_i/\Gamma_d-1)^2}+\sqrt{\frac{\pi\Gamma_d}{B_\xi}}\frac{\sigma_s}{\sigma_{z0}}(\Gamma_i/\Gamma_d-1)\right), \qquad (18)$$

where the residual oscillation rms without the feedback, σ_{z0}, is given by Eq. (17).

Taking from [1] that $\Gamma_i/\Gamma_d = 4\pi$ and setting the sensor bandwidth in Eq. (18) to one synchrotron frequency, $B_\xi = \omega_s$, we obtain the final result shown in Fig. 3. It plots the beam arrival position jitter vs. the feedback sensor noise for representative levels of RF phase noise.

The figure illustrates that, e.g. for σ_ϕ= 0.1° rms RF noise level, the sensor noise of 0.11 mm rms (or 0.078° of RF phase), is sufficient to maintain the arrival position jitter less than a factor of two of the 0.33 mm rms crab cavity limit derived earlier. This sensor noise level is a factor of 16 higher than what is measured at NSLS-II [2], so we consider it readily achievable. Separately, similar or better bunch-by-bunch feedback BPM noise levels have been demonstrated elsewhere [9]. If more stringent RF phase jitter specs can be achieved in the ESR, the requirement for the feedback sensor noise will be even more relaxed.

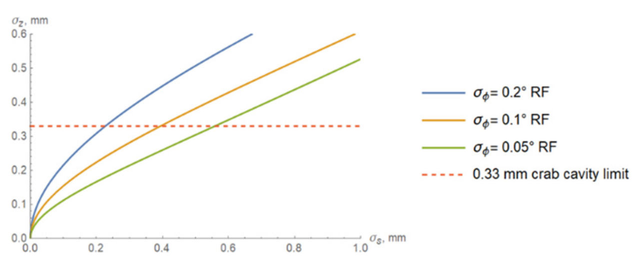

Figure 3: Arrival position jitter vs. feedback sensor noise for several rms values of RF phase noise, σ_ϕ. Bandwidth of [0.5 1.5] ω_s is assumed for both the RF and sensor noise.

CONCLUSION

To avoid unacceptable hadron emittance blow-up, the electron bunch arrival position jitter in the crab cavities must be maintained below 0.33 mm rms. This specification can be met in the presence of longitudinal coupled bunch instability by using a feedback damper. Our analysis predicts fairly relaxed specs for the RF phase noise (achieved at NSLS-II and elsewhere) as well as for the maximum allowable feedback sensor noise (also achieved). In addition to damping the unstable mode(s) the feedback will greatly reduce the amplitude of the (stable) m = 0 mode that usually dominates the noise in the longitudinal plane.

An alternative option is to passively damp the instability (through the cavity damper redesign) but this will also require meeting much more challenging RF phase noise specs. This option is a seemingly inferior one.

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB254

REFERENCES

[1] J. Beebe-Wang, "Electron Ion Collider: Conceptual Design Report", Brookhaven National Laboratory, Upton, NY, USA, Feb. 2021. https://www.bnl.gov/ec/files/EIC_CDR_Final.pdf

[2] B. Podobedov and M. Blaskiewicz, "Limiting Coherent Longitudinal Beam Oscillations in the EIC Electron Storage Ring", Brookhaven National Laboratory, Upton, USA, Rep. IC-APG-TN-001, Apr. 2020.

[3] M. Blaskiewicz, "Beam-Beam Damping of the Ion Instability", in *Proc. North American Particle Accelerator Conf. (NAPAC'19)*, Lansing, MI, USA, Sep. 2019, paper TUPLM11, pp. 391-394.

[4] R. Brinkmann, "Proton Emittance Growth Caused by Electron rf-Noise and the Beam-Beam Interaction in HERA", in *Proc. 1st European Particle Accelerator Conf. (EPAC'88)*, Rome, Italy, Jun. 1988, pp. 657-660.

[5] P. Baudrenghien and T. Mastoridis, "Transverse emittance growth due to rf noise in the high-luminosity LHC crab cavities", *Phys. Rev. ST Accel. Beams*, vol. 18, p. 101001, 2015. doi:10.1103/PhysRevSTAB.18.101001

[6] G. P. Conangla, F. Ricci, M. T. Cuairan, A. W. Schell, N. Meyer, and R. Quidant, "Optimal Feedback Cooling of a Charged Levitated Nanoparticle with Adaptive Control", *Phys. Rev. Lett.*, vol. 122, no. 22, 2019. doi:10.1103/physrevlett.122.223602

[7] S. Y. Lee, *Accelerator Physics*, Singapore: World Scientific, 2011.

[8] D. Teytelman, "Coupled-Bunch Instabilities in Storage Rings and Feedback Systems", in *Synchrotron Light Sources and Free-Electron Lasers*, E. J. Jaeschke *et al.*, Eds. New York, USA: Springer, 2016, pp. 605-628.

[9] D. Teytelman, private communication, 2020.

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2021). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI