# PERIODIC TRANSIENT BEAM LOADING EFFECTS PREDICTED BY A SEMI-ANALYTICAL METHOD

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Abstract

In this paper, we improve a semi-analytical method, which can be not only used for bunch lengthening under equilibrium conditions, but also applied to the prediction of a periodic transient beam loading effect. This periodic transient is induced by the presence of the passive harmonic cavity and might be encountered under specific conditions for an ultra-low emittance storage ring with a higher beam current.

#### INTRODUCTION

The higher harmonic cavities, mostly operated in passive mode, are widely used to stretch the bunches in the fourthgeneration storage ring light sources, in order to improve the beam lifetime and suppress the intra-beam scattering. Up to now there have been several semi-analytical methods [1–3] used to study the bunch lengthening under equilibrium conditions in the presence of harmonic cavities. It was reported that they [1, 2] can not obtain convergent results in some cases, which were attributed to possible instabilities [1], or the failure of the approach itself [2]. We will show later that some of the cases without a convergent solution may indicate a new instability, namely a periodic transient beam loading effect. Because when we use the semi-analytical method [3] to calculate some examples, non-convergent cyclic solutions will appear during the iterations, which are in good agreement with the tracking results. Therefore, our semianalytical method can be used not only for the calculation of bunch profiles under equilibrium state, but also for the prediction of periodic transient effects.

### ALGORITHM MODIFICATION

In Ref. [3], we proposed a self-consistent semi-analytical algorithm which can be used efficiently to study the longitudinal equilibrium density distribution under arbitrary fill patterns. This algorithm mainly includes an iterative loop to calculate the density distributions, as well as a sub-loop to calculate the synchronous phase deviations. With only the consideration of the beam loading effect of the harmonic cavity, this algorithm has been proved to work well for several common fill patterns such as uniform fill, uniformly distributed gaps, a long gap and so on. Recently, we have extended it to include the beam loading effect of the main cavity based on a simple feedback compensation model. This model can be given by the form

$$\widetilde{V}_{g,1} = \widetilde{V}_{g,0} + 0.1 \times (\widetilde{V}_c - < \widetilde{V}_b > -\widetilde{V}_{g,0}), \tag{1}$$

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where  $\widetilde{V}_{g}$  is the generator voltage phasor, and the subscripts 1 and 0 denote the updated and the previous phasors, respectively,  $\widetilde{V}_c$  is the desired cavity voltage phasor,  $\langle \widetilde{V}_b \rangle$  is the average of loading voltage phasors of all bunches. The number 0.1 represents an iterative coefficient used for updating the generator voltage phasor, which is adjustable and needs to be selected properly. Here 0.1 is a suitable value for the HALF to obtain satisfactory results.

With the inclusion of this compensation model, it is found that the semi-analytical algorithm might fail at the calculation of the synchronous phase deviations for the fill pattern with a long gap. In addition, it is also found that the comparison reference value set for the sub-loop is large so that the synchronous phase deviations are actually calculated only one time during each iteration. Nevertheless, still good results are obtained. In order to simplify the semi-analytical algorithm and enhance its robustness, it is necessary to modify the part of algorithm for the synchronous phase deviations:

- The sub-loop is removed.
- During each iteration, the latest synchronous phase deviations are calculated once by the Newton method and then checked for their values, if there is one whose absolute value is obviously large or NaN appears, this round results should be replaced by the previous ones.

After the modifications, it is found that for the case of a long gap, the results obtained by the semi-analytical algorithm are in good agreement with the tracking results.

#### **CALCULATION PARAMETERS**

The main calculation parameters used in this paper are summarized in Table 1 [4]. The energy loss per turn from the bending magnets is 198.8 keV, to keep the momentum acceptance of 5%, the main voltage should be at least 0.85 MV. We assume both the main and the harmonic cavities are superconducting, both of which have R/Q  $\sim 90 \Omega$  (in circuit definition). The main cavity is detuned to keep the load angle close to -20 deg, while the harmonic cavity is detuned to meet the near-optimum lengthening condition [3].

#### PERIODIC TRANSIENT EFFECTS

With the inclusion of the beam loading of main cavity, we study the bunch lengthening and accidentally find the periodic transient effect under the uniform fill pattern. This periodic transient still occurs even with a lower beam current of 300 mA. For the current of 300 mA, the detunings of the main and the harmonic cavities are set to -19.0 kHz and 147.444 kHz, respectively. We will first present the periodic transient results calculated by the semi-analytical method,

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Table 1: The Parameters of HALF Used for Calculation

Parameter	Value	Unit
Circumference	480	m
Beam energy	2.2	GeV
Harmonic number	800	
Nominal beam current	350	mA
Longitudinal damping time	22	ms
Momentum compaction	$8.1 \times 10^{-5}$	
Natural energy spread	$6.45 \times 10^{-4}$	
Energy loss per turn	198.8	keV
Voltage of MC	0.85	MV
<i>RoverQ</i> of MC	90	Ω
Loaded quality factor of MC	40350	
Load angle of MC	-20	deg
<i>RoverQ</i> of HHC	90	Ω
Quality factor of HHC	$2 \times 10^{8}$	

then benchmark them against the corresponding tracking results. Note that a lower quality factor of  $5 \times 10^5$  instead of the true value of  $2 \times 10^8$  is taken to accelerate the convergence of the tracking results.

#### Periodic Semi-Analytical Results

Figure 1 shows the results obtained by the semi-analytical method. We can see that both RMS lengths and centroid positions vary with the bunch number, although the fill pattern is uniform. For the results of  $4000^{th}$  iteration, some bunches around the bunch 600 have RMS lengths more than 30 ps, which means they are over-stretched, as shown in Fig. 2. The longest RMS length reaches up to 70 ps, which signifies some bunches are so over-stretched that two sub-bunches are separately formed. Most bunches have RMS lengths ranged in 15-20 ps, which means they are less-stretched.

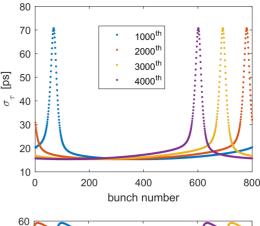
According to the results distribution at  $1000^{th}$  –  $4000^{th}$ iterations, we can know that the results are non-convergent and vary periodically with the iteration number. This phenomenon indicates a periodic transient effect. When it happens, all bunches can not reach equilibrium and their RMS lengths vary in the range of 15-70 ps with time.

Note that our semi-analytical algorithm builds on the basis of the equilibrium state. Therefore, whether this nonequilibrium state that predicted by it is correct remains to be further checked. Next, we will use the tracking method to study this non-convergent case.

#### Benchmark Against the Tracking Results

We use the STABLE code [5] to do the tracking simulation for the case mentioned above. Every bunch is represented by a collection of 4000 macro-particles. In order to see the periodic phenomenon obviously, the total tracking turns are set to 5 million.

Figure 3 shows the tracking results varied along the bunch number at tracking turns of  $3000000^{th}$ ,  $4000000^{th}$ and  $5000000^{th}$ , respectively. It can be seen that with the tracking turns, the whole results shift to the left along the



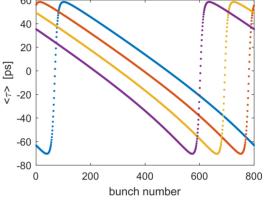


Figure 1: RMS length (top) and centroid position (bottom) vs. bunch number, calculated by the semi-analytical method, at 300 mA and in uniform fill pattern. The results of  $1000^{th}$  $2000^{th}$ ,  $3000^{th}$  and  $4000^{th}$  iterations are displayed.

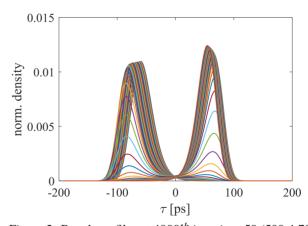


Figure 2: Bunch profiles at  $4000^{th}$  iteration. 50 (500:4:700) bunches are chosen to be displayed.

bunch number, which is consistent with that shown in Fig. 1. This variation phenomenon reflects a periodicity. According to that the result distribution translated about 20 bunches from  $3000000^{th}$  to  $5000000^{th}$  tracking turns, we know that the period for one cycle takes about 40 million turns, namely about 64 seconds.

Compared with that shown in Fig. 1, it is clear that the results given by both methods are in good agreement.

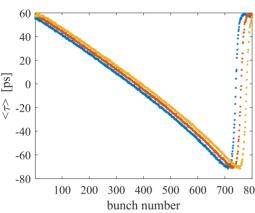


Figure 3: RMS length (top) and centroid position (bottom) vs. bunch number, obtained by the tracking code [3], at 300 mA and in uniform fill pattern.

So the periodic transient effect, which corresponds to a non-equilibrium state, can be predicted well by our semianalytical algorithm.

In order to show more clearly the bunch variations in the tracking process, we present the results varied with the turns, as can be seen in Fig. 4. The variation of bunch centroid is relatively fast from the negative to the positive value, and conversely, it is relatively slow. Furthermore, during the process from the negative to the positive, the corresponding bunch length will be stretched up to about 70 ps first and then shortened low to about 15 ps.

## **CONCLUSION**

In this paper, we presented a periodic transient beam loading effect which might be encountered by HALF with bare lattice parameters under the current of 300 mA and the uniform fill pattern. It was shown that our semi-analytical algorithm can predict this periodic transient effect well. Both the semi-analytical and tracking results are consistent. For future work, we will go deep into the periodic transient effect and analyze the conditions for its occurrence.

#### ACKNOWLEDGEMENTS

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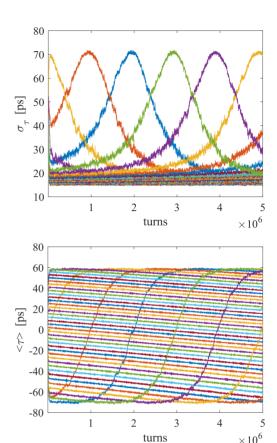


Figure 4: RMS length (top) and centroid position (bottom) vs. turns, obtained by the tracking code [5], at 300 mA and in uniform fill pattern. 40 (1:20:800) bunches are chosen to be displayed.

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