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# TRANSVERSE IMPEDANCE OF LOSSY CIRCULAR METAL-DIELECTRIC WAVEGUIDES

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*Abstract*

The properties of the transverse impedance of a dielectric-loaded metallic circular waveguide are investigated with special attrition to losses in the outer metallic pipe and in the inner dielectric layer. The dispersion curves, impedances and wake functions for dipole TM modes are analysed and compared for thin and thick dielectric layer cases. The correspondence of the resonant frequencies of the longitudinal monopole and the transverse dipole impedances is established.

## INTRODUCTION

The application of two-layer metal-dielectric compounds as accelerating structure [1, 2] and as radiators for the generation of intense wakefield radiation [3], especially in the THz frequency range, are recognized as promising areas in which intensive theoretical and experimental research is being carried out.

For simplicity, theoretical studies of the radiation characteristics of metal-dielectric structures are often limited to ideal metallic waveguides covered on the inside with a lossless dielectric layer. Meanwhile, we have demonstrated the fundamental importance of the finite conductivity of the metallic waveguide and of losses in the dielectric layer, by means of a comprehensive study of the longitudinal impedances of such structures [4]. Ignoring those leads to a rough interpretation of the radiation processes, and to a misinterpretation of many regularities.

Obviously, transverse effects in a two-layer structures demand similar detailed considerations. In this report, an attempt is made to comprehensively study the regularities in the transverse characteristics of the radiation of a two-layer metal-dielectric waveguide, such as the transverse impedances and wake functions, as well as the regularities of the dispersion curves as the losses are varied. In combination with similar results obtained in [4] for the longitudinal characteristics of radiation, an integral understanding of how to optimize the mechanical and electromagnetic characteristics of a waveguide for an application (generation of single-mode radiation, spatial or energy modulation of a bunch [5], creation of superradiant [3] and narrow directed monochromatic radiation sources, etc.) can be obtained.

## THE PROBLEM

A cylindrical metal waveguide with an internal dielectric coating is considered (Fig.1). It is advisable to study the transverse characteristics of radiation for the same waveguide parameters as they were assumed in the study of the longitudinal characteristics in [4]. Thus, the inner radius of the waveguide is $a_1 = 2$ mm, the real component, $\varepsilon_1'$, of the relative dielectric constant $\varepsilon_1 = \varepsilon_1' + j\varepsilon_1''$ of the inner dielectric coating is assumed to be 10 and a highly conductive metal of the outer wall (copper with a conductivity $\sigma = 58 \cdot 10^6$ Ω⁻¹ m⁻¹) is assumed. The properties of the radiation characteristics are compared for a relatively thick $(d = a_2 - a_1 = 200\ \mu\text{m})$ and sufficiently thin $(d = 2\ \mu\text{m})$ dielectric layer. Values assigned to the imaginary part of the dielectric constant $\varepsilon_1'' = 0, 0.1, 0.5, 3$: range from 0 (no losses) to 3 (significant losses). The outer wall is unbounded $(a_3 \to \infty)$.

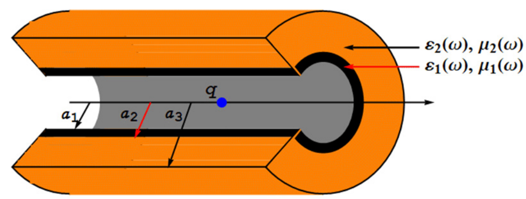


Figure 1: Metal-dielectric waveguide.

The transverse impedances were obtained with an algorithm developed in [6] for a multilayer cylindrical waveguide. A similar algorithm was developed for calculating the transverse eigenvalue $\nu_{n,i}(\omega)$ ($n = 0$ for the longitudinal and $n = 1$ for the transverse impedances, $i$ denotes the resonance number) of hybrid TM and TE eigenmodes of a multilayer waveguide. It is used to construct dispersion curves. The wake functions presented in the article were calculated by a numerical Fourier transform.

## IMPEDANCES AND WAKES

In case of an ideally conducting outer wall and the absence of an imaginary part of the dielectric constant of the inner layer, both longitudinal and transverse impedances develop singularities with an infinite number of delta-like resonances. The wake function turns into a cosine-like nondecaying function at an arbitrary value of the dielectric constant and arbitrary thickness of the inner layer. The canonical form with commensurate real and imaginary components of the impedance is acquired only in the presence of losses in at least one of the layers (finite conductivity of the metal or losses in the dielectric). A realistic attenuation in both components of the two-layer structure will thus be considered in the following.

Figures 2 and 3 show the real part of the transverse impedances (top) and the wake functions (bottom) for thick (Fig. 2, $d = 200\ \mu\text{m}$) and thin (Fig. 3, $d = 2\ \mu\text{m}$) dielectric layers. In the first case (Fig. 2, top) with a low damping $(\varepsilon_1'' = 0, 0.5)$ the impedance contains numerous resonances. As can be seen from the shape of the wake

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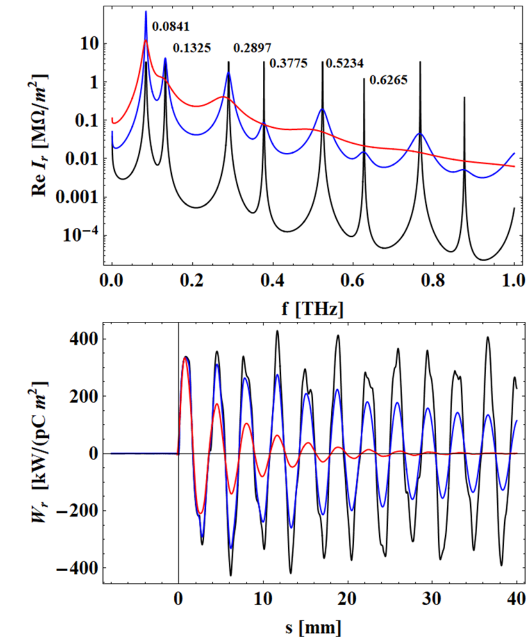


Figure 2: Impedance (top) and wake function (bottom) for the case of thick $(d = 200\mu m)$ inner layer: $\varepsilon_1'' = 0$ (black), 0.5 (blue) and 3 (red).

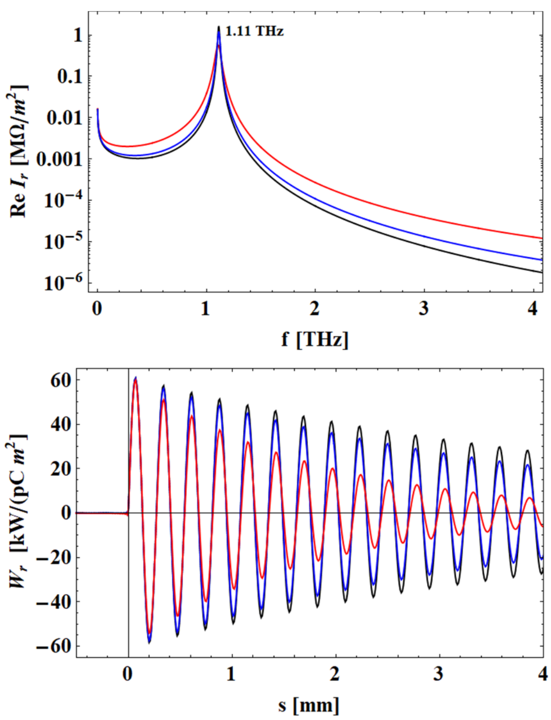


Figure 3: Impedance (top) and wake function (bottom) for the case of a thin $(d = 2\mu m)$ inner layer: $\varepsilon_1'' = 0$ (black), 0.5 (blue) and 3 (red).

function (Fig. 2, bottom), the first few resonances are involved in its formation. The transverse wake function is non-monochromatic in this case. With a significant damping ($\varepsilon_1'' = 3$), the impedance contains also three notable resonances, but the corresponding wake function seems quasiperiodic, which indicates its monochromaticity: it can be assumed that radiation is formed mainly due to the first resonant frequency. Obviously, only modes with synchronous frequencies contribute to the wake function. The analysis of the dispersion curves performed in the next section allows to distinguish synchronous and non-synchronous modes. The contributions of each synchronous mode to the wake function in the case of a thick inner layer will also be highlighted.

In the case of a thin dielectric layer (Fig. 3) the impedance features a single resonant frequency in all considered cases. Accordingly, the wake function has a quasiperiodic character, which indicates the participation of a single resonant frequency in the formation of the field and ensures the monochromaticity of the radiation.

## DISPERSION CURVES

In contrast to the longitudinal case [4], both hybrid $TM_{11}$ and $TE_{11}$ eigenmodes of a two-layer waveguide are involved in the formation of the transverse characteristics of radiation. As in the longitudinal case, the eigenmodes of the resonance frequencies have a phase velocity equal to the speed of light $c$, i.e. they propagate synchronously with the emitting particle. The dispersion curves are given by:

$$\Delta k(\omega) = \omega/c - Re\sqrt{\omega^2/c^2 - \nu_{n,i}^2(\omega)} \quad (1)$$

The synchronous frequencies correspond to the zero crossing of this function which is displayed in Fig. 4 for the case of a thick dielectric layer with low losses.

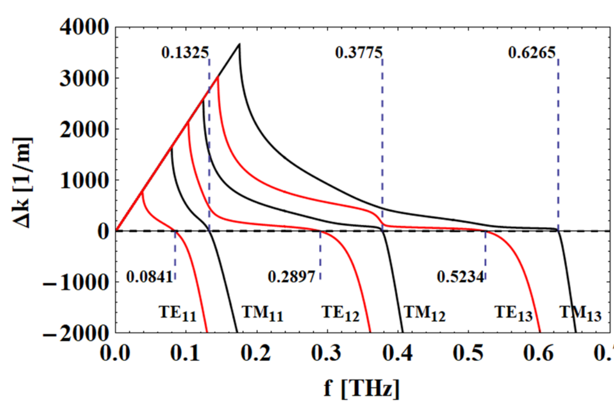


Figure 4: Overlapping dispersion curves for $\varepsilon_1'' = 0$ and 0.1 for a thick inner layer. The resonant frequencies are given for the lossless dielectric.

The curves of the first six modes overlap despite the variation of the damping and the frequencies, which satisfy Eq. (1), coincide with the resonant frequency of one of the transverse impedance resonances (Fig. 2, top). Thus, all modes generated in a thick-coated waveguide have synchronous frequencies, which ensures their interaction with the test particle. Resonant frequencies are generated by alternating TE and TM hybrid modes. Figure 5 shows the contributions of the first four resonances to the total wake function and demonstrates that only the first three resonances contribute significantly.

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In the case of larger losses ($\varepsilon_1'' = 3$) in a thick dielectric layer (Fig. 6) only two TE modes, $TE_{11}$ and $TE_{12}$ (corresponding to the first two resonance peaks in Fig. 2 are synchronous and are thus involved in the formation of the wake function. The predominant influence of the $TE_{11}$ mode (Fig. 7), determines the quasi-monochromaticity of the transverse wake function (Fig. 2, bottom, red).

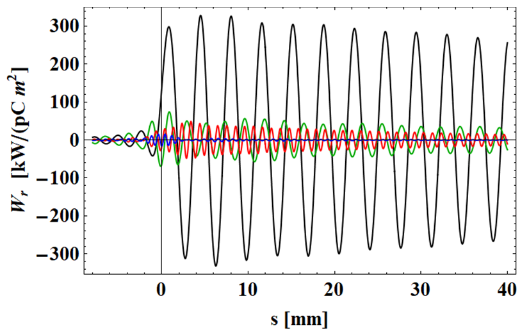


Figure 5: Contributions of the first four resonances (thick dielectric layer, Fig. 2, top) to the total wake function (Fig. 2, bottom): 1) 0.0841 THz, (black); 2) 0.1325 THz (green); 3) 0.2897 THz (red); 4) 0.5234 THz (blue); $\varepsilon_1'' = 0$.

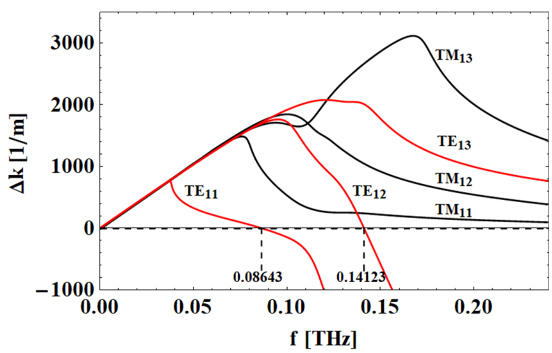


Figure 6: Dispersion curves for $\varepsilon_1'' = 3$, thick inner layer.

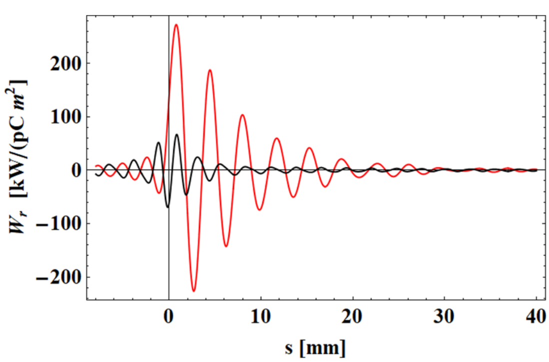


Figure 7: Contributions of the two synchronous TE modes to the wake function (thin dielectric layer, $\varepsilon_1'' = 3$, Fig. 2, top, red) to the total wake function (Fig. 2, bottom, red): 1) 0.08643 THz, (red); 2) 0.14123 THz (black).

With a thin dielectric layer, only a single resonance exists (Fig. 3, top) and the dispersion curves are simplified: in all considered cases of damping, only one $TE_{11}$ mode at $f = 1.11$ THz is synchronous and forms the wake function, as shown in Fig. 8.

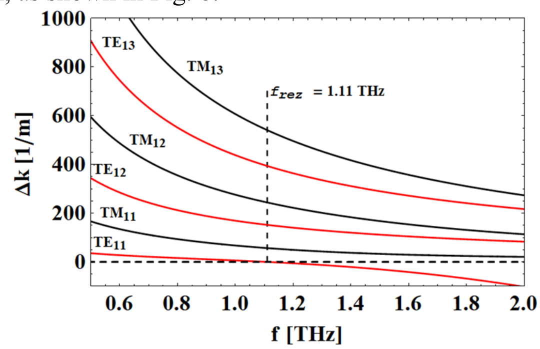


Figure 8: Overlapping dispersion curves for $\varepsilon_1'' = 0$, $0.1, 0.5$ and 3 and a thin inner layer. The resonance frequency is given for a lossless dielectric ($\varepsilon_1'' = 0$).

Comparing the resonance frequencies for longitudinal [4] and transverse wake fields, we note, that transverse fields are formed by both $TM_{1i}$ and $TE_{1i}$ hybrid modes, while only $TM_{0i}$ modes contribute in the longitudinal case. Thus, the number of resonant frequencies is doubled in the transverse case in comparison to the longitudinal case, both for thick and thin layers. The first few frequencies for the longitudinal [4] and the transverse impedances are summarized in Table 1 for comparison.

Table 1: Resonant Frequencies (THz) of Longitudinal $I_{||}$ and Transverse $I_r$ Impedances, $\varepsilon_1'' = 0$, Thick Layer.

| $i$ | $I_{\|\|}, TM_{0,i}$ | $I_r, TE_{1,i}$ | $I_r, TM_{1,i}$ |
|---|---|---|---|
| **1** | 0.0877 | 0.0841 | 0.1325 |
| **2** | 0.2901 | 0.2897 | 0.3775 |
| **3** | 0.5234 | 0.5234 | 0.6265 |

In the case of a thin dielectric layer, the resonances of the longitudinal mode, formed by $TM_{01}$ [4] coincides with the transverse mode resonance, formed by $TE_{11}$: in both cases $f_{res} = 1.11$ THz for $\varepsilon_1'' = 0$. For a thin dielectric layer with low losses and high conductivity the coinciding resonant frequency is approximately determined by the formula

$$f_{res} = \frac{c}{2\pi}\sqrt{\frac{\varepsilon_1'}{\varepsilon_1'-1}\frac{2}{a_1 d}}, \qquad (2)$$

for a perfectly conducting wall with a lossless dielectric layer.

## CONCLUSION

This work is devoted to the development of a technique for studying the transverse characteristics of the radiation of a particle in a metal-dielectric waveguide. In combination with the work in [4], where its longitudinal characteristics are considered, it provides an opportunity for a comprehensive study of radiation characteristics with the inclusion of losses in the metal wall and in the dielectric layer, which is necessary for the development of new applications.

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