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COHERENT EXCITATIONS AND CIRCULAR ATTRACTORS IN COOLED ION BUNCHES*

S. Seletskiy†, A.V. Fedotov, D. Kayran
Brookhaven National Laboratory, Upton, USA

Abstract

In electron coolers, under certain conditions, a mismatch in either γ-factors or trajectory angles between an electron and an ion beam can cause formation of a circular attractor in the ion beam phase space. This leads to coherent excitations of the ions with a small synchrotron or betatron amplitude and results in an unusual beam dynamics, including bifurcations. In this paper we consider the effect of coherent excitations and discuss its implications both for Low Energy RHIC Electron Cooler (LEReC) and for high energy electron coolers proposed for the Electron Ion Collider (EIC).

THEORY OF COHERENT EXCITATIONS

Electron cooling [1, 2] is an established method of increasing the phase space density of bunches of heavy particles through their interaction with copropagating electron beam. Ions interact with electrons in a straight section of the ion storage ring, called the cooling section (CS). In the CS dynamical friction [3] created by the "cold" e-beam reduces a velocity spread of the ion bunch.

In a beam-frame, the friction force acting on an ion is given by [3, 4]:

$$\vec{F} = -C_0 \int \frac{\vec{v}-\vec{v_e}}{|\vec{v}-\vec{v_e}|^3} f(v_e) d^3 v_e, \qquad (1)$$

where $C_0 = \frac{4\pi n_e e^4 Z^2 L_C}{m_e}$, e and m_e are electron charge and mass, n_e is electron bunch density in the beam frame, Z is ion's charge number, L_C is Coulomb logarithm, v_e and v are respectively electron and ion velocities in the beam frame and $f(v_e)$ is the electron bunch velocity distribution.

Each component (F_x, F_y, F_z) of the friction force (1) can be represented by a 1-D integral, which substantially simplifies numerical studies of the beam dynamics. To obtain such expressions (called Binney's formulas [5]) we assume $f(v_e)$ to be Gaussian:

$$f(v_e) = \frac{1}{(2\pi)^{3/2}\Delta_x\Delta_y\Delta_z} \exp\left(-\frac{(v_{ex}-\mu_x)^2}{2{\Delta_x}^2} - \frac{(v_{ey}-\mu_y)^2}{2{\Delta_y}^2} - \frac{(v_{ez}-\mu_z)^2}{2{\Delta_z}^2}\right)$$

where $\Delta_{x,y,z}$ are the horizontal, vertical, and longitudinal velocity spreads and $\mu_{x,y,z}$ are respective velocity offsets. We introduce an effective potential in a velocity-space:

$$U = C_0 \int \frac{f(v_e)}{|\vec{v}-\vec{v_e}|} d^3 v_e \,,$$

such that $F_{x,y,z} = \partial U / \partial v_{x,y,z}$.

After a few algebraic manipulations [6] we obtain:

$$\begin{cases} F_{x,y} = -C(v_{x,y} - \mu_{x,y}) \int_0^\infty g_t(q) dq \\ F_z = -C(v_z - \mu_z) \int_0^\infty g_z(q) dq \end{cases}. \qquad (2)$$

Here $C = 2\sqrt{2\pi} n_e {r_e}^2 m_e c^4 Z^2 L_C$, it was assumed that $\Delta_x = \Delta_y \equiv \Delta_t$ and

$$g_t(q) = \frac{\mathrm{E}}{{\Delta_t}^2 (1+q)^2 \sqrt{{\Delta_t}^2 q + {\Delta_z}^2}} \,,$$

$$g_z(q) = \frac{\mathrm{E}}{(1+q)\left({\Delta_t}^2 q + {\Delta_z}^2\right)^{\frac{3}{2}}} \,,$$

$$\mathrm{E} = \exp\left[-\frac{(v_x-\mu_x)^2 + (v_y-\mu_y)^2}{2{\Delta_t}^2(1+q)} - \frac{(v_z-\mu_z)^2}{2({\Delta_t}^2 q + {\Delta_z}^2)}\right].$$

Consider an ion motion in one plane (a horizontal one for example) in the presence of the friction force from electron beam. In Courant-Snyder coordinates ($\xi = \frac{x}{\sqrt{\beta_T}}, \zeta = \frac{\alpha_T}{\sqrt{\beta_T}} x + \sqrt{\beta_T}\theta_x$, with α_T, β_T being Twiss parameters and $\theta_x = dx/ds$) the equation of motion is:

$$\xi'' + \xi = \frac{L_{CS}\sqrt{\beta_{CS}}}{m_i c^2 \beta^2} F_x(v_x(\zeta)) \mathbb{C}(\psi) \,. \qquad (3)$$

Here, differentiation is performed with respect to the betatron phase ψ, L_{CS} is the length of the cooling section, β_{CS} is Twiss β-function, m_i is the ion's mass, c is the speed of light, $v_x(\zeta) = \zeta\gamma\beta c/\sqrt{\beta_{CS}}$, γ and β are relativistic factors and the comb function $\mathbb{C}$ is given by:

$$\mathbb{C} = \sum_{n=0}^{\infty} \delta_D(\psi - 2\pi n Q_x) \,, \qquad (4)$$

where δ_D is the Dirac delta function, and Q_x is a horizontal tune of the ion storage ring. In derivation of Eq. (3) we assumed that on its pass through the cooling section the ion gets an angular kick $d\theta = \frac{L_{CS} F_x}{m_i c^2 \beta^2}$, that the ions displacement does not change, and that $\alpha_{CS} = 0$.

We integrate Eq. (3) numerically with an explicit, exactly simplectic, third order method [7]. To accelerate the numerical studies in this section we use a large friction kick and, without a loss of generality, use $1 + \nu_x$ instead of Q_x (where ν_x is a partial tune).

Figure 1 shows an angular kick from the horizontal component of the friction force, normalized by ions' initial angular spread σ_θ, for the electron velocity distribution having μ_x offset.

If the shift μ_x is smaller than the velocity (v_0) at which the first derivative of the friction force changes sign, then the betatron oscillations of ions in the bunch are getting damped (Fig. 2). If, on the other hand, $\mu_x \geq v_0$ then the friction force brings amplitudes of the oscillations of all the

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† seletskiy@bnl.gov

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ions in the bunch to the same value (Fig. 3). This is the effect of coherent excitations [8].

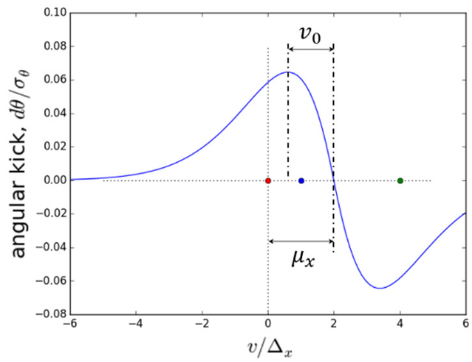

Figure 1: Angular kick in the CS from the horizontal component of the friction force. The red, the blue and the green dots represent initial amplitudes of three "test" ions.

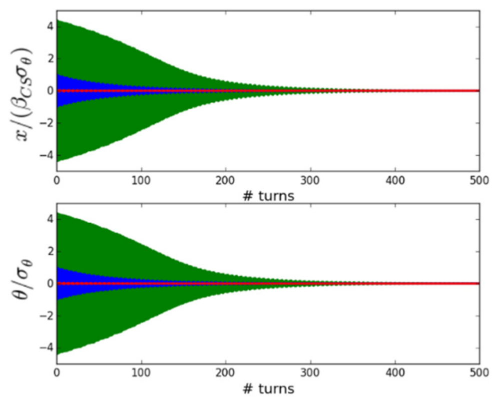

Figure 2: Cooling of the three "test" ions for $\mu_x < v_0$.

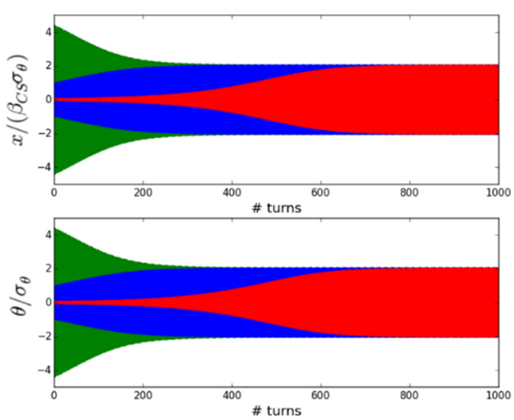

Figure 3: Cooling of the three "test" ions for $\mu_x \geq v_0$.

The coherent excitations effect is possible because the friction force is a non-monotonic function of the ion velocity. Figure 4 explains how the coherent excitations occur.

The electron bunch with the coherent offset in its v-distribution creates a friction force that causes an excitation of the ion's oscillations for ions with $v \in [0, \mu_x]$. We denote the friction force in this velocity range as F_+. The same ion going through the cooling section when its velocity $v < 0$ will experience a damping force F_-. For $\mu_x \geq v_0$ there is a range of velocities for which $F_+ > F_-$. Therefore, the average friction force acting on an ion with the amplitude belonging to this range is an exciting one. For ions with velocities outside of this range the friction force is a damping one.

Hence, instead of driving the amplitudes of ion oscillations to zero the friction force with a sufficient enough offset makes all the ions to oscillate with the same non-zero amplitude.

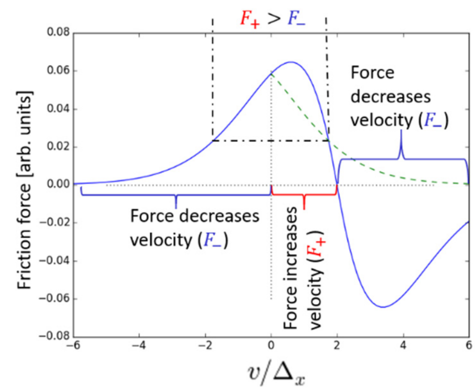

Figure 4: Friction force for $\mu_x \geq v_0$. The green dashed line is a vertical reflection of the friction force for $v < 0$.

In other words, for $\mu_x \geq v_0$ the friction force creates a circular attractor in the phase space and pulls all the ions to it.

IMPLICATIONS FOR LEReC AND OTHER COOLERS

LEReC [9-15] is the world's first electron cooler utilizing RF based acceleration of electron bunches. It is the first cooler applied directly to the ions in the collider at top energy and it is the first cooler that utilize a non-magnetized electron beam.

The LEReC accelerator consists of a 375 keV photo-gun followed by the SRF Booster, which accelerates the beam to 1.6-2 MeV, the transport beamline, the merger that brings the beam to the two cooling sections (in the Yellow and in the Blue RHIC rings), the cooling sections (CSs) separated by the 180° bending magnet and the extraction to the beam dump. The LEReC layout is schematically shown in Fig. 5.

LEReC design parameters are chosen so that electron bunch transverse and longitudinal root mean square (rms) velocity spread is equal to respective ion velocity spread.

Requirements to electron bunch relative momentum spread and angular spread in the laboratory frame are: $\sigma_\delta = 5 \cdot 10^{-4}$ and $\sigma_\theta = 150\ \mu$rad.

By design LEReC cooling time is several minutes and the IBS-driven growth rate is on average about 70% of the cooling rate during an operational RHIC store. In a typical non-relativistic (with $\gamma \lesssim 1.5$) cooler the cooling rate is much larger than the rate of diffusive processes in the ion beam. This difference in operational parameters of LEReC and non-relativistic coolers defines the different response to the presence of a circular attractor.

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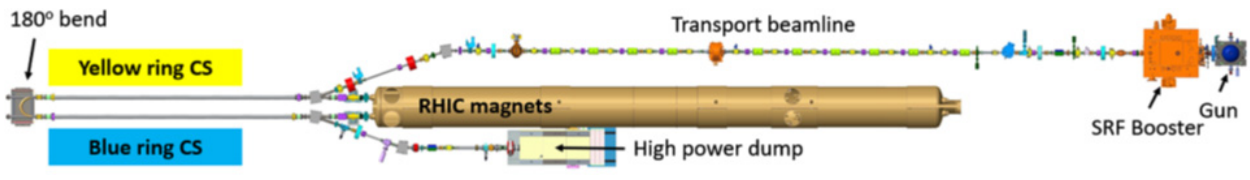

Figure 5: LEReC layout.

Figure 6 shows the effect of the circular attractor for a case of the rms diffusive kick over one turn being five times smaller than $d\theta$. Here the ions form a “doughnut” shape in the phase space and its projection on the physical space results in a “two-hump” density distribution of the ions. Examples of experimental observations of this effect can be found in [16].

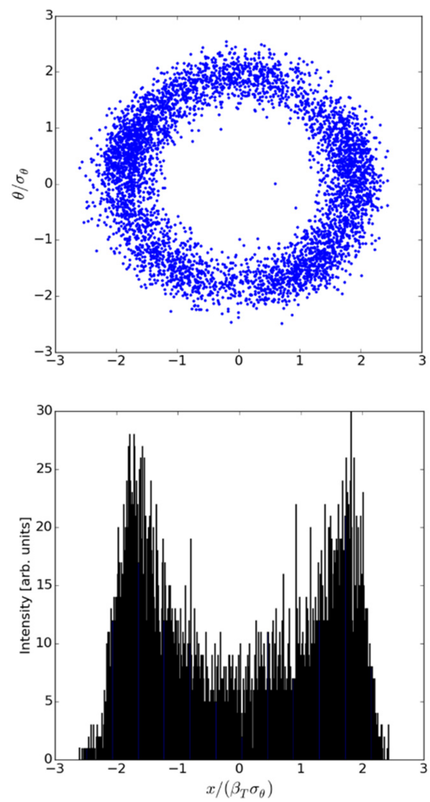

Figure 6: Ion bunch distribution in the presence of circular attractor when the diffusive heating is five times weaker than the cooling.

If, for the same case, the coherent velocity offset is introduced after the ion bunch has been precooled to a small emittance one can observe the effect of density bifurcations [17].

For a case of the rms diffusive kick being just half as strong as the cooling kick in the CS, the presence of the circular attractor is masked by the diffusion (Fig. 7). Then, qualitatively the coherent offset in the v-distribution of electron bunch behaves as an additional heating.

Such a “heating” was observed in LEReC several times in the longitudinal plane during operations. The LEReC dedicated studies of coherent offsets in both the transverse and longitudinal plane are scheduled for the near future.

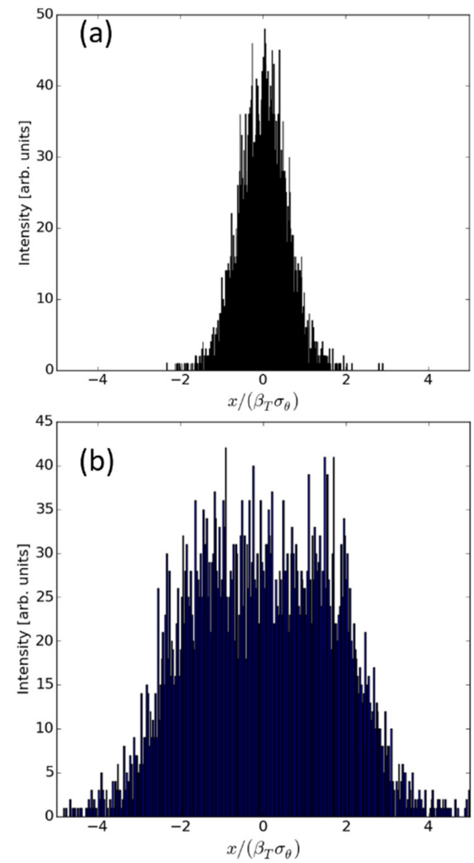

Figure 7: Ion bunch density distribution when the diffusive heating is two times weaker than the cooling for $\mu_x = 0$ (a) and for $\mu_x \geq v_0$ (b).

LEReC approach to the design of electron coolers is directly scalable to high energy applications. For example, preliminary design of the Electron Ion Collider cooler [18], which provides required cooling of protons at $\gamma = 25.4$ and $\gamma = 43.7$, is based on LEReC-type cooling. For this cooler's design parameters, it was found that $\theta_0 = \frac{v_0}{\gamma\beta c} =$ 25 µrad. Hence, the effect of coherent excitations sets 25 µrad requirement to the angular alignment of the electron and proton trajectories in the respective cooling section.

CONCLUSION

A set of formulas useful for fast simulations of the effect of coherent excitations in the electron coolers was derived.

Numerical studies of this effect in LEReC were performed and compared to the coherent excitations in a typical non-relativistic electron cooler.

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