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LOSS OF TRANSVERSE LANDAU DAMPING BY DIFFUSION IN HIGH-ENERGY HADRON COLLIDERS*

S. V. Furuseth†[1], X. Buffat, CERN, 1211 Geneva 23, Switzerland
[1]also at EPFL, 1015 Lausanne, Switzerland

Abstract

Circular hadron colliders rely on Landau damping to stabilize the beams. Landau damping depends strongly on the bunch distribution, which is often assumed to be Gaussian in the transverse planes. In this paper, we introduce and explain an instability mechanism observed in the LHC, where Landau damping is eventually lost due to a diffusion that modifies the transverse bunch distribution. The mechanism is caused by a wide-spectrum noise that excites the transverse motion of the beam, which consequently produces wakefields that drive a narrow-spectrum diffusion. It is shown that this diffusion efficiently lowers the stability diagram at the frequency of the least stable coherent mode, leading to a loss of Landau damping after a latency. A semi-analytical model agrees with measurements in dedicated latency experiments performed in the LHC. This instability mechanism explains the need for a stability margin in octupole current in the LHC, relative to the amount needed to stabilize a Gaussian beam. We detail the impact of this mechanism and possible mitigations for the LHC and HL-LHC.

INTRODUCTION

The linear Vlasov equation has been invaluable in both studying the destabilizing impact of wakefields and quantifying the amount of transverse detuning needed to ensure beam stability in high-energy hadron colliders. However, there are observations that have not been explained by the linear theory. Of consequential interest, was the trend that the Landau octupole current for operation of the Large Hadron Collider (LHC) [1] during Run 2 was about a factor 2 larger than the predicted threshold [2]. Of diagnostic interest, were the observations of instabilities with high latencies in the LHC, both in regular operation [3] and in dedicated experiments with artificial noise introduced on the beam [4]. Such observations suggested that the culprit was a mechanism that caused a slow evolution of the beam distribution.

The mechanism causing the loss of Landau damping has been identified as diffusion driven by noise excited wakefields, which is explained further in the following. This mechanism was first modeled numerically in [3, 5]. It was recently also modeled analytically [6–10]. One analytical model [6–8] was able to reproduce the latencies in the dedicated experiment that is illustrated in Fig. 1.

NOISE EXCITED WAKEFIELDS

The beam is initially stable, meaning that infinitesimal excitations will be damped by decoherence. However, noise excites the beam to nonzero oscillation amplitudes, whereupon it acts back on itself through wakefields. The wakefields amplify the stochastic excitation of the beam by noise.

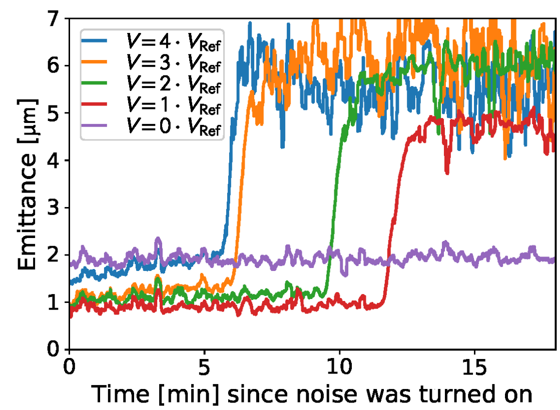

Figure 1: Evolution of normalized horizontal emittance for 5 bunches in beam 2 during a dedicated latency experiment in the LHC [4]. The bunches were affected by linearly spaced external noise amplitudes, given by the legend, until they became unstable after a latency. Courtesy of [11].

The impact of noise excited wakefields can be modeled by a perturbed Hamiltonian $\mathscr{H} = \mathscr{H}_0 + \Delta\mathscr{H}$ as in [12]. The equilibrium Hamiltonian $\mathscr{H}_0$ models e.g. the synchrotron motion with tune Q_s and the detuning by octupole magnets

$$\Delta Q_x = a_x J_x + b_x J_y, \tag{1}$$

where J are the transverse actions. The perturbation models both the horizontal noise and wakefields

$$\Delta\mathscr{H} = -\frac{x F_x^{\mathrm{coh}}}{p_0} = -\frac{x}{p_0}\left(F_x^{\mathrm{noise}} + F_x^{\mathrm{wake}}\right), \tag{2}$$

where $p_0 = \gamma m_0 v$ is the momentum of the particle.

By assuming weak wakefields without coupling between modes of different azimuthal mode numbers, the effective wake force averaged over time can be written [7, 10]

$$\begin{aligned} F_x^{\mathrm{wake}}(r_z,\phi;t) &= -\frac{p_0}{\beta_x}\sum_m 2\omega_0\Delta Q_m^{\mathrm{coh}}\left\langle x(r_z,\phi;t)\right\rangle_m \\ &\equiv -\frac{p_0}{\beta_x}\sum_m 2\omega_0\Delta Q_m^{\mathrm{coh}}\chi_m(t)m_m(r_z,\phi), \end{aligned} \tag{3}$$

where β_x is an effective optical beta-function, ω_0 is the (angular) revolution frequency, $\langle x(r_z,\phi;t)\rangle_m$ is the horizontal offset of mode m as a function of the longitudinal coordinates, and $\Delta Q_m^{\mathrm{coh}}$ is the coherent tune shift of the mode driven by the wakefields. The transverse offset has been decomposed into a mode amplitude $\chi_m(t) \propto \exp\left[-i\omega_0(Q_{x0} + l_m Q_s + \Delta Q_m^{\mathrm{coh}})t\right]$ and normalized mode function $m_m(r_z,\phi)$. The mode is stable if the tune shift $\Delta Q_m^{\mathrm{coh}}$ is inside the stability diagram as in Fig. 2.

* Research supported by the HL-LHC project
† s.v.furuseth@gmail.com

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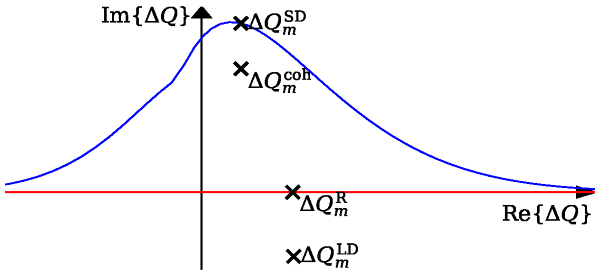

Figure 2: Tuneshifts: The blue line is the stability diagram ΔQ^{SD}. The red line are free real tunes, ΔQ^{R}. Each ΔQ^{SD} corresponds to a ΔQ^{R}, given by the dispersion integral [13]. Similarly, the tune shift of the wakefield mode $\Delta Q_m^{\mathrm{coh}}$ is in the underdamped stochastic harmonic oscillator (USHO) theory approximated to correspond to a single Landau damped tune shift ΔQ_m^{LD}, obtained with a Taylor expansion from ΔQ_m^{SD} into the interior of the stability diagram. The point ΔQ_m^{SD} has the same real part as $\Delta Q_m^{\mathrm{coh}}$.

The noise, modeled as a single kick per turn, can also be decomposed into the wakefield eigenmode functions m_m as

$$F_x^{\mathrm{noise}} = \frac{p_0}{\beta_x}\xi(z;t), \tag{4}$$
$$\xi(z;t) = \sum_i \xi_i(t)\Xi_i(z) = \sum_i \xi_i(t)\sum_m \eta_{im} m_m(r_z,\phi),$$

where Ξ_i are normalized noise shape functions and η_{im} are noise-mode-moments. Most noise has the shape $\Xi_0 = 1$ over the length of a bunch, while crab cavity (CC) amplitude noise has the shape $\Xi_1 \propto z$. The CCs are not currently installed in the LHC, but are foreseen in the HL-LHC. The larger the noise-mode-moments η_{im} are, the more mode m is excited by noise type i. The sum of $|\eta_{im}|^2$ for all modes with the same azimuthal mode number l_m is displayed in Fig. 3. A mode with $l_m = 0$ and zero radial dependence can have a large η_{0m} and a small η_{1m} at low chromaticities.

The combination of noise and wakefields drives a diffusion that can be modeled by [10, 14, 15]

$$\frac{\partial \Psi_{\mathrm{eq}}}{\partial t} = \frac{\partial}{\partial J_x}\left[J_x D_x \frac{\partial \Psi_{\mathrm{eq}}}{\partial J_x}\right], \tag{5}$$

$$D_x = \sum_m D_{xm}(Q_x) = \sum_m \frac{\sum_i \left|\hat{\xi}_i(Q_x + l_m Q_s)\right|^2 |\eta_{im}|^2}{2\beta_x} \tag{6}$$
$$\times \left|1 - \frac{2\omega_0 \Delta Q_m^{\mathrm{coh}}\hat{\chi}_m(Q_x + l_m Q_s)}{\sum_i \hat{\xi}_i(Q_x + l_m Q_s)\eta_{im}}\right|^2,$$

where the hats (as in $\hat{\chi}$) denote a Fourier transform. Note that the diffusion coefficient is decomposed into a sum of parts due to each mode separately. The diffusion of the individual particles is a function of their tunes Q_x, which depend on the transverse actions as in Eq. (1). If the noise signals are white, the power spectral densities of the noise types are flat and equal to the noise amplitudes squared $|\hat{\xi}_i|^2 = \sigma_{\xi i}^2 \propto \beta_x$ (in units of the beam size).

The key question that remains is how much the wake eigenmodes are excited by the noise. Equivalently to how the

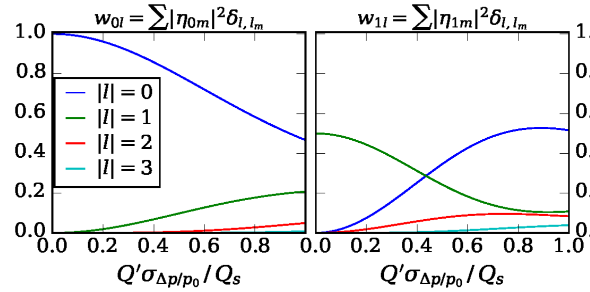

Figure 3: Weights of sidebands excited by rigid noise $\propto \Xi_0$ on the left and headtail noise $\propto \Xi_1$ on the right. The chromatic headtail phase shift changes the weights with linear chromaticity Q'. The wakefields are here neglected.

beam transfer function (BTF) is a measure for the excitation of the rigid beam motion by the noise, the wakefield beam eigenmode transfer function (MTF) is a measure for the excitation of the mode amplitude χ_m by the noise.

Two theories exist for the MTF. The first, which models an almost unstable mode as an USHO, gives [6, 7]

$$\left.\frac{\hat{\chi}_m(Q_x)}{\sum_i \hat{\xi}_i(Q_x)\eta_{im}}\right|^{\mathrm{USHO}} = \frac{1}{2\omega_0\left[\Delta Q_m^{\mathrm{LD}} - \Delta Q_x\right]}, \tag{7}$$

where $\Delta Q_x = Q_x - Q_{x0}$. The second, which models the dynamics of any mode by the Vlasov equation, gives [10]

$$\left.\frac{\hat{\chi}_m(Q_x)}{\sum_i \hat{\xi}_i(Q_x)\eta_{im}}\right|^{\mathrm{Vlasov}} = \frac{1}{2\omega_0\left[\Delta Q_m^{\mathrm{coh}} - \Delta Q^{\mathrm{SD}}(Q_x)\right]}. \tag{8}$$

The difference between these two expressions for the MTF can be appreciated by studying Fig. 2. The derivation of Eq. (8) was based on a similar idea for a rigid mode [9].

LOSS OF LANDAU DAMPING

The diffusion driven by noise excited wakefields related to an almost unstable mode is peaked and narrow in action space. It causes a local flattening of the distribution as the almost resonant particles are evenly distributed in this narrow space [7]. This can lead to a loss of Landau damping. Using the USHO model, one can derive an analytical estimate for the latency of a Gaussian bunch excited by white noise [7]

$$L = \frac{\left|\Delta Q_m^{\mathrm{SD}} - \Delta Q_m^{\mathrm{coh}}\right|}{2.5\,\mathrm{Im}\left\{\Delta Q_m^{\mathrm{SD}}\right\}} \frac{\mathrm{Im}\left\{\Delta Q_m^{\mathrm{LD}}\right\}^4}{a_x^2 J_{\mathrm{eff}}\left|\Delta Q_m^{\mathrm{coh}}\right|^2 \sum_i \sigma_{\xi i}^2 |\eta_{im}|^2} I_{\mathrm{int}}, \tag{9}$$

where $I_{\mathrm{int}} \in [1, 1.25]$ is an integral, J_{eff} is an effective action, and it has been assumed that $|\mathrm{Im}\left\{\Delta Q_m^{\mathrm{coh}}\right\}/\mathrm{Re}\left\{\Delta Q_m^{\mathrm{coh}}\right\}| \ll 1$.

We consider here an illustrative example for the LHC in 2018 with white rigid noise of amplitude $\sigma_{\xi 0} = 10^{-4}\sigma_x$, a linear chromaticity of $Q' = 15$, a feedback giving a damping time of 100 turns, a detuning coefficient a_x that is 25% larger than the initial stability threshold, and a ratio between the detuning coefficients of $b_x/a_x = -0.7$. The total and the largest individual diffusion coefficients are shown in Fig. 4.

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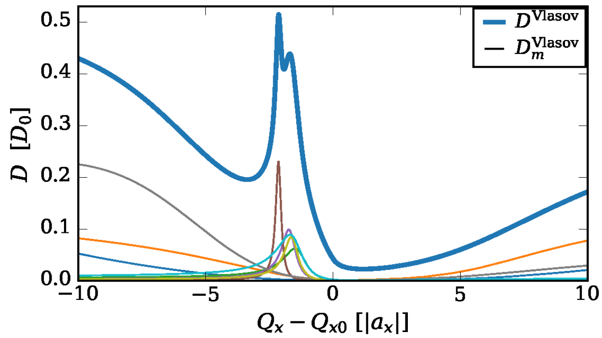

Figure 4: The largest single-mode diffusion coefficients (thin lines) and the total diffusion coefficient (thick line) for an LHC-relevant case, based on the Vlasov MTF in Eq. (8). The unit $D_0 = \sigma_{\xi 0}^2/2\beta_x$ is the flat diffusion coefficient expected without feedback or wakefields. The least stable mode, with a tune shift $\Delta Q_m^{\mathrm{coh}} = -8.4\times10^{-5} + i4.4\times10^{-6}$, has the tallest and most narrow D_m^{Vlasov} (brown thin line).

Table 1: Latencies Found with Different Approaches

Approach	Modes	Latency
Analytical, Eq. (9)	The least stable one	144 s
USHO, simplified [1]	The least stable one	122 s
USHO	The least stable one	42 s
Vlasov	The least stable one	24 s
Vlasov	All	37 s

[1] Same assumptions as when deriving Eq. (9). See [7].

The latency can be found either by Eq. (9) or by numerically solving the diffusion equation self-consistently, which can be done with the code PyRADISE [7, 16]. The latencies found with the different approaches and theories are gathered in Table 1. The evolution of the stability diagram until the beam is unstable is illustrated in Fig. 5.

The Vlasov theory with all the modes is considered the most accurate modeling of the physics. The latency is in this case slightly longer because the diffusion of the more stable modes counteract the flattening driven by the least stable mode. Nevertheless, all the un-simplified approaches give similar latencies, considering how fast the latency scales with different variables, as given by Eq. (9). For example, the same configuration, but with a twice as large stability margin (50%), has approximately 24 times longer latencies.

MITIGATION

The best approach to mitigate this mechanism is to maximize the analytical latency given in Eq. (9). In short, that means maximizing the stability margin and minimizing the impedance, noise, and noise-mode-moments. In addition, since the loss of Landau damping is linked to the local flattening of the distribution, it is preferable to have a minimal in-plane detuning coefficient a_x. To still achieve the necessary amount of detuning with a smaller a_x, one needs to have either a larger cross-plane detuning coefficient b_x or to generate detuning related to the longitudinal action, such as that by RF quadrupoles [17].

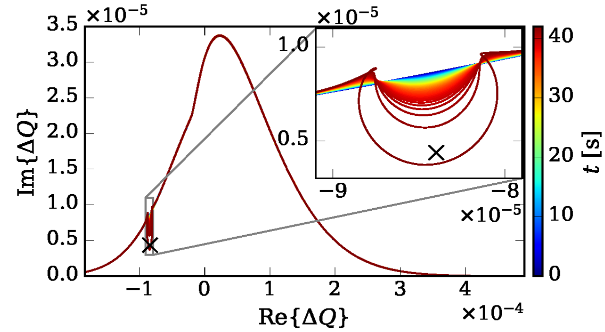

Figure 5: Evolution of the stability diagram due to the least stable mode for an LHC-relevant case, based on the USHO MTF in Eq. (7). When the mode (black ×) is outside the stability diagram, it is unstable. The evolution speeds up towards the end as the mode approaches the stability limit.

A transverse feedback can also mitigate this mechanism by suppressing several of the least stable modes with large η_{0m}, as long as the feedback noise is low enough to avoid driving a worse diffusion with the remaining modes. An optimization of the chromaticity could also maximize the latency. In the LHC, featuring only rigid bunch noise (Ξ_0), an optimal chromaticity was found around 5 units [7, 8], whereas the LHC is usually operated with a higher chromaticity [2]. Modes that are sensitive to CC amplitude noise (Ξ_1) cannot in general by suppressed with a standard transverse feedback. Such modes could lead to instabilities with short latencies in the HL-LHC. To mitigate this effect, it is therefore advisable to avoid enabling CC on non-colliding bunches with a small stability margin. Once in collision, the stability margin is larger thanks to the head-on tune spread [18], such that a significantly higher noise amplitude can be tolerated.

CONCLUSION

In this paper, we have reviewed the current understanding of loss of transverse Landau damping by diffusion in high-energy hadron colliders. The cause is a wide-spectrum noise that excites the transverse motion of the beam, which consequently produces wakefields that drive a narrow-spectrum diffusion. A key question is how much the noise excites the wakefield eigenmodes, modeled through the MTF. Two theories for the MTF have been presented: the simplified USHO theory, which enabled the derivation of an analytical expression for the latency, and the more self-consistent Vlasov theory. Both depend strongly on the noise-mode-moments η_{im}. It is often sufficient to model only the least stable mode when trying to predict the latency of a given configuration.

This destabilizing impact of noise excited wakefields explains the need for a significant stability margin in the LHC. Mitigating the instabilities requires combined efforts on minimizing the noise and impedance, as well as well-informed operational settings to maximize the stability margin and minimize the noise-mode-moments of the least stable modes.

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