

# BEAM DYNAMICS STUDY IN A DUAL ENERGY STORAGE RING FOR ION BEAM COOLING\*

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## Abstract

A dual energy storage ring designed for beam cooling consists of two closed rings with significantly different energies: the cooling and damping rings. These two rings are connected by an energy recovering superconducting RF structure that provides the necessary energy difference. In our design, the RF acceleration has a main linac and harmonic cavities both running at crest that at first accelerates the beam from low energy  $E_L$  to high energy  $E_H$  and then decelerates the beam from  $E_H$  to  $E_L$  in the next pass. The purpose of the harmonic cavities is to extend the bunch length in a dual energy storage ring as such a longer bunch length may be very useful in a cooling application. Besides these cavities, a bunching cavity running on zero-crossing phase is used outside of the common beamline to provide the necessary longitudinal focusing for the system. In this paper, we present a preliminary lattice design along with the fundamental beam dynamics study in such a dual energy storage ring.

## INTRODUCTION

A dual energy storage ring design consists of two loops at markedly different energies, one at low energy and the other at high energy as shown in Fig. 1. These two rings are connected by an energy recovering superconducting RF structures, which provide the necessary energy difference. In our design, the RF system consists of main cavities and harmonic cavities, both running on crest, and a bunching cavity running at a zero-crossing phase that is outside of the common beamline to provide the necessary longitudinal focusing for the system. The main cavity at first accelerates the electron beam from low energy  $E_L$  to high energy  $E_H$  and then decelerates the beam from  $E_H$  to  $E_L$  in the next pass. Harmonic cavities among the main cavities extends the bunch length which helps to get the longer bunch length required for a cooling application. Here, we use a 3<sup>rd</sup> harmonic cavity which provides the desirable bunch length of the order of cm for the better cooling application. Another compensating RF cavity should be used if we include the effects of synchrotron radiation. For the beam dynamics study, we present the preliminary lattice design of dual energy

storage ring as shown in Fig. 2. This idea can be applied equally in the case of multiple energy storage rings [1].

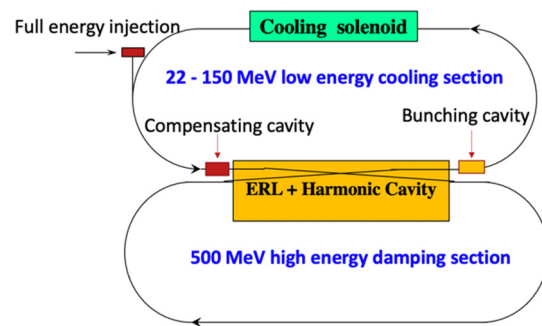


Figure 1: Schematic drawing of a dual-energy storage ring cooler.

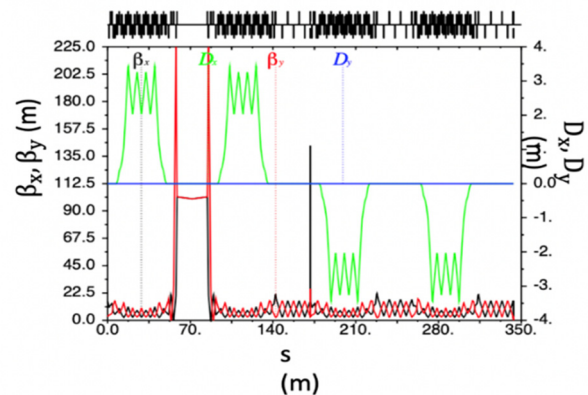


Figure 2: Preliminary optics design of a dual-energy storage ring cooler.

## LONGITUDINAL STABILITY

Some work has already done and presented on longitudinal stability in a dual energy storage ring [2, 3]. In our baseline design, we have the main RF cavities and third harmonic RF cavities adjacent to the main cavities both running on crest. The main cavities accelerate the electron beam from low energy in the low energy ring to a high energy in the high energy ring during one accelerating pass. Use of a third harmonic cavity in our design is to increase the bunch length. In the decelerating pass, the RF phases of both the main cavity and third harmonic cavity is opposite to that of accelerating pass. A bunching cavity

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running on zero-crossing phase is placed outside of the common beamline to provide the necessary longitudinal focusing on the system.

With this RF cavities set-up, we do single particle tracking using ELEGANT [4] starting at the beginning of low energy ring. This ensures that there exists longitudinal stability in a dual energy storage ring.

### DYNAMIC APERTURE

The maximum phase-space area that particles can survive in an accelerator is called the Dynamic Aperture (DA) [5]. Nonlinearities can come from different sources and greatly affects the DA in a storage ring. We introduce sextupoles in our dual energy storage ring to correct the chromaticity. Further explorations on the DA with different sextupoles distributions in a dual energy storage ring system were carried out. The maximum DA is obtained with sextupoles being placed in both low energy ring and high energy ring to compensate the chromaticity. The DA for two set of energies is shown in Fig. 3.

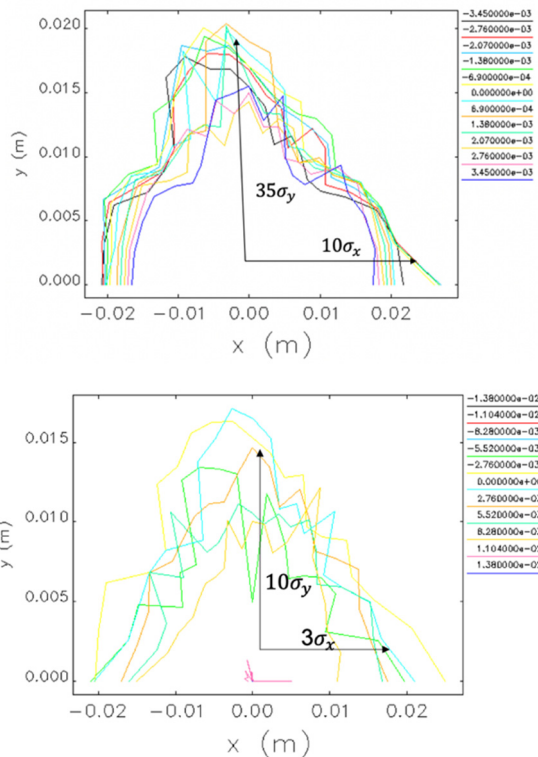


Figure 3: (top) DA for low energy ring at 150 MeV and high energy ring at 500 MeV. (Bottom) DA for low energy ring at 150 MeV and high energy ring at 1000 MeV.

Figure 3 shows that the DA for high energy ring at 500 MeV is bigger than the DA for high energy ring at 1000 MeV. Elegant tracking code has been used to get these DA plots.

### MOMENTUM APERTURE

The Momentum Aperture (MA) is defined as the maximum momentum deviation that a particle can have

without becoming unstable and being lost by colliding with the vacuum chamber of the storage ring [6]. The momentum aperture is determined by the complex 6-dimensional dynamics of the particle. In a dual energy storage ring, we do particle tracking simulation using ELEGANT to determine the momentum aperture.

The momentum aperture run using ELEGANT is carried out starting at the beginning of the Low Energy Ring (LER) and going to the end of the High Energy Ring (HER) as shown in Fig. 4. The total circumference of the dual energy ring is 343.3 m and both LER and HER are of equal length in circumference (i.e., 171.7 m each). We can see that the LER momentum acceptance is bigger than the HER momentum acceptance. We have RF cavities to increase the electron beam energy going from LER to HER, so there exists adiabatic damping [5] of the phase space due to RF acceleration. Hence, the high energy section will have its fractional momentum acceptance reduced by the ratio of the energies if all other parameters are equal [7].

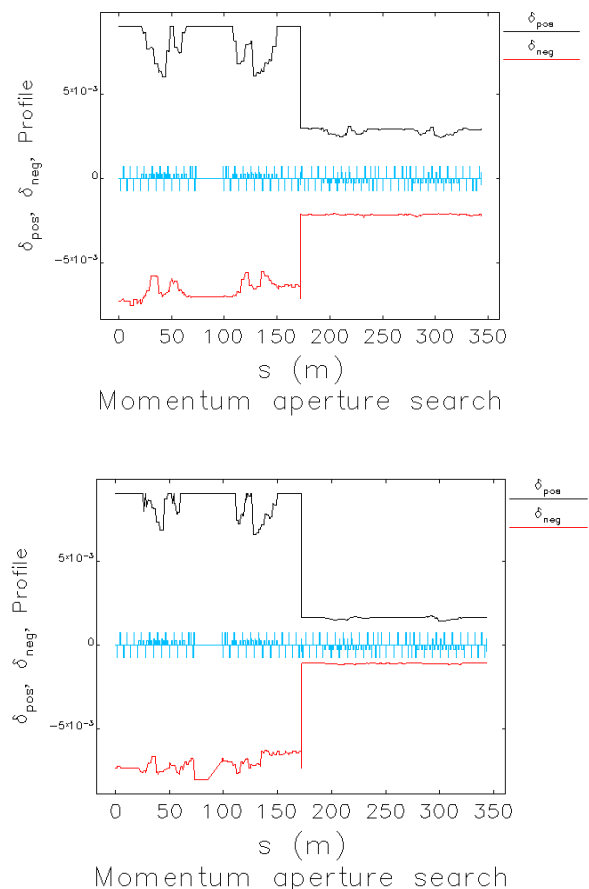


Figure 4: (top) MA for low energy ring at 150 MeV and high energy ring at 500 MeV. (Bottom) MA for low energy ring at 150 MeV and high energy ring at 1000 MeV in the dual energy storage ring cooler.

### TOUSCHEK LIFETIME

Coulomb scattering of charged particles traveling together causes an exchange of momentum between the transverse and longitudinal directions [8]. Touschek

scattering is a phenomenon described by a collision of two electrons inside a bunch with transfer of transverse momentum into longitudinal momentum [9]. When the change of longitudinal momentum exceeds the momentum acceptance of the electron storage ring, both electrons get lost.

The Touschek scatter rate  $\alpha$  and the lifetime  $\tau$  (where lifetime is defined for the beam intensity to decay down half of its initial value) is given by [10, 11]

$$\alpha = \frac{1}{\tau} = \frac{r_e^2 c q}{8 \pi e \gamma^3 \sigma_s} \cdot \frac{1}{C} \oint \frac{F \left( \left[ \frac{\delta_{acc}(s)}{\gamma \sigma_{x'}(s)} \right]^2 \right)}{\sigma_x(s) \sigma_{x'}(s) \sigma_y(s) \delta_{acc}^2(s)} ds. \quad (1)$$

The integration in the formula above is along the circumference of the ring, where  $r_e$  is the classical electron radius,  $q$  the bunch charge,  $\sigma_s$  the rms bunch length, assumed to be constant along the ring,  $C$  the circumference of the ring,  $\sigma_x(s), \sigma_y(s)$  are the rms horizontal and vertical beam radii including the dispersion terms. The variable  $c$  is the speed of light,  $e$  the electron charge,  $\gamma$  the relativistic Lorentz factor of the beam.  $\delta_{acc}(s)$  is the local relative momentum acceptance along the ring which can be printed out using ELEGANT tracking output result. The beam sizes  $\sigma_x(s)$  and  $\sigma_y$  can be calculated using the formula

$$\sigma_x(s) = \sqrt{\epsilon_x \beta_x(s) + (\sigma_\delta \eta(s))^2}, \sigma_y(s) = \sqrt{\epsilon_y \beta_y(s)}$$

with  $\epsilon_x = \frac{\epsilon_{x0}}{1+\kappa}$ ,  $\epsilon_y = \frac{\kappa \epsilon_{x0}}{1+\kappa}$  and there is no dispersion in vertical dimension.

Where  $\epsilon_{x0}$  is the natural horizontal emittance and  $\kappa$  is the emittance coupling factor.  $\eta, \eta'$  are the horizontal dispersion and the slope of horizontal dispersion respectively. There is no dispersion in the vertical dimension. The quantity  $\sigma_{x'}(s)$  is the beam divergence which can be calculated using the formula

$$\sigma_{x'}(s) = \frac{\epsilon_x}{\sigma_x(s)} \sqrt{1 + \frac{H(s) \sigma_\delta^2}{\epsilon_x}}$$

where  $H(s)$  is the chromatic invariant given by

$$H(s) = \gamma_x \eta^2 + 2 \alpha_x \eta \eta' + \beta_x \eta'^2.$$

The special function  $F(x)$  is defined by

$$F(x) = \int_0^1 \left( \frac{1}{u} - \frac{1}{2} \ln \left( \frac{1}{u} \right) - 1 \right) \cdot \exp \left( -\frac{x}{u} \right) du.$$

The Touschek lifetime in the dual energy storage ring is obtained through  $\frac{1}{\tau} = \frac{1}{2\tau_{LER}} + \frac{1}{2\tau_{HER}}$ , where  $\tau_{LER}$  and  $\tau_{HER}$  are the corresponding Touschek lifetime for LER and HER respectively. Similarly, ELEGANT particle tracking uses

Piwinski's formula [8, 11] to compute Touschek lifetime. Table 1 lists the Touschek lifetimes for the dual energy storage ring cooler with two sets of energy combinations in the low and high rings respectively: (150 MeV, 500 MeV) and (150 MeV, 1000 MeV). In a conclusion, the Touschek lifetime in the dual energy storage ring is dominated by the high energy ring in both cases.

Table 1: Touschek Lifetimes in a Dual Energy Storage Ring with Two Sets of Energy Combinations in the Low and High Energy Rings Respectively: (150 MeV, 500 MeV) and (150 MeV, 1000 MeV)

Method	Touschek lifetime (h)					
	Option1			Option2		
	$\tau_{LER}$	$\tau_{HER}$	$\tau$	$\tau_{LER}$	$\tau_{HER}$	$\tau$
Elegant	0.67	0.31	0.42	14.79	3.70	5.92
Formula	0.68	0.23	0.34	13.27	2.88	4.73

In the above Table 1, Option1 has low energy ring at 150 MeV and high energy ring at 500 MeV. Option2 has low energy ring at 150 MeV and high energy ring at 1000 MeV respectively. The number of particles in the bunch is  $6.9 \times 10^{10}$  and the bunch length ( $\sigma_s$ ) used in this calculation is 2.5 cm. The damped equilibrium emittance and energy spread values are used with the emittance coupling factor  $\kappa = 0.05$  in each case. Calculations show that higher the energy value of high energy ring, longer the Touschek lifetime in a dual energy storage ring.

## CONCLUSION

We have developed the dual energy storage ring lattice and particle tracking simulation were carried out with the correct set up of RF structures. With this baseline design, we established that the dual ring is longitudinally stable. Further, particle tracking simulation were carried out to find the corresponding dynamic aperture and momentum aperture in dual energy storage ring. Touschek lifetime calculations were carried out both analytically and using ELEGANT code. All these beam dynamics studies show that the dual energy storage ring is a stable design, and it can be considered as a possible cooler for the future colliders.

## ACKNOWLEDGMENTS

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