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EFFECT OF CHROMATICITY AND FEEDBACK ON TRANSVERSE HEAD-TAIL INSTABILITY*

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Abstract

The head-tail instability caused by the beam interaction with short-range wakefields is a major limitation for the single-bunch beam intensity in circular accelerators. The combined effect of the transverse feedback systems and chromaticity suppressing the instability is discussed. Theoretical and experimental studies of the head-tail instability and methods of its mitigation are reviewed. Results of experimental studies of the transverse mode coupling carried out at NSLS-II are compared with the theoretical model and numerical simulations.

INTRODUCTION

The interaction of a single bunch with short-range transverse wakefields (broadband impedance) results in head-tail instability. The wakefields induced by the head of a bunch act on the particles of its tail; the head and tail of the bunch interchange periodically due to synchrotron oscillations; the instability occurs if certain conditions of resonance excitation exist. Suppression of the single-bunch instability by feedback and chromaticity is discussed in this paper.

Early studies of head-tail instability are summarized in the review [1] presented at the Particle Accelerator Conference in 1969. The results of experimental studies of the feedback performance with varied chromaticity carried out at the ADONE storage ring are compared with analytical estimations. The first theoretical explanations of the head-tail effect published in [2, 3] include two-particle and multi-particle models, as well as the formulation and the solution of an eigenvalue problem to determine the growth rates and frequency shifts of the head-tail modes.

Later, advanced theories were developed using a number of approaches, such as macro-particle models, linearizing the Vlasov equation, applying perturbation theory [4–14]. These comprehensive studies cover almost all aspects of the head-tail effect, including mode coupling, various impedance models, chromaticity, and Landau damping. The multi-mode analysis of eigenvalue problems is efficient because only the lowest modes are essential.

Transverse feedback systems were proposed to cure the beam instabilities almost immediately after they were observed. The mode-coupling theory of the fast head-tail (transverse mode coupling) instability was expanded by including feedback [8, 11–14], which was introduced as an equivalent transverse impedance and the feedback term was added to the matrix equation. If the bunch is not very long, which is always true for electron rings, the feedback acts on the beam center of mass only, so the feedback can be modeled by a matrix with a single non-zero element related to the coherent mode. Mathematical models of the head-tail instability have been developed on the basis of the multi-mode analysis of the eigenvalue problem taking into account both chromaticity and feedback [11–14]. As concluded in [12], resistive feedback in combination with negative chromaticity can effectively damp the instability, increasing the threshold beam current by a factor of 3 to 5, the same conclusion was reached in the simulation studies of damper efficiency for the Advanced Photon Source (Argonne National Laboratory, USA) and the Large Hadron Collider (CERN) [15].

The efficiency of transverse feedback in combination with varied chromaticity was experimentally studied at several accelerator facilities. A few confirmations of feedback efficiency with negative chromaticity were demonstrated [16, 17]. However, experimental results obtained at several electron storage rings show that feedback is even more efficient at positive chromaticity [18–25]. Some of the experimental results are summarized in Fig. 1. Here, it looks like the threshold beam current is higher with positive chromaticity, both with and without feedback.

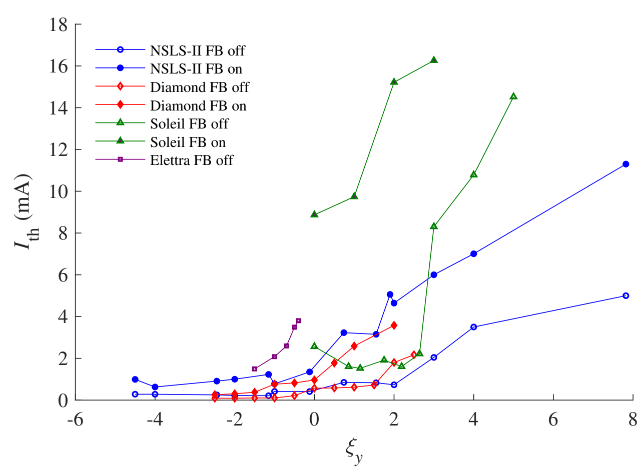

Figure 1: Measured single-bunch threshold current as a function of chromaticity, with and without feedback.

NSLS-II STUDIES

The effects of chromaticity and feedback on the transverse beam instabilities were studied at the National Synchrotron Light Source II (NSLS-II) in Brookhaven National Laboratory. The stabilizing effect of positive chromaticity was confirmed, the single-bunch threshold current of about 0.8 mA

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was measured at zero chromaticity, and the bunch current of 11 mA was achieved at chromaticity $\xi_x = 7.0$, $\xi_y = 7.8$ with the reduced gain of resistive feedback.

For our analysis, we follow the mode-coupling theory [11, 12], which takes into account chromaticity and feedback. The analysis of beam stability is reduced to a system of algebraic equations and the values of the frequency and decrement/increment of the oscillation modes can be found as eigenvalues of the set of equations. According to [11, 12], the symmetric mode expansion is efficient because only a few lowest oscillation modes are significant. This simplification is done with several assumptions: the longitudinal oscillations of the beam particles are much slower than the transverse ones; the longitudinal particle distribution in a bunch is Gaussian; the impedance is broadband; the motion is represented as a sum of oscillation modes $e^{i\Omega t}$, where Ω is the complex oscillation frequency.

In our case, we use a single-mode broadband resonator as the impedance model. The beam-impedance interaction is characterized by the integral parameters, such as the longitudinal loss factor, transverse kick factor, and effective impedance. If the bunch is not much shorter than the vacuum chamber aperture, these parameters can be calculated with acceptable accuracy using the broadband impedance approximated by a single resonator [26, 27]. We also analyze the horizontal and vertical oscillations separately assuming the motion is transversely uncoupled.

For NSLS-II, the following machine and beam parameters are used: $\nu_x = 0.217$ and $\nu_y = 0.262$ are the non-integer parts of the horizontal (x) and vertical (y) betatron tune $\nu_{x,y} = \omega_{x,y}/\omega_0$; $\nu_s = \omega_s/\omega_0 = 0.008$ is the synchrotron tune; ω_0 is the revolution frequency; $E = 3$ GeV is the energy; $\beta_{xav} = 13.1$ m and $\beta_{yav} = 15.3$ m are the average beta functions. Both the horizontal and vertical broadband impedance is approximated by a single broadband resonator with the quality factor $Q = 1$, resonance frequency $\omega_{rx} = \omega_{ry} = 2\pi \cdot 28.6$ GHz, and shunt resistance $R_x = 0.3$ M/m (horizontal), $R_y = 0.6$ M/m (vertical).

Solving the eigenvalue problem [11, 12] for specific impedance and beam parameters, one can derive the intensity-dependent shift λ of complex oscillation frequencies for the head-tail modes; $\mathrm{Re}\lambda = \Delta\omega_{x,y}/\omega_s$ is the normalized frequency shift per se and $\mathrm{Im}\lambda = 2\pi/\tau\omega_s$ is the normalized damping/growth rate. The positive rate corresponds to the damping and the negative rate corresponds to the growth of the mode.

For electron storage rings with short bunches, the intensity-dependent bunch lengthening caused by the combined effect of intrabeam scattering and beam interaction with longitudinal broadband impedance, is significant. The bunch length can increase by a factor of 2 per 1 mA of the single-bunch current or even more. The bunch lengthening weakens collective effects resulting in the head-tail instability, so it is very important to take it into account while solving the eigenmode problem. This is illustrated by Fig. 2 showing 10 lowest vertical modes calculated by solving the eigenvalue problem for NSLS-II. The real and imaginary parts of the normalized complex frequency shift were calculated with the constant zero-current bunch length (left) and with the realistic bunch lengthening (right). As one can see in the graphs on the right side, the beam becomes unstable at a much higher current if the bunch lengthening is taken into account.

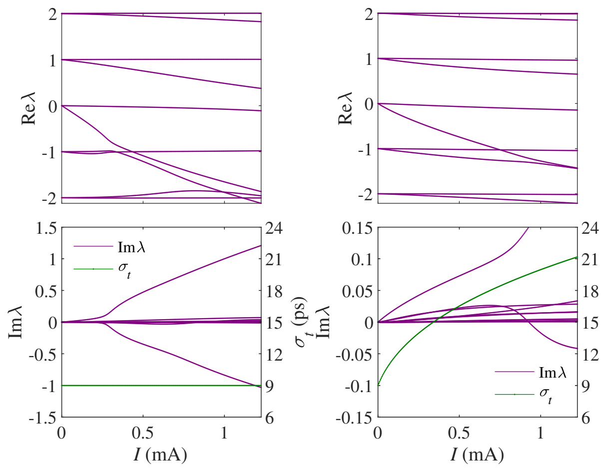

Figure 2: Effect of bunch lengthening on the frequencies (top) and damping/growth rates (bottom) of the 10 lowest vertical modes.

The frequency shifts and damping/growth rates of the low-order horizontal and vertical head-tail modes were measured at NSLS-II with chromaticity varied in the range from −1 to +7. Coherent beam oscillations were excited by a pulse kicker and measured by button-type beam position monitors (BPMs). The measurements were done with and without the bunch-by-bunch feedback system [28]. The oscillation frequencies were obtained by spectral analysis of turn-by-turn BPM data using refined discrete Fourier transform, the damping/growth rate was obtained by exponential fit of the oscillation envelope. To minimize the oscillation decoherence caused by the betatron tune shift with amplitude, the kick strength was adjusted to excite beam oscillations with the amplitude well below 1 mm. An example of a vertical turn-by-turn oscillation and its spectrum is shown in Fig. 3.

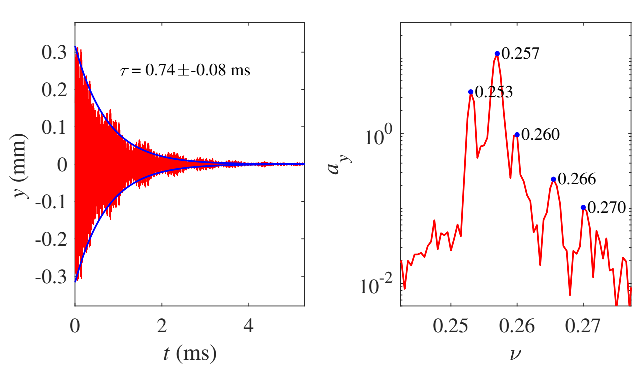

Figure 3: Vertical turn-by-turn oscillation (left) and its spectrum (right); $\xi_y = 2.1$; $I_b = 0.5$ mA.

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The measured data were compared with the solution of the eigenmode problem [11, 12] using the machine and beam parameters listed above. The set of equations was truncated to 5 azimuthal modes and 2 radial ones. Multi-particle numerical simulations were also carried out using the ELEGANT tracking code [29], with and without transverse feedback. The simulations were done using the same machine and beam parameters as were used for the mode-coupling equation, the impedance parameters were adjusted to fit the measured data.

Figure 4 shows an example of measured and calculated frequencies (top) and damping/growth rates (bottom) of the 0-th, −1-st, and +1-st vertical head-tail modes. The dots with error bars represent the measured data and the solid lines represent the real and imaginary parts of the solution of the mode-coupling equations. The horizontal and vertical linear chromaticities were equal to 2.1, the bunch-by-bunch feedback was turned off.

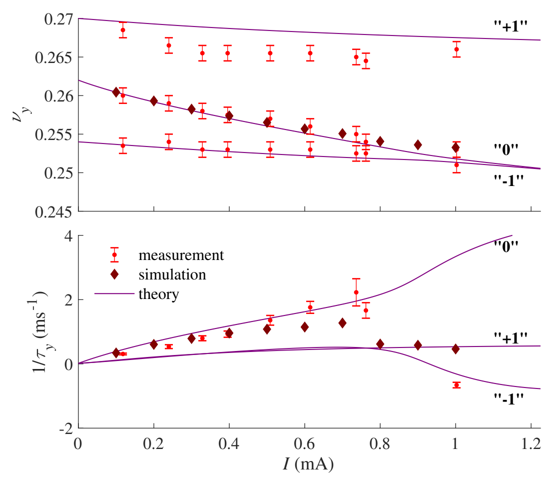

Figure 4: Frequencies (top) and damping/growth rates (bottom) of vertical head-tail modes; $\xi_y = 2.1$; feedback off.

With transverse bunch-by-bunch feedback, the set of equations truncated to 10 lowest modes was solved with the same machine and beam parameters as for the case without feedback. For these calculations, the equivalent impedance of the NSLS-II bunch-by-bunch feedback system is represented by a simple integrator with the cut-off frequency characterizing the bandwidth of the real system. Figure 5 shows an example of measured and calculated frequencies (top) and damping/growth rates (bottom) of the 0-th, −1-st, and +1-st vertical head-tail modes. As one can see, the feedback decouples the 0-th and −1-st modes and increases the damping rate by about an order of magnitude. However, the mode-coupling model does not show the saturation-like behavior of the damping rate with the beam current, which is seen in the measurement results and in multi-particle simulations.

CONCLUSION

Since the head-tail instability is a severe effect limiting the beam intensity and stability in particle accelerators, the instability has been studied theoretically and experimentally

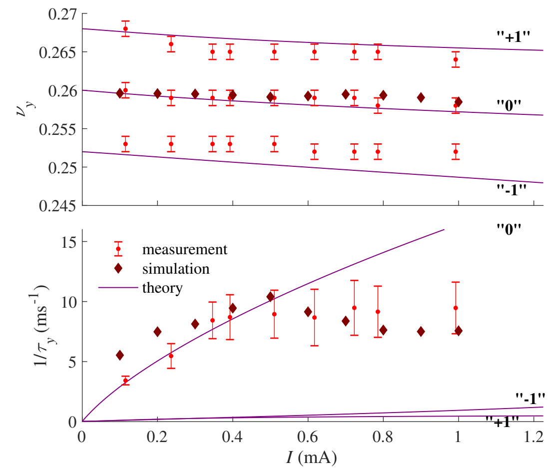

Figure 5: Frequencies (top) and damping/growth rates (bottom) of vertical head-tail modes; $\xi_y = 2.1$; feedback on.

ever since it was first observed. The beam-based feedback was proven as an effective way to suppress the instability; the bunch-by-bunch feedback systems are now implemented at many accelerator facilities.

The mode-coupling theory based on multi-mode analysis of the eigenvalue problem is an effective tool to describe the beam dynamics with the head-tail instability and feedback, taking into account the chromaticity. A simplified representation of the machine impedance as a broadband resonator is accurate enough to obtain solutions consistent with experimental data. Taking the intensity-dependent bunch lengthening into account is important for electron machines with short bunches.

Both the mode-coupling theory and experiments show that positive chromaticity helps increase the instability threshold even without feedback. For electron storage rings, operation with negative chromaticity does not look practical, the combination of positive chromaticity and feedback looks more robust. The other factor affecting single-bunch instabilities, which is not taken into account in the mode-coupling model, is the nonlinear spread of oscillation frequencies of particles caused by higher-order chromaticity, machine nonlinearity, and beam-beam interaction in colliders. For accurate analysis of the beam dynamics with nonlinear effects, self-consistent simulations using advanced computer codes are necessary.

More details of these studies can be found in the article recently published in Physical Review Accelerators and Beams [30].

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