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# COMPLEX UNIT LATTICE CELL FOR LOW-EMITTANCE SYNCHROTRONS*

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*Abstract*

To reach the real diffraction-limited emittance, it is generally required to increase the number of bends in multi-bend achromat (MBA) lattices that are used in the designs of fourth-generation synchrotron light sources. For an MBA lattice with distributed chromatic correction, more bends mean much tighter space and much stronger magnets. Inspired by the hybrid MBA lattice concept, in this paper we propose a new lattice concept called complex unit lattice cell, which can save the space and reduce magnet strengths. A 17BA lattice based on the complex unit cell concept is designed for a 3 GeV synchrotron light source with circumference of 537.6 m, which reaches a natural emittance of about 21 pm·rad. And a comparison is also made between this 17BA lattice and the 17BA lattice with distributed chromatic correction to demonstrate the merit of the complex unit cell concept.

## INTRODUCTION

During past few decades of synchrotron light source evolution, the emittance has been decreased by 2-3 orders of magnitude. As a study for future MAX IV upgrade, a 19BA lattice was designed to achieve a natural emittance of 16 pm·rad [1]. The beam emittance of a storage ring can be written as

$$\varepsilon_x = F\cdot\frac{E^2}{J_x N_B^3}, \tag{1}$$

where $F$ is a coefficient that depends on the specific lattice, $E$ is the electron energy, $J_x$ is the horizontal damping partition number, and $N_B$ is the number of bending magnets in the ring. Increasing the number of bends is the main way to reduce emittance, since the theoretical minimum emittance is inversely proportional to the third power of the number of bends, which however requires stronger focusing to limit the dispersive orbit amplitudes. This leads to smaller dispersion outside the bends and higher natural chromaticity. As a consequence, stronger sextupoles for chromaticity correction are required. The equations for chromaticity compensation by sextupoles can be represented as

$$\begin{cases} 4\pi(\xi_{x1}-\xi_{x0}) = \oint \lambda(s)\eta\beta_x ds \\ 4\pi(\xi_{y1}-\xi_{y0}) = \oint \lambda(s)\eta\beta_y ds \end{cases}. \tag{2}$$

If we only use two families of sextupoles to correct chromaticities in a lattice cell, it can be written as:

$$\begin{cases} 4\pi(\xi_{x1}-\xi_{x0}) = \eta_F\beta_{xF}I_{SF} - \eta_D\beta_{xD}I_{SD} \\ 4\pi(\xi_{y1}-\xi_{y0}) = \eta_F\beta_{yF}I_{SF} - \eta_D\beta_{yD}I_{SD} \end{cases}, \tag{3}$$

where $\beta_{xF}$, $\beta_{yF}$, $\eta_F$, $\beta_{xD}$, $\beta_{yD}$, $\eta_D$ are the optical functions at the locations of the focusing and defocusing sextupoles respectively. The horizontal and vertical chromaticities are corrected to ($\xi_{x1}$, $\xi_{y1}$) from ($\xi_{x0}$, $\xi_{y0}$). $I_{SF}$ and $I_{SD}$ are the integrated strengths of sextupoles. By solving the equations, we can get:

$$\begin{cases} I_{SF} = \dfrac{4\pi[\beta_{yD}(\xi_{x1}-\xi_{x0}) + \beta_{xD}(\xi_{y1}-\xi_{y0})]}{2\eta_F(\beta_{xF}\beta_{yD} - \beta_{yF}\beta_{xD})} \\ I_{SD} = \dfrac{4\pi[\beta_{yF}(\xi_{x1}-\xi_{x0}) + \beta_{xF}(\xi_{y1}-\xi_{y0})]}{2\eta_D(\beta_{yF}\beta_{xD} - \beta_{xF}\beta_{yD})} \end{cases}. \tag{4}$$

As shown in the formula (4), the integrated strength of sextupoles used is inversely proportional to the dispersion function of its location. Installing the chromatic sextupoles in the dispersion bumps helps to keep its integrated strength small. The variation of the dispersion function in the straight section can be described as:

$$\eta_1 = \eta_0 + \eta_0' \cdot l_D\,. \tag{5}$$

Increasing the length of the drift section is beneficial to magnify the dispersion bump. Inspired by ESRF-EBS lattice [2], we propose a new lattice unit cell concept aimed to decrease the strengths of multipole magnets, which we call "complex unit cell". We will describe this concept in detail in the following sections.

## COMPLEX UNIT CELL

If we take the part between two focusing sextupoles as a lattice unit cell, then the hybrid 7BA lattice can be regarded as a combination of a 5-bend cell and two matching sections. Similar to the 5-bend cell, as shown in Fig. 1, several bends are combined into a lattice unit cell and share two families of chromatic sextupoles which can save the space. In this way, it helps to form the dispersion bump in the cell and leads to smaller magnet strengths. We call this kind of unit cell as complex unit cell.

We use a simple ring with identical unit cells to study the characteristics of the complex unit cell. Its parameters are as follows:

- The ring consists of 360 bends;
- The circumference of the ring is 432 m;
- The normalized phase advance $\begin{cases} \nu_x = 108 \\ 0.1\cdot\nu_x \le \nu_y \le 0.5\cdot\nu_x \end{cases}$;
- The horizontal damping partition number $J_x \le 2$.

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As shown in Fig. 2. Compared with the standard unit cell, the complex unit cell can effectively reduce the integrated strengths of sextupoles. There is no significant difference in the integrated strengths of sextupoles for complex unit cells with three and four bends. When the number of bends in a unit cell increases, the chromaticity will be magnified at the same time. This offsets the decrease in magnet strength caused by the increase in the straight section.

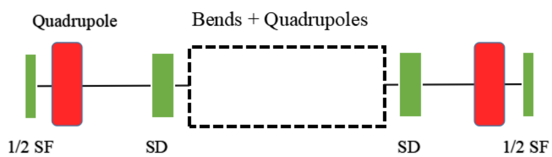
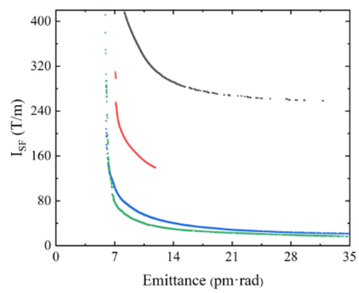
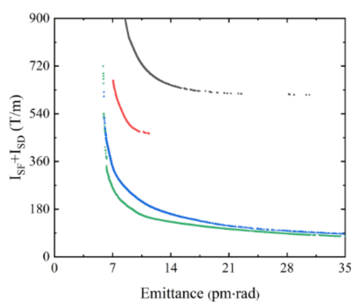



Figure 1: The magnet layout of complex unit cell. (SF: focusing sextupole; SD: defocusing sextupole).




(a)




(b)

Figure 2: Pareto front of the integrated strength of sextupoles and beam emittance. (Black: standard unit cell; Red: Complex unit cell with two main bends; Blue: Complex unit cell with three main bends; Green: Complex unit cell with four main bends).

More elements in the unit cell mean more free knobs, and the complex unit cell can achieve more feasibility, which helps to balance the parameters, such as momentum compaction factor, $J_x$ and chromaticities. In addition, for the complex unit cell, the phase advance over a unit cell is no longer limited to lower than $0.5{\cdot}2\pi$, which means that the complex unit cell can choose the working point more freely.

## LATTICE DESIGN AND COMPARISON

In this section, an attempt of lattice design for future diffraction limited storage rings based on a complex unit cell with three main bends (case 1), is presented, which is a 17BA lattice. As a comparison, we also designed another 17BA lattice with standard unit cells (case 2). The cell tunes of the two kinds of unit lattices are respectively taken as $\nu_{cell}$ = [4/5, 1/5] and $\nu_{cell}$ = [4/15, 1/15] to achieve higher-order achromat. A natural emittance of 21.68 pm·rad (ignoring the impact of IBS) was obtained as a preliminary lattice with the complex unit cell concept.

The main parameters of the two lattices are summarized in Table 1 and the optical functions are shown in Figs. 3-6. Those two lattices have similar emittance, but the $J_x$ of case 1 is much lower than case 2. And the momentum compaction factors of the two lattices are reasonable.

Table 1: Parameters of 17BA Lattices: Complex Unit Cell (case 1) vs. Standard Unit Cell (case 2)

| | Case 1 | Case 2 |
|---|---|---|
| Circumference | 537.6 m | |
| Energy | 3 GeV | |
| Number of cells | 20 | |
| Nat. emittance | 21.68 pm·rad | 21.91 pm·rad |
| Transverse tunes | 96.14/27.14 | 96.16/27.14 |
| Nat. chromaticities | -116.2/-128.8 | -92.7/-123.1 |
| Mom. comp. factor | $4.1\times10^{-5}$ | $5.4\times10^{-5}$ |
| Damping partitions | 1.77/1.0/1.23 | 2.12/1.0/0.88 |
| Nat. damping times | 13.9/24.6/19.9 | 12.0/25.4/28.8 |

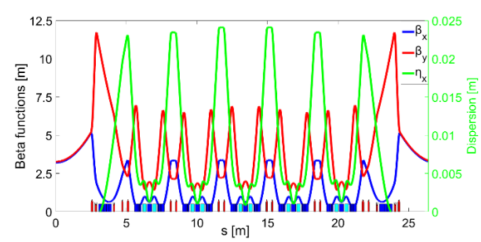



Figure 3: Linear optical functions of the 17BA lattice based on the complex unit cell.

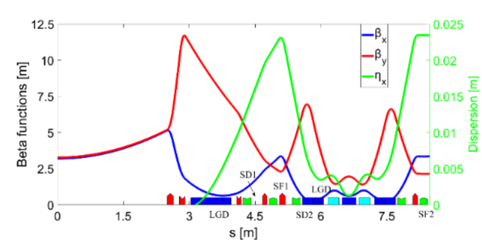



Figure 4: Enlarged view of the complex unit cell and matching section of the 17BA lattice (case 1).

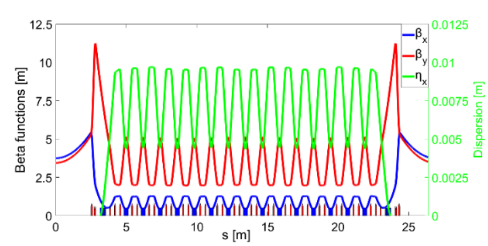



Figure 5: Linear optical functions of the 17BA lattice based on the standard unit cell.

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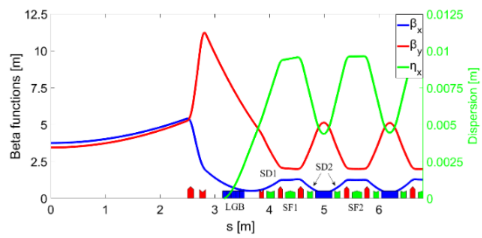


Figure 6: Enlarged view of the standard unit cell and matching section of the 17BA lattice (case 2).

The OPA code [3] is the tool we used for preliminary nonlinear dynamic optimization, which is also used to evaluate the required strengths of sextupoles. There are 4 families of sextupoles, 2 in unit cells and 2 near the matching bends. In the optimization, chromaticities are corrected to (3, 3). As shown in Table 2, the 17BA lattice based on the complex unit cell obtains much lower integrated strengths of sextupoles. The optimized DAs of the two lattices are shown in Figs. 7 and 8, respectively. And Figs. 9 and 10 show the momentum dependent tune footprints, with on-momentum transverse tunes of (96.14, 27.14) and (96.16, 27.14). The nonlinear dynamics performances of the two lattice are similar. The horizontal DA is about 1.5 mm and the horizontal tune crosses the half-integer resonance line at up to 4.0%.

Table 2: Integrated Strengths (T/m) of Sextupoles

| | Case 1 | Case 2 |
|---|---|---|
| SF1 | 2193.5 | 6840.0 |
| SD1 | 1620.9 | 3286.0 |
| SF2 | 2801.6 | 6622.8 |
| SD2 | 2221.6 | 3831.2 |

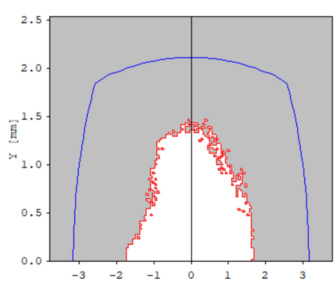


Figure 7: DA of the 17BA lattice based on the complex unit cell.

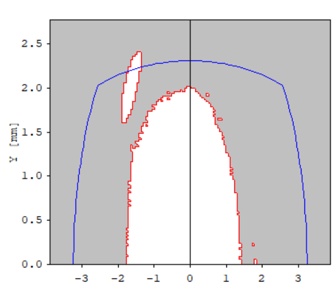


Figure 8: DA of the 17BA lattice based on the standard unit cell.

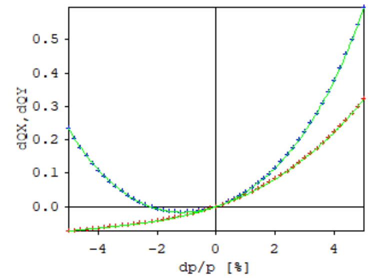


Figure 9: Momentum dependent tune footprints for the 17BA lattice based on the complex unit cell.

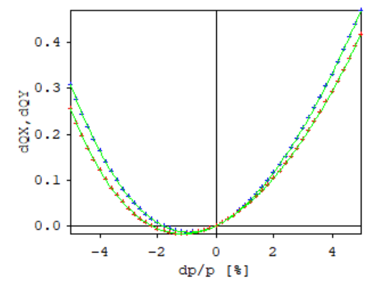


Figure 10: Momentum dependent tune footprints for the 17BA lattice based on the standard unit cell.

## CONCLUSION

In this paper, we have proposed the concept of complex unit lattice cell. We briefly studied the complex unit cell with a simple model. And then, we compared the properties of two lattices based on different concepts (complex unit cell and standard unit cell) and found that the former can save the space and reduce magnet strengths. It also showed an advantage in balancing the lattice parameters. The complex unit cell has the potential for the design of future diffraction-limited storage rings, and we will do further study for the complex unit cell.

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