

BEAM-BEAM SIMULATIONS FOR LEPTON-HADRON COLLIDERS: ALOHEP SOFTWARE

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Abstract

It is known that rough luminosity estimations for ll , lh and hh colliders can be performed easily using nominal beam parameters. In principle, more precise results can be obtained by analytical solutions. However, beam-dynamics is usually neglected in this case since it is almost impossible to cope with beam size fluctuations. In this respect, several beam-beam simulation programs for linear e^+e^- and photon colliders have been proposed while no similar open-access simulation exists for all types of colliders (i.e. linac-ring ep colliders). Here, we present the software ALOHEP (A Luminosity Optimizer for High Energy Physics), a luminosity calculator for linac-ring and ring-ring colliders, which also computes IP parameters such as beam-beam tune shift, disruption arising out of electromagnetic interactions. In addition, the program allows to take crossing-angle effects on luminosity into account.

INTRODUCTION

Colliders can be classified by different aspects; such as colliding beam types (i.e. lepton-lepton, hadron-hadron, lepton-hadron), utilized accelerator mechanisms (i.e. linear collider, ring-ring, linac-ring) or its main function (i.e. particle factories, discovery machines). When we make the classification according to the colliding beam types, lepton-hadron colliders manifest their superiority in some certain areas. For example; lepton-hadron scattering has been playing a vital role in order to understand the inner structure of the matter since earlier attempts (see [1, 2]) to HERA [3] which provided precision PDFs for adequate interpretation of Tevatron and LHC data. Then, THERA [4] project (combination of TESLA and HERA) was proposed which offers a linac-ring type lepton hadron collider dedicated to exploration on deeper kinematic region of proton. Today, LHeC [5] is being planned to operate as a lepton-hadron collider by construction of electron linac tangential to the LHC. LHeC is expected to reach high Q^2 region which will be an opportunity to analyze parton distribution functions in depth. For history of lepton-hadron collider proposals see reviews of [6, 7].

In Table I we present correlations between colliding beams and collider types for energy frontier colliders, where symbol “+” implies that given type of collider provides maximal center of mass energy for this type of colliding particles (for example; linac-ring type colliders will give opportunity to achieve highest center of mass energy for ep collisions).

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Table 1: Energy Frontier Colliders: Colliding Beams vs Collider Types

Colliders	Ring	Linac	Linac-Ring
Hadron	+		
Lepton (e^+e^-)		+	
Lepton ($\mu^+\mu^-$)	+		
Lepton-Hadron (eh)			+
Lepton-Hadron (μh)	+		
Photon-Hadron (γh)			+
Photon-Lepton (γe)		+	
Photon-Photon ($\gamma\gamma$)		+	

Considering Table 1, authors have shown that future ring and linac-ring colliders also provide a great enhancement on physics search potential [8, 9] by a possible combination of lepton (ILC [10], PWFA [11], MAP [12]) and hadron colliders (FCC [13] and LHC or HE-/HL-LHC [14]). These combinations push discovery limits of BSM particles higher and therefore show the necessity of dedicated studies on possible future eh and μh constructions [8, 9, 15–18].

For all collision schemes, interaction region comes up as the most important part regarding luminosity since the whole effort is to realize the desired beam-beam interactions there. Therefore, beam parameters at IP should be optimized carefully and necessary calculations should be performed regarding parameters at the interaction region. The basics of luminosity calculations are well-known and rough estimations can be done easily for the steady beams approaching to the IR. Inhomogeneous distributions of particles through the bunches within the beams bring analytical challenges which can be overcome by certain approximations. However, reciprocal interactions of colliding beams usually lead the particle distributions to have amorphous forms. When revolving beams in opposite directions are too much close to each other they may encounter parasitic interactions which may cause offsets [19, 20]. Thus, it is almost impossible to calculate the precise luminosity by only analytical methods. Several software programs were proposed in this regard [8, 21–26]. CAIN [21] based on ABEL (Analysis of Beam-Beam Effects in Linear colliders) [24] was proposed exhibiting effects of focusing/disruption on ee in addition to $\gamma\gamma$ and $e\gamma$ interactions. Another simulation was presented in a similar manner, the collider software of linear accelerators GUINEA-PIG [25], to show the strong

beam-beam effects¹ on luminosity. It also serves as a tool for background computation regarding the same effects.

The range of computational approaches to the collider designs enhance day by day [26]. However, there are still gaps corresponding to the current and future accelerator schema. For example, still there is no computer code for all types of collisions, such as ring-ring μp or linac-ring ep . Acar *et al.* and Canbay *et al.* have shown the great potential of these type collider based lepton-hadron collisions, such that discovery limits of predicted beyond the standard model (BSM) particles are superior compared to the ll and competitive to hh colliders [8, 9]. The same studies also show the effect of integrated luminosity on the search of BSM physics. Therefore, luminosity values carry a crucial role and requires precise calculations considering non-negligible beam-beam interactions. In this regard authors proposed an earlier version of a software program, AloHEP (A Luminosity Optimizer for High Energy Physics), to reveal the luminosity of future linac-ring type electron-proton colliders [8].

In this study, we extended the collision types from only linac-ring ep to any e, μ, p, Pb combinations for any collider scheme options. Current version of AloHEP is available to access at <http://alohep.hepforge.org> and our research group web page http://yef.etu.edu.tr/ALOHEP_eng.html. A novel algorithm is being developed to reduce run-time dramatically. Moreover, current program is planned to allow taking offset and crossing-angle effects into account.

In the next section we briefly mention the main accelerator physics phenomena that have effect on luminosity. Afterwards, the computation algorithm considering these effects is described briefly. Finally, we give the conclusion with the physical comments on the software program, possible further upgrades and future accelerator physics.

METHOD

Physics background of the code is mainly based on electromagnetism and relativistic effects which are the building blocks of almost all accelerator physics phenomena. Therefore, we begin this section by the physical background and continue by presenting how the physics was implemented to the code.

Luminosity Calculation

Even though the physics background is well-known by the high energy physics community, we approve giving brief explanations, basic assumptions and notations used in this article.

Beam Parameters Firstly, we assume that initial (before beam-beam interactions) particle densities have Gaussian distributions in all axes throughout the bunches

$$\rho(x, y, z) = A_0 e^{-\frac{(x-x_c)^2}{2(\sigma_x)^2}} e^{-\frac{(y-y_c)^2}{2(\sigma_y)^2}} e^{-\frac{(z-z_c)^2}{2(\sigma_z)^2}}. \quad (1)$$

x_c, y_c and z_c stand for the center of the bunch. Head-on collision with zero offset and zero crossing-angle ϕ is the simplest case where $z_c = z_0 \pm ct$. If crossing-angle is non-zero then we assume crossing-angle belongs to the xz plane and operating a rotation matrix on these coordinates is useful for both analytical and numerical calculations:

$$R_{1,2} = \begin{pmatrix} \cos(\phi/2) & \pm \sin(\phi/2) \\ \mp \sin(\phi/2) & \cos(\phi/2) \end{pmatrix}. \quad (2)$$

From now on $\sigma_{x,y}$ represents the bunch size through the transverse dimension and $\sigma_{x,y} = \sqrt{(\epsilon_{x,y})^N (\beta_{x,y}) / \gamma}$. Here, ϵ^N is the normalized emittance, γ is the Lorentz factor and the β function denotes the envelope function of the beam size. A_0 is a constant which should be set to match the total number of particles per bunch, N :

$$N = \int_{-\infty}^{\infty} \rho \, dv = \frac{A_0}{(2\pi)^{\frac{3}{2}} \sigma_x \sigma_y \sigma_z}. \quad (3)$$

Revolution frequency, $f_{rev} \approx 2\pi r/c$, is one of the main characteristic structures of a beam. Here r is the radius of the ring and c is the speed of light. Considering linear machines, beams are either single pass or reused for energy recovery purpose.

Luminosity Concept The most general definition of the luminosity is the ratio between detected events per time and total cross section.

$$L = \frac{dR/dt}{\sigma}. \quad (4)$$

For particle colliders dR/dt is proportional to $N_1 N_2 f_{coll}$, where f_{coll} is the collision frequency. It depends on f_{rev}, f_p , number of bunches per beam (n_b) and collider type:

- i) $f_{coll} = n_b f_{rev}$ for ring (if there are two different rings as in the μp collider case, minimum of f_{coll} should be taken into account),
- ii) $f_{coll} = n_b f_p$ for linac and
- iii) $f_{coll} = \min(n_b f_{rev}, n_b f_p)$ for linac-ring type colliders.

The last case requires a rearrangement on one of the beam parameters of linear or ring accelerator and f_{coll} can be increased by these arrangements. However we keep this out of the scope of the current study.

Mathematical background of luminosity is given in many previous studies in the literature and therefore we leave the details of the derivations to the readers [27–29]. Equation below gives the general form of L including non-zero crossing angle:

$$L = 2\cos^2(\phi/2) N_1 N_2 f_{coll} \int \rho'_1 \rho'_2 \, dv', \quad (5)$$

where ρ' stands for particle density distribution. These Gaussian distributions are spoiled under beam-beam interactions.

¹ Beamstrahlung, bremsstrahlung and pair production are dominant effects beside dynamic beam sizes (namely disruption in this case) in linear e^+e^- colliders

This is one of the reason why a computational approach is required and we address this issue in the next subsections. However, the results are reasonable when beam-beam interactions are weak and the integral leads us to the definition of nominal luminosity:

$$L = \frac{f_{coll} N_1 N_2 \cos(\phi/2) / 2\pi}{\sqrt{\sigma_{1y}^2 + \sigma_{2y}^2} \sqrt{(\sigma_{1x}^2 + \sigma_{2x}^2) \cos^2(\frac{\phi}{2}) + (\sigma_{1z}^2 + \sigma_{2z}^2) \sin^2(\frac{\phi}{2})}} \quad (6)$$

In the most general case, beam-beam interactions, which directly effect L , should not be neglected. In other words, analytical model developing for the luminosity change due to beam-beam interaction is possible only for small disruption/beam-beam parameters. Otherwise, computational methods should be used to simulate interaction [22]. We developed the AloHEP code to perform also these simulations. In the AloHEP, bunches of the beams are represented by a number of gaussian distributed macroparticles. We assumed that both beams travel with speed of light and therefore the fields of the particles are transverse. The encountering bunches are divided into slices through the z -axis to handle the problem as two-dimensional in which slices are divided by rectangular grid cells [23]. The charges of particles are distributed into these cells. To avoid calculation noises caused by point-like macroparticle assumption, particle-in-cells model is used [24]. The force field of slices are calculated such that a slice interacts only with the slice of the other bunch at the same z -position and a particle is affected only by the fields of other bunch. The macroparticles are moved only in transverse plane under force and a slice moves step by step through the z -axis to interact with the next slice. The intersection of density distributions of the encountering slices at same z coordinate is calculated for each time interval to reach the enhanced or reduced luminosity due to beam-beam interactions.

There are several other accelerator physics phenomena which makes the most of the parameters given in luminosity equation both time and position dependent. Below, we list and briefly explain the most crucial ones.

Hourglass Effect A particle beam with Gaussian distribution is actually exposed to periodical focusing and defocusing while propagating in beampipes. This is similar to the case of light propagation through periodical lenses but linearly gradient magnetic fields are used for beam optics [30]. External fields usually vanish in collision region which is one of the drift spaces of beampipes. In this case, an initially focusing beam continues focusing to a certain spot size at interaction point and a fully symmetrical defocusing is realized until the beam is encountered by external field again. This is called the hourglass effect and can be arranged to optimize the luminosity. One should note that, nominal luminosity values are calculated by the parameter values taken at the exact interaction point while they are position dependent throughout the interaction region.

Disruption and Beam-Beam Tune Shift A similar focusing like hourglass-effect occurs when two charged bunches encounter within a linear accelerator. Each bunch acts as a focusing/defocusing lens with the focal lengths of:

$$f_{1,x,y} = \frac{\gamma_1 \sigma_{2,x,y} (\sigma_{2y} + \sigma_{2x})}{2N_2 r_1} \quad (7)$$

where N_2 is number of particles per bunch of the counter beam and r_1 is the classical radius of the particle that is exposed to the force. The definition of the disruption can be derived by this focal length as

$$D_{1,x,y} = -\sigma_{2,y} / f_{1,x,y} = \Delta r / r. \quad (8)$$

In other words, disruption states a particle's total deflection from its original trajectory afterwards passing through the counter bunch. In a ring-ring collider, this directly corresponds to a beam-beam parameter of

$$\xi_{x,y} = -\frac{\beta^*_{x,y}}{4\pi f_{x,y}}, \quad (9)$$

which is expected to stay below 0.1 [31].

Time Structure of Unstable Particles Muon has a distinct place among the all colliding particle types due to its short life-time. Therefore, average number of particles per bunch, N_{avg} , should be calculated for muon beams first:

$$N_{avg} = N \int \frac{e^{-t/\tau}}{t} dt, \quad (10)$$

where τ denotes the life-time of muons (Please see [12] for calculation of total run time of muon beams).

CONCLUSION

Beam-beam interactions have an obvious direct effect on the luminosity of colliders and should be included in luminosity estimations of future collider plans. Strong beam interactions show themselves as luminosity enhancement factors on opposite charge collisions for any type of colliding particles. Even though dynamic focusing scheme is already a well-known method to increase the luminosity of lepton-hadron colliders [32], an optimal value should be set to the focusing strength in order to avoid extreme pinching effects. In this point of view, we have developed an already available beta version of luminosity calculation software, AloHEP, to current version in which any type of collision in any type of collider can be simulated. The release version of the software will include time structure of muons, hourglass effect and crossing-angle options in addition to disruption and beam-beam tune shift. A possible further improvement may bring a chance to enhance AloHEP including beamstrahlung/bremsstrahlung and intra-beam scattering effects which may be effective on luminosity calculations in some specific cases.

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