ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-WEPAB400

FORCED COUPLING RESONANCE DRIVING TERMS

A. Wegscheider, R. Tomás, CERN, Geneva, Switzerland

Abstract

At CERN's Large Hadron Collider (LHC), coupling is routinely measured using forced oscillations of the beam through excitation with an AC-dipole. The driving of the particle motion has an impact on the measurement of resonance driving terms. Recent findings suggest that the current models describing the forced motion are neglecting a local effect of the AC-dipole, creating a jump of the amplitude of the resonance driving terms. This work presents a study of the improvement of coupling measurements for typical LHC optics as well as its upgrade project, the High Luminosity LHC (HL-LHC), by using the new model.

INTRODUCTION

Precise control of linear coupling in an accelerator is important for operational control and machine safety. For LHC, coupling measurements were steadily improved over the years [1–4].

The coupling Resonance Driving Terms (RDTs) $f_- = f_{1001}$ and $f_+ = f_{1010}$ are defined as [5]

$$f_\pm(s_i) = \frac{\sum_w^W \delta J_{1,w}\sqrt{\beta_{x,w}\beta_{y,w}}e^{i\pi(\varphi_{x,wi}\pm\varphi_{y,wi})}}{8\left(1-e^{2\pi i(Q_x\pm Q_y)}\right)}, \quad (1)$$

with $J_{1,w}$ denoting the skew quadrupolar error of element w, $\beta_{x/y,w}$ its horizontal and vertical β functions, $\varphi_{x/y,wi}$ the hor. and vert. phase advances between elements w and i and $Q_{x/y}$ the hor. and vert. tunes, s_i is the longitudinal position of element i.

RDTs are calculated from the spectral lines of the turn-by-turn data. Since the particle's motion is affected by the driving of forced oscillation by the AC-dipole, the spectrum also undergoes a change and the measured RDTs are not equal to the free ones.

In the past, two methods were used to model the effect of the driven motion, the first is a simple rescaling of the tune dependent denominators of the RDTs, this method will therefore be called *rescaling* method. Using this rescaling, the driven RDTs read

$$f^{\mathrm{drv}}_{\pm,x} = \frac{\sin(Q_x\pm Q_y)}{\sin(Q^d_x\pm Q_y)}f_\pm, \quad f^{\mathrm{drv}}_{\pm,y} = \frac{\sin(Q_x\pm Q_y)}{\sin(Q_x\pm Q^d_y)}f_\pm, \quad (2)$$

where $f^{\mathrm{drv}}_{\pm,x}$ denotes the driven RDT as measured from horizontal turn-by-turn data and analogously for $f^{\mathrm{drv}}_{\pm,y}$. $Q^d_{x/y}$ are the hor. and vert. driven tunes.

The second one [6] applies the findings of a detailed study of the driven particle motion using the equations of motion of the particle in the accelerator with AC-dipole excitation. This method provides reconstruction and compensation formulae for all optics parameters, including those that enter into the coupling terms.

$$\begin{aligned} f_{\pm,x} = &\frac{1}{\sqrt{1-\lambda_x^2}}\frac{\sin\left[\pi(Q_x\mp Q_y)\right]}{\sin\left[\pi(Q^d_x\mp Q_y)\right]}\Big\{ \\ &e^{i(\varphi^{\mathrm{d},x}_{s_ds}-\varphi^{x}_{s_ds})}f_\mp + \lambda_x\lambda_c e^{i(\varphi^{\mathrm{d},x}_{s_ds}-\varphi^{x}_{s_ds})}f^*_\pm \\ &+ 2i\sin(\pi(Q^-_x))e^{i(\varphi^{\mathrm{d},x}_{s_ds}-\varphi^{x}_{s_ds})}f_\pm(s;s,s_d) \\ &+ 2i\lambda_c^{\mp1}\sin(\pi(Q^-_x))e^{i(\varphi^{\mathrm{d},x}_{s_ds}+\varphi^{x}_{s_ds})}f_\mp(s;s,s_d)\Big\}, \end{aligned} \quad (3)$$

for the f terms as measured from the signal in the horizontal plane, and

$$\begin{aligned} f_{\pm,y} = &\frac{1}{\sqrt{1-\lambda_y^2}}\frac{\sin\left[\pi(Q_x\mp Q_y)\right]}{\sin\left[\pi(Q_x\mp Q^d_y)\right]}\Big\{ \\ &e^{i(\varphi^{\mathrm{d},x}_{s_ds}-\varphi^{x}_{s_ds})}f_\mp + \lambda_x\lambda_c e^{i(\varphi^{\mathrm{d},x}_{s_ds}-\varphi^{x}_{s_ds})}f^*_\pm \\ &+ 2i\sin(\pi(Q^-_x))e^{i(\varphi^{\mathrm{d},x}_{s_ds}-\varphi^{x}_{s_ds})}f_\pm(s;s,s_d) \\ &+ 2i\lambda_c^{\mp1}\sin(\pi(Q^-_x))e^{i(\varphi^{\mathrm{d},x}_{s_ds}+\varphi^{x}_{s_ds})}f_\mp(s;s,s_d)\Big\} \quad . \end{aligned} \quad (4)$$

in the vertical plane. We used the following set of definitions:

$$\lambda_z = \frac{\sin\left[\pi(Q^d_z-Q_z)\right]}{\sin\left[\pi(Q^d_z+Q_z)\right]}, \quad \lambda_c = \frac{\sin\left[\pi(Q_x-Q_y)\right]}{\sin\left[\pi(Q_x+Q_y)\right]},$$

$$Q^\pm_z = Q^d_z \pm Q_z, \text{ for } z\in\{x,y\} \quad (5)$$

and

$$\begin{aligned} f_\mp(s;s,s_d) = &\frac{1}{8i\sin\left[\pi(Q_x\mp Q_y)\right]} \\ &\times\sum_{j=1}^N\Big\{\Theta(s_j;s,s_d)J_1\sqrt{\beta_{x,j}\beta_{y,j}} \\ &\times e^{-[\varphi_{x,ss_j}\mp\varphi_{y,ss_j}-(Q_x\mp Q_y)\mathrm{sgn}(s_j-s)]}\Big\}. \end{aligned} \quad (6)$$

where Θ denotes the step function

$$\Theta(x;a,b) = \begin{cases} 1 & \text{if } b<x<a \\ -1 & \text{if } a<x<b \\ 0 & \text{else} \end{cases} \quad (7)$$

An asterisk (f^*) denotes complex conjugation.

This method will be called *formula method* in the following.

Recent findings [7] show that the AC-dipole locally affects RDTs and introduces a jump in amplitude of the RDTs at its location. Even the detailed considerations of the formula method presented above lack such an effect.

In [8] the calculations have been extended to coupling RDTs f_- and f_+.

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-WEPAB400

Driven Coupled Motion

The AC-dipole inflicts a kick on the beam at each passing, in Courant-Snyder coordinates $h_z^{\pm} = z \mp ip_z$ this kick can be described as follows [9]:

$$\Delta h_z^{\pm} = \pm i A_\theta \beta(s_d) \cos(2\pi Q_z^d N), \tag{8}$$

where $\beta(s_d)$ is the β function at the position of the AC-dipole, A_θ denotes the AC-dipole kick amplitude and N is the turn number.

In normal form coordinates $\zeta_z^{\pm}$, the one-turn evolution of the particle is just a simple rotation. Normal form coordinates are calculated from Courant-Snyder coordinates (and vice-versa) by

$$\begin{aligned} \zeta_z^{\pm} &= h_z^{\pm} + [-F, h_z^{\pm}] + O(f^2), \\ h_z^{\pm} &= \zeta_z^{\pm} + [F, \zeta_z^{\pm}] + O(f^2), \end{aligned} \tag{9}$$

where F denotes the generating function. In our case (with coupling as the only non-model contribution to RDTs), F reads

$$F = f_- \zeta_x^+ \zeta_y^- + f_+ \zeta_x^+ \zeta_y^+ + f_-^* \zeta_x^- \zeta_y^+ + f_+^* \zeta_x^- \zeta_y^-. \tag{10}$$

Consecutive application of AC-dipole kick, transformation to normal-form coordinates, one-turn rotation and transformation back to Courant-Snyder coordinates yields the particle's Courant-Snyder coordinate at turn N:

$$\begin{aligned} h_x^+(s,N) = h_x^{d+}(s,N) & \\ &+ 2if_{1001}^* h_y^{d+}(s,N) + 2if_{1010}^* h_y^{d-}(s,N) \\ &- 2if_{1001}^*(s_d) h_{y,x}^+(s,N) \\ &- 2if_{1010}^*(s_d) h_{y,x}^-(s,N), \end{aligned} \tag{11}$$

From a spectral analysis of the driven signal in Eq. (11) we get the driven coupling terms

$$\begin{aligned} f_{-,x}^{\mathrm{drv}*} = f_-^* &+ \lambda_y f_+^* e^{2i\left[\varphi^y_{s_d s} + \pi Q_y\right]} \\ &- f_-^*(s_d) \frac{\sin \pi Q_y^-}{2 \sin\left(\pi Q_{y-}^d\right)} e^{i\left[\varphi^x_{s_d s} - \varphi^y_{s_d s} + \pi Q_- \mathrm{sgn}(s - s_d)\right]} \\ &- f_+^*(s_d) \frac{\sin \pi Q_y^-}{2 \sin\left(\pi Q_{y+}^d\right)} e^{i\left[\varphi^x_{s_d s} + \varphi^y_{s_d s} + \pi Q_+ \mathrm{sgn}(s - s_d)\right]}, \end{aligned} \tag{12}$$

where $Q_{\pm} = Q_y \pm Q_x$ and $Q_{y\pm}^d = Q_y^d \pm Q_x$. For the signal, as measured from the vertical plane, we get a similar expression:

$$\begin{aligned} f_{-,y}^{\mathrm{drv}} = f_- &+ \lambda_y f_+^* e^{2i\left[\varphi^y_{s_d s} + \pi Q_y\right]} \\ &- f_-(s_d) \frac{\sin \pi Q_y^-}{2 \sin\left(\pi Q_{x-}^d\right)} e^{i\left[\varphi^x_{s_d s} - \varphi^y_{s_d s} + \pi Q_- \mathrm{sgn}(s - s_d)\right]} \\ &- f_+^*(s_d) \frac{\sin \pi Q_y^-}{2 \sin\left(\pi Q_{x+}^d\right)} e^{i\left[\varphi^x_{s_d s} + \varphi^y_{s_d s} + \pi Q_+ \mathrm{sgn}(s - s_d)\right]}. \end{aligned} \tag{13}$$

Analogously to before, we define $Q_{x\pm}^d = Q_x^d \pm Q_y$. The major difference is the absence of the complex conjugate of $f_{-,y}^{\mathrm{drv}}$ and f_-.

Equations (12) and (13) yield the analytical tools needed to improve coupling measurements in future runs of LHC and its High Luminosity upgrade project.

This method will be called *new formula method* in the following.

In this work a comparison between the rescaling method, formula method and the newly calculated coupling terms is shown in the frame of LHC and HL-LHC simulations. A comparison between rescaling and formula method for different phase measurement noise levels can be found in [10].

EFFECT OF COUPLING AT THE POSITION OF THE AC-DIPOLE

As Eq. (12) shows, the jump at the position of the AC-dipole is proportional to the strength of the coupling term $f_-(s_d)$ at this point. The performance of the new formula compared to the previous methods is shown in a set of simulations and discussed in detail in the following subsections.

Note that, in this work, only the *reconstruction* of the driven coupling term is considered and not the *compensation* – i.e. the inversion of this process – needed to obtain the actual coupling term from measurement data.

In order to simulate coupling measurements, MADX tracking is used to produce turn-by-turn data followed by an analysis of the tracked turn-by-turn data using our analysis tool set. The momentum reconstruction uses a second BPM and picks up coupling terms between the two BPMs. This creates an offset in the measured values.

LHC Collision Optics

First, we consider LHC collision optics that were used at the end of Run 2 in 2018.

Figure 1 shows the coupling term f_-. A closed coupling bump was introduced. The AC-dipole is located outside the bump and no change in f_- can be observed at its location.

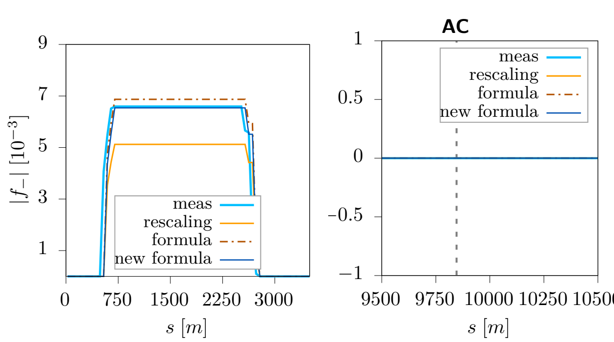

Figure 1: Closed coupling bump in LHC Run 2 collision optics at $\beta^* = 30$ cm. The coupling term f_- is zero outside the bump. At the position of the AC-dipole, the term is not affected. The formula method reconstructs well the driven coupling term f_- whereas the rescaling method shows poorer performance.

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-WEPAB400

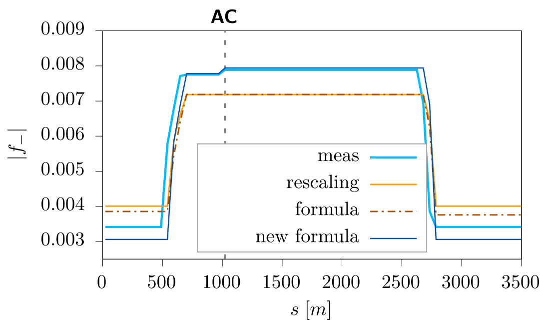

Figure 2: Putting the AC-dipole inside of a coupling bump shows the effect of the AC-dipole on the local coupling terms. This plot shows round LHC collision optics as in Fig. 1. The new formula perfectly reconstructs the jump in amplitude of the coupling term at the position of the AC-dipole whereas rescaling and the previous formula method show a small discrepancy with respect to the real value.

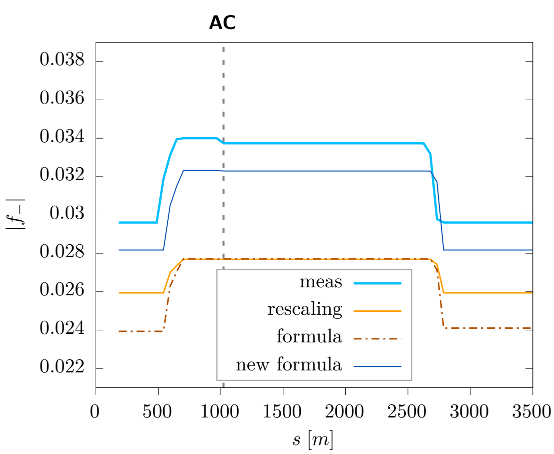

Figure 3: This plot shows round HL-LHC collision optics with $\beta^* = 15$ cm. A similar coupling source is placed in the lattice as for Fig. 2 which has a larger effect on the coupling term than in the LHC case. The agreement between the new formula and the real coupling term is slightly deteriorated but still better than the previous methods.

If the AC-dipole is placed inside of the coupling bump, on the other hand, a jump at its position can clearly be observed, as shown in Fig. 2. The overall reconstruction of the previous methods is poor in comparison to the new method.

HL-LHC optics

The HL-LHC has more challenging optics and a similar coupling source as for the LHC simulations in the previous section, this creates considerably larger coupling as can be seen in Fig. 3. The new formula cannot perfectly reconstruct the real driven coupling but is still superior in comparison to the old methods. Note that the new formula shows only a minuscule jump at the position of the AC-dipole while still keeping a better agreement with the real value than the previous methods.

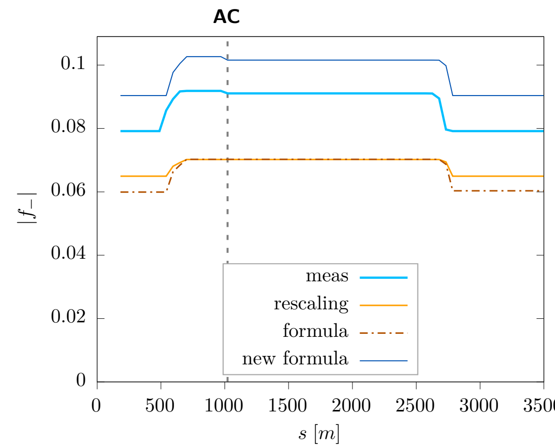

Figure 4: This plot shows the same configuration as Fig. 3 but with a stronger coupling source. The agreement between new method and real value is similar in amplitude but now the new formula yields a coupling term that is larger than the real one.

If we introduce an even stronger coupling source, the agreement starts to deteriorate. Figure 4 shows the same configuration as before but with larger coupling source.

CONCLUSION AND OUTLOOK

The HL-LHC optics is more challenging for our measurement and correction tools than any configuration of the LHC. Novel measurement methods are in order to maintain the needed accuracy and precision and to meet the requirements imposed on optics corrections.

This paper demonstrates a new formalism to calculate the particle's driven coupled motion. The next step is to inverse Equations (12) and (13) in order to obtain corrections that then can be applied to our analysis steps to calculate free coupling terms from the measured ones.

Since the effect of the driven motion on the coupling terms $f_\pm$ is proportional to the individual terms, only an iterative approach seems possible for the correction.

ACKNOWLEDGMENTS

Research supported by the HL-LHC project. We want to thank R. Miyamoto for explaining his AC-dipole compensation formalism to us and A. Franchi for introducing one of the authors to the normal form approach. Thanks go also to our fellow members of the LHC Optics Measurements and Corrections team, especially to T. Persson for fruitful discussions and their time and patience.

REFERENCES

[1] T. Persson and R. Tomás, "Improved control of the betatron coupling in the Large Hadron Collider", *Phys. Rev. ST Accel. Beams*, vol. 17, p. 051004, 2014. doi:10.1103/PhysRevSTAB.17.051004

[2] T. Persson *et al.*, "Transverse Coupling Measurements With High Intensity Beams Using Driven Oscillations", in *Proc.*

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-WEPAB400

9th Int. Particle Accelerator Conf. (IPAC'18), Vancouver, Canada, Apr.-May 2018, pp. 208–211. doi:10.18429/JACoW-IPAC2018-MOPMF047

[3] E. H. Maclean, F. S. Carlier, M. Giovannozzi, T. Persson, and R. Tomas, "Effect of Linear Coupling on Nonlinear Observables at the LHC", in *Proc. 8th Int. Particle Accelerator Conf. (IPAC'17)*, Copenhagen, Denmark, May 2017, pp. 3151–3154. doi:10.18429/JACoW-IPAC2017-WEPIK092

[4] R. Tomás, M. Aiba, A. Franchi, and U. Iriso, "Review of Linear Optics Measurement and Correction for Charged Particle Accelerators", *Phys. Rev. Accel. Beams*, vol. 20, p. 054801, 2017. doi:10.1103/PhysRevAccelBeams.20.054801

[5] R. Bartolini and F. Schmidt, "Normal form via tracking or beam data", *Part. Accel.*, vol. 59, pp. 93–106, 1998.

[6] R. Miyamoto, "Measurement of Coupling Resonance Driving Terms with the AC Dipole", BNL, Upton, NY, USA, Rep. BNL-94350-2010-IR, Oct. 2020.

[7] F. S. Carlier, "A Nonlinear Future - Measurements and corrections of nonlinear beam dynamics using forced transverse oscillations", Ph.D. thesis, Univ. Amsterdam, 2020.

[8] A. Wegscheider, "Development of Optics Measurement Methods for Circular Accelerators", Ph.D. thesis, Universität Hamburg, 2020.

[9] S. Peggs and C. Tang, "Nonlinear diagnostics using an AC Dipole", BNL, Upton, NY, USA, Rep. RHIC/AP/159, 1998.

[10] E. J. Hoydalsvik and T. H. B. Persson, "Reaching the Sub Per Mil Level Coupling Corrections in the LHC", presented at the 12th Int. Particle Accelerator Conf. (IPAC'21), Campinas, Brazil, May 2021, paper THPAB001, this conference.