ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-WEPAB411

ION COULOMB CRYSTALS IN STORAGE RINGS FOR QUANTUM INFORMATION SCIENCE*

K. A. Brown† [1,2], G. Mahler, T. Roser, T. Shaftan, Z. Zhao
Brookhaven National Laboratory, Upton, NY, USA
R. Chen[2], B. Huang[3], T. Robertazzi[2,3], Stony Brook University, Stony Brook, NY, USA
A. Aslam, S. G. Biedron, T. Bolin, S. Sosa Guitron
University of New Mexico, Albuquerque, NM, USA
C. Gonzalez-Zacarias[2], University of Southern California, Los Angeles, CA, USA
[1]also at Stony Brook University, Stony Brook, NY, USA
[2]at Department of Electrical and Computer Engineering
[3]at Department of Applied Mathematics and Statistics

Abstract

We discuss the possible use of crystalline beams in storage rings for applications in quantum information science (QIS). Crystalline beams have been created in ion trap systems and proven to be useful as computational basis for QIS applications. The same structures can be created in a storage ring, but the ions necessarily have a constant velocity and are rotating in a circular trap. The basic structures that are needed are ultracold crystalline beams, called ion Coulomb crystals (ICC's). We will describe different applications of ICC's for QIS, how QIS information is obtained and can be used for quantum computing, and some of the challenges that need to be resolved to realize practical QIS applications in storage rings.

INTRODUCTION

Quantum information science is a growing field that promises to take computing into a new age of higher performance and larger scale computing, able to solve problems classical computers are incapable of solving [1]. The outstanding issue in practical quantum computing today is scaling up the system while maintaining interconnectivity of the qubits and low error rates in qubit operations, to be able to implement error correction and fault-tolerant operations [2]. Trapped ion qubits offer long coherence times that allow error correction [3, 4]. Error correction algorithms require large numbers of qubits. We can potentially create many thousands (or more) of qubits with long coherence states in a storage ring [5]. A circular radio-frequency quadrupole (CRFQ) (shown in Fig. 1) is a large circular ion trap that could enable larger scale quantum computing [6]. Such a Storage Ring Quantum Computer (SRQC) would be a scalable and fault-tolerant quantum information system [7, 8]. With computing demands potentially outpacing the supply of high-performance systems, quantum computing could bring innovation and scientific advances to particle physics and other DOE-supported programs [9].

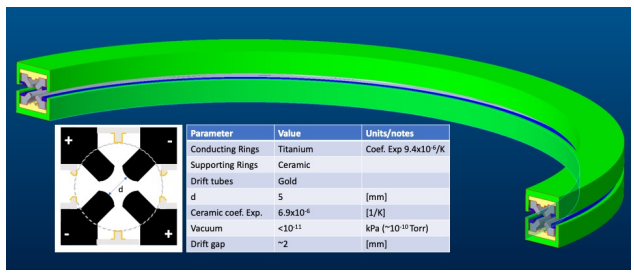

Figure 1: Circular Radio Frequency Quadrupole. Four flat vanes are precision shaped and fitted into ceramic guides. Drift tubes are distributed around the ring and are used for orbit correction and can be used to accelerate the beams. This design is modeled on the PALLAS CRFQ [10].

Overview of Ion Coulomb Crystals and Applications

An ion Coulomb crystal [11–13] is an ultracold crystalline beam [14–17]. It needs to be cooled to the point where the de Broglie wavelength is greater than or equal to the particle thermal oscillation amplitude, but not to the point of the Lamb-Dicke limit [18]. This is something of a 'Goldilocks' regime, where couplings between internal and external quantum states are not strongly suppressed and is well above a Bose-Einstein condensate [19]. In this regime, thermal vibrations are small enough to distinguish the external quantum modes of the crystalline structure.

Related Work in Ion Traps

Trapped Ion quantum computers generally use Paul Traps or similar devices that use an electric field oscillating at RF frequencies to prevent ions from drifting out of the trap while they are cooled and stopped in the trap [20]. Many designs and methods are being developed to resolve issues that can lead to practical quantum computers using trapped ions [21]. These issues are usually described as the DiVincenzo's criteria, which state that a system is scalable, has means for initializing qubits, allows operations within decoherence times, has methods for a universal set of operations, and allow qubits to be easily read [22]. What is now significant is the technologies are well enough advanced that companies

* This work was performed under Contract No. DE-SC0012704 with the auspices of the U.S. Department of Energy.
† kbrown@bnl.gov

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-WEPAB411

are forming to market ion trapped based quantum computing systems [23].

The use of ICCs goes beyond just quantum computing. They can be used for quantum memory (note the no cloning theorem) and for error correction. ICCs are also of interest, as poorly studied states of matter may exist on the surface and atmosphere of neutron stars.

ENTANGLEMENT AND GATES

Entanglement plays an essential role in quantum computing. What is entangled are the complex vectors describing the wavefunction states in two quantum systems. In our case, we exploit the hyperfine states of electrons in the upper energy level shells of atoms in an ICC. Our qubit states are the hyperfine spin states. Even when the ions are separated by a large distance, but the hyperfine quantum states can be coupled. If we had two particles with entangled hyperfine states, we could put one into a spin-up state, by exciting it with a laser pulse. The other ions spin state will then go into a spin-down state. We can observe this by measuring the states and seeing that if one is fluorescing when illuminated by laser light, the other one will always be dark. These are our canonical qubit $|1\rangle$ and $|0\rangle$ states for the ions in an ICC. The two ions were placed in such a state by performing operations that create an entangled wavefunction, such as $\psi = |01\rangle - |10\rangle$. Such a state does not factorize into a product of the single particle wavefunctions, so $\psi \neq \psi_1\psi_2$, meaning the two are entangled.

In quantum computers changing the state of an entangled qubit, which then changes the state of the paired qubit (almost) immediately, improves the processing speed of the quantum computation. This is necessary in order to achieve exponential speed-up over classical algorithms [24].

To entangle the two wavefunctions, we need to perform an operation that will create a new wavefunction that cannot factorize into the original two. This is the role of quantum gate operations. One of the first, and most important quantum gates is the CNOT gate. The CNOT gate is well-known in classical computing, but the quantum version operates on the wavefunctions, not a single state. In our hyperfine spin states example, we can describe the wavefunction of two ion qubits as $\psi_1 = \alpha_0|0\rangle + \alpha_1|1\rangle$ and $\psi_2 = \beta_0|0\rangle + \beta_1|1\rangle$, where $|\alpha_0|^2 + |\alpha_1|^2 = |\beta_0|^2 + |\beta_1|^2 = 1$ (since the probability amplitudes can be complex quantities). A CNOT operation will change the states of the combined wavefunctions as $|00\rangle \rightarrow |00\rangle$, $|01\rangle \rightarrow |01\rangle$, $|10\rangle \rightarrow |11\rangle$, and $|11\rangle \rightarrow |10\rangle$. If we have a third wavefunction, we will call a control bit, $\psi_C = 1/\sqrt{2}(|0\rangle + |1\rangle)$ and our target qubit is $|0\rangle$, a new wavefunction will be created that will entangle the two wavefunctions to a state $\psi = 1/\sqrt{2}(|00\rangle + |11\rangle)$. We use as our control bit an external eigenstate from the quantum motional modes of the ion chain. Through a series of laser excitations, the CNOT gate operation is performed using this control bit to produce entanglement of the two internal eigenstates [25].

Quantum circuits are interconnections [26] of quantum gates that can accomplish some desired overall function [27, 28]. There are well known quantum circuits for purposes such as teleportation, super dense coding and error correction. Quantum gates include well known ones such as the H, CNOT, SWAP, I, Z, X and Y gates. Sometimes a gate may be expressed as an equivalent interconnection of more basic gates [29]. Sometimes gates entangle multiple qubits. Finally each logical qubit may be represented by several physical qubits (i.e., ions and their states) [30].

MOTIONAL MODES

A classical object, an ion Coulomb crystal can be described as a string of charged masses acting as simple harmonic oscillators. This structure can be described in one dimension as masses coupled by springs. Here the motion is small compared to the distances between the ions and so the spring coefficient, $m\omega_0^2$, is constant. The fundamental frequency, ω_0, for the chain of ions and is a function of the Coulomb and other potentials holding the ions in the chain. In this case, the motion is seen as modes in the axial motion of the ions. Looking at it in three dimensions [31], there are N independent harmonic oscillators each of charge +1 and mass m, the potential is the sum of applied voltages and the Coulomb potential of the adjacent ions,

$$V = \frac{m}{2}\sum_{j=1}^{N}(\omega_x^2 x_j^2 + \omega_y^2 y_j^2 + \omega_z^2 z_j^2) + \sum_{n,j}\frac{e^2}{4\pi\epsilon_0 r_{n,j}}, \quad (1)$$

where $r_{n,j}$ is the distance between ions n and j and ω_u is the center of mass frequency (of the system) along the u direction. We define d to represent a unitless scale length, $d_n = z_n^0/l$, where $l^3 = e^2/(4\pi\epsilon_0 m\omega_z^2)$. The potential can be expanded out as a Taylor series, where the first order is zero (by definition) and the second order represents a harmonic approximation of the potential seen by the ions.

$$V = \frac{1}{2}\sum_{\xi,n,j} q_n^\xi q_j^\xi \left[\frac{\partial^2 V}{\partial\xi_n \partial\xi_j}\right]_{\xi_n=\xi_n^0}. \quad (2)$$

Expanding around the ion equilibrium positions, $\xi_n^0 = (x_n^0, y_n^0, z_n^0)$, such that $\xi_n = \xi_n^0 + q_n^\xi$, q_n^ξ represents the position of ion n in the direction ξ. In this case, the quantized vibrational energy per mode n, is

$$E_n = < n|H_0|n > = (n + \frac{1}{2})\hbar\omega_0. \quad (3)$$

As the number of ions increase, the number of modes also increase. An ensemble of ions of size $N + 1$ has $3N + 3$ motional degrees of freedom, so as N becomes large, cooling and controlling the chain becomes very difficult. However, in a storage ring, we can imagine isolating two small ensemble of ions, cooling them independently, and then adiabatically merging the two chains, each already in a ground state.

Tracking large numbers of ions may seem to be more an engineering problem. But given that we need to cool all ions down to stable ICCs of variable lengths, we need to quickly and precisely locate each ion. When two ions are entangled we want to be able to identify them, especially in

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-WEPAB411

an environment where many entangled ions may exist in a single volume.

The equilibrium positions of the ions $z_n^0 (x_n^0 = y_n^0 = 0)$ are determined from the first derivative of the potential. Using the scaling parameter, u_n, the longitudinal equilibrium positions of the ions will be given by

$$u_j - \sum_{n=1}^{j-1} \frac{1}{(u_j - u_n)^2} + \sum_{n=j+1}^{N} \frac{1}{(u_j - u_n)^2} = 0. \quad (4)$$

The scaling parameter, $u_n = z_n^0/l$, where $l = [e^2/(4\pi\epsilon_0 m\omega_z^2]^{1/3}$. Equation (4) is a set of N coupled algebraic equations and can only be solved numerically. To determine the positions precisely as N becomes large, the computation time becomes exceedingly long, scaling by $O(N^2)$. The calculation can be parallelized, but gets very CPU greedy as a function of N. On a GPU implementation, the data transfers limit the wall clock time. We studied the use of machine learning and doing parametrized fittings to find solutions [32]. If we can eliminate the number of variables to calculate, we can reduce the computation time. This can be done by re-expressing Eq. (4) in terms of the ion spacings and then only calculating the increases in spaces that are significant. This approximation method may be sufficient for the purposes of quickly finding and labeling ion positions experimentally.

$$-[\frac{N-1}{2} - (j-1)]V_{min} - \sum_{i=j}^{\lfloor \frac{N}{2} \rfloor - 1} V_i - \sum_{n=1}^{j-1} \frac{1}{[(j-n)V_{min} + \sum_{i=n}^{j-1} V_i]^2} + \sum_{n=j+1}^{\lceil \frac{N}{2} \rceil} \frac{1}{[(n-j)V_{min} + \sum_{i=j+1}^{n-1} V_i]^2} + \sum_{n=\lceil \frac{N}{2} \rceil + 1}^{N} \frac{1}{[(n-j)V_{min} + \sum_{i=j}^{\lfloor \frac{N}{2} \rfloor - 1} V_i + \sum_{k=N-(n-1)}^{\lfloor \frac{N}{2} \rfloor - 1} V_k]^2} = 0 \quad (5)$$

Figure 2 shows the new variables. Expressing Eq. (4) in terms of these variables, we find Eq. (5), where, $(j = 1, 2, 3, ..., \lceil \frac{N}{2} \rceil)$. The variables, V_{min}, V_i, and V_k are in the same units as the u_n. The problem looks more complicated, but as N becomes large, V_{min} asymptotically approaches a constant value and the values of V_i in the middle of the chain asymptotically become constant and small, compared to V_{min}. We are still investigating this approach, but with the promise of reducing the number of variables we can speed up the calculation.

3D COOLING

The main difficulty with transverse cooling in a ring is the transverse velocity components are not accessible. To expose them requires adding some method of kicking the ions tangentially, as in sympathetic cooling, electron cooling,

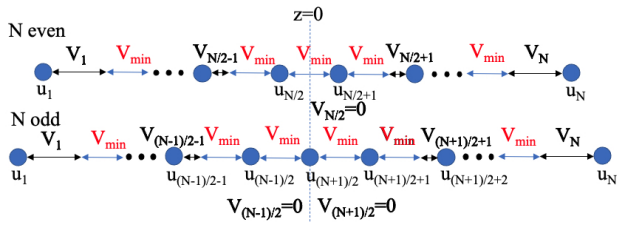

Figure 2: Equilibrium spacings variables.

or even stochastic cooling, or finding a way to expose the transverse components to a longitudinal laser pulse. This would simulate cooling by optical molasses. To this end, we are investigating the use of an electrostatic wiggler (both in a simple configuration or as a helical wiggler).

For example, consider a CRFQ style drift tube configured to be used as a deflector. We will assume it acts very close to a simple parallel plate deflector. The drift tube is 8.5 mm long with a gap of 10 mm. The design is such that the field only penetrates at 0.5%, for a 500 V applied potential, the ions only see a 2.5 V potential.

Consider a Ytterbium ion, singly charged, enters the drift tube on axis with a velocity of 1000 m/s. It will see an electric field of $E = V_{eff}/h = 2.5/10^{-2} = 250$ V/m and will experience a force of $F = eE = eV/h = 4x10^{-17}$ N. This will introduce an acceleration of $a = F/m = 1.4x10^8$ m/sec^2 (mass = 173.04 u). A simple simulation shows we can obtain rotation angles of over 0.15 radian, which could be made variable over all smaller angles. With the wiggler contained within the CRFQ lattice, particles will remain confined by the transverse RF linear focusing.

CONCLUSION

The use of ICCs in storage rings can potentially allow scaling of QIS applications beyond what is available today or even in the near future. Since the SRQC is nothing more than an unbounded ion trap, the same technologies and techniques can be applied. The challenges in such an approach are in developing new methods to track ions with non-zero velocities, cooling a large number of ions, and controlling the computational work as many lasers are applied to write and measure qubit states.

REFERENCES

[1] L. Gyongyosi and S. Imre, "A Survey on quantum computing technology", *Computer Science Review*, vol. 31, pp. 51–71, 2019. doi:10.1016/j.cosrev.2018.11.002

[2] J. M. Auger, H. Anwar, M. Gimeno-Segovia, T. M. Stace, and D. E. Browne, "Fault-tolerant quantum computation with nondeterministic entangling gates", *Phys. Rev. A* , vol. 97, p. 030301(R), 2018. doi:10.1103/PhysRevA.97.030301

[3] A. Steane, "The ion trap quantum information processor", *Appl. Phys. B.*, vol. 64, pp. 623–643, 1997. doi:10.1007/s003400050225

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-WEPAB411

[4] D. J. Wineland *et al.*, "Experimental Primer on the Trapped Ion Quantum Computer", *Fortschr. Phys.*, vol. 46, pp. 363-390, 1998. doi:10.1002/3527603093.ch3

[5] D. Aharonov and M. Ben-Or, "Fault-Tolerant Quantum Computation With Constant Error Rate", in *Proc. 29th annual ACM symposium on Theory of computing (STOC'97)*, ACM, New York, NY, USA, pp. 176-188, 1997. doi:10.1137/S0097539799359385

[6] U. Schramm, M. Bussmann, D. Habs, and T. Schaetz, "Storage of Crystalline Ion Beams", in *Proc. 20th Particle Accelerator Conf. (PAC'03)*, Portland, OR, USA, May 2003, paper TOAA004, pp. 112–116.

[7] K. A. Brown and T. Roser, "Towards storage rings as quantum computers", *Phys. Rev. Accel. Beams*, vol. 23, p. 054701, May 2020. doi:10.1103/physrevaccelbeams.23.054701

[8] T. Shaftan, "On challenges with using crystalline beams of ions as a medium for Quantum Computers", BNL, Upton, NY, USA, Rep. NSLSII-ASD-TN-309, Jul. 2019.

[9] T. Boccali, "Computing models and resource needs for ATLAS and CMS during HL_LHC ", presented at the CERN OpenLab workshop, Quantum Computing for High Energy Physics, 2019, unpublished.

[10] U. Schramm, T. Schätz, and D. Habs, "Bunched Crystalline Ion Beams", *Phys. Rev. Lett.*, vol. 87, p. 184801, Oct. 2001. doi:10.1103/PhysRevLett.87.184801

[11] J. Wei and A.M. Sessler, "Crystalline Beams", SLAC, Menlo Park, CA, USA, Rep. SLAC-Report-574, 2003.

[12] A. M. Sessler, "Methods of Beam Cooling", LBNL, Berkeley, CA, USA, Rep. LBL-38278, 1995.

[13] T. Schätz, U. Schramm, and D. Habs, "Crystalline ion beams", *Nature*, vol. 412, pp. 717–720, 2001. doi:10.1038/35089045

[14] U. Schramm, T. Schätz, M. Bussmann, and D. Habs, "The quest for crystalline ion beams", *Plasma Phys. Control. Fusion*, vol. 44, pp. B375–B387, 2002. doi:10.1088/0741-3335/44/12B/326

[15] U. Schramm, T. Schätz, M. Bussmann, and D. Habs, "Cooling and heating of crystalline ion beams", *J. Phys. B: At. Mol. Opt. Phys.*, vol. 36, pp. 561–571, 2003. doi:10.1088/0953-4075/36/3/314

[16] U. Schramm and D. Habs, "Crystalline ion beams", *Progress in Particle and Nuclear Physics*, vol. 53, pp. 583-677, 2004. doi:10.1016/j.ppnp.2004.03.002

[17] U. Schramm, T. Schätz, and D. Habs, "Three-dimensional crystalline ion beams", *Phys. Rev. E*, vol. 66, p. 036501, Sep. 2002. doi:10.1103/PhysRevE.66.036501

[18] D. J. Wineland, "Experimental Issues in Coherent Quantum-State Manipulation of Trapped Ions", *J. Research National Institute Standards Tech.*, vol. 103. p. 259, 1998. doi:10.6028/jres.103.019

[19] C. J. Foot, *Atomic Physics*, Oxford, UK: Oxford University Press, 2014.

[20] W. Paul, "Electromagnetic traps for charged and neutral particles", *Rev. Mod. Phys.*, vol. 62, p. 531, 1990. doi:10.1103/RevModPhys.62.531

[21] B. Tabakov *et al.*, "Assembling a ring-shaped crystal in a microfabricated surface ion trap", *Phys. Rev. Applied*, vol. 4, p. 031001, 2015. doi:10.1103/PhysRevApplied.4.031001

[22] D. P. DiVincenzo, "The Physical Implementation of Quantum Computation", *Fortschritte der Physik*, vol. 48, pp. 771–783, 2020. doi:10.1002/3527603182.ch1

[23] S. K. Moore, "Rapid Scale-Up of Commercial Ion-Trap Quantum Computers", Nov. 2020, unpublished.

[24] R. Jozsa and N. Linden, "On the Role of Entanglement in Quantum-Computational Speed-Up", *Proceedings: Mathematical, Physical and Engineering Sciences*, vol. 459, no. 2036, pp. 2011-2032, 2021. doi:10.1098/rspa.2002.1097

[25] J. I. Cirac and P. Zoller, "Quantum Computations with Cold Trapped Ions", *Phys. Rev. Lett.*, vol. 74, p. 4091, May 1995. doi:10.1103/PhysRevLett.74.4091

[26] K. A. Brown and T. Robertazzi, "Mapping Quantum Circuits to Ions in a Storage Ring Quantum Computer", *IEEE Transactions on Quantum Engineering*, submitted for publication.

[27] C. Bernhardt, *Quantum Computing for Everyone*, MIT, Massachusetts, USA: MIT Press, 2019.

[28] M. Nielsen and I. Chuang, *Quantum Computation and Quantum Information*, Cambridge, UK: Cambridge University Press, 2000.

[29] J. H. Bae *et al.*, "Quantum Circuit Optimization using Quantum Karnaugh Maps", *Scientic Reports, Nature Research*, vol. 10, no. 1, Sep. 2020. doi:10.1038/s41598-020-72469-7

[30] D. Kielpinski, C. Monroe, and D. J. Wineland, "Architecture for a large-scale ion-trap quantum computer", *Nature*, vol. 417, no. 6890, pp. 709-711, Jul. 2002. doi:10.1038/nature00784

[31] S. Zhu, C. Monroe, and L.-M. Duan, "Trapped Ion Quantum Computation with Transverse Phonon Modes", *Phys. Rev. Lett.*, vol. 97, p. 050505, Aug. 2006. doi:10.1103/PhysRevLett.97.050505

[32] B. Huang *et al.*, "AI assisted design and virtual diagnostic for the initial condition of a storage ring quantum information system", *IEEE Access*, submitted for publication.