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T-BMT SPIN RESONANCE TRACKER CODE FOR He3 WITH SIX SNAKES*

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Abstract

Polarization lifetime for He3 using two and six snakes are studied using the T-BMT Spin Resonance Tracker code. This code integrates a reduced spinor form of the T-BMT equation including only several spin resonances and the kinematics of synchrotron motion. It was previously benchmarked against RHIC polarization lifetime under the two snake system.

INTRODUCTION

The proposed Electron Ion Collider (EIC) will collide polarized Helium 3 (He3) with polarized electrons at energies above 180 GeV. The larger magnitude of the anomalous magnetic moment coefficient ($G = \frac{g-2}{2}$) of -4.184 compared to protons of 1.7928474 He3 result in stronger spin depolarizing resonances. To assist in managing the strong spin resonances an additional 4 snakes will be installed in the upgrade of the Yellow RHIC ring, yielding a total of 6 snakes. While preliminary spin tracking results have shown that the additional snakes will ensure high polarization transmission over the acceleration cycle from injection to 183 GeV, assessing the polarization lifetime for He3 beam under in the new EIC hadron lattice is still challenging. This is due to the fact that direct spin tracking requires tracking over 300 million turns to estimate 1 hour of beam polarized lifetime. This is computationally prohibitive given current speed of existing direct tracking codes. Additionally for these long times the issue of numerical noise and round off error can become very important.

LATTICE INDEPENDENT INTEGRATION

There has been some apparent success in estimating polarization lifetimes using a reduced spin tracking model [1]. This approach uses a multiple spin resonance model including simplified longitudinal effects. However when benchmarked against measured polarization lifetimes, this approach appears to over-estimate lifetime decay beyond 1 hour of beam time. The approach integrates a form of the Thomas-BMT equation recast using a two component spinors Φ,

$$\frac{d\Phi}{d\theta} = -\frac{i}{2}\begin{pmatrix} G\gamma & -\zeta(\theta) \\ -\zeta(\theta)^* & -G\gamma \end{pmatrix}\Phi. \tag{1}$$

Here G is the anomalous magnetic moment coefficient and $\zeta(\theta) = \sum_K \mathrm{w}_K e^{-iK\theta}$ with w_K the complex spin resonance amplitude at precession frequency K. Using a 4th order Magnus Gaussian quadrature integrator described in [2] Eq. (1) can be integrated for an arbitrary $\zeta(\theta)$. In the code the effect of snakes are handled separately and are added at the appropriate place in θ as optically transparent thin spin kicks using the matrix,

$$T_s = e^{-i\frac{\pi}{2}\hat{n}_s\cdot\vec{\sigma}}, \tag{2}$$

with $n_s = \hat{e}_1\cos\phi_s + \hat{e}_2\sin\phi_s$ and ϕ_s the snake angle.

The effects of longitudinal motion are included via a modulation of the betatron phase component of the spin resonance phase. yielding a new $\hat{\zeta}$ term,

$$\begin{aligned}\hat{\zeta}(\theta) &= \zeta(\theta)g(\theta) \\ &= \sum_K \mathrm{w_K e^{-iK+i\frac{00}{}(1-\cos(Q_s))}}.\end{aligned} \tag{3}$$

Here the initial betatron phase is absorbed into the complex phase of $\mathrm{w}_K = a_K e^{-i\phi_K}$. We choose the initial $\delta(0) = 0$ since that will only alter the constant initial phase and shouldn't contribute to the dynamics which drive polarization lifetime.

In addition to the phase effect there is also a direct energy modulating effect which has previously been described by Lee and Berglund [3]. This effect modifies $G\gamma$ as follows,

$$G\gamma(\theta) = (G\gamma_0 + \alpha\theta)(1 + \delta(\theta)). \tag{4}$$

NUMERICAL INTEGRATION SET UP

The code was set up to integrate Eq. (1) with $\hat{\zeta}(\theta)$ given by:

$$\hat{\zeta}(\theta, m) = \sum_K \mathrm{w_K e^{-iK+i\frac{00}{}(1-\cos(2Q_s m))}}. \tag{5}$$

Here m is the turn number and the phase due to the synchrotron motion is updated only once per turn as is the value of $G\gamma$. Prior to integration Eq. (1) was transformed to the interaction frame using $U = e^{\frac{-i}{2}G\gamma\theta\sigma_z}$. The new equation written in matrix form, with its initial conditions is,

$$\Psi' = A(\theta)\Psi, \quad \Psi(\theta_0) = \Psi_0. \tag{6}$$

The 4th order Magnus Gaussian quadrature integrator evaluates the matrix $A(\theta)$ at two orbital locations $\theta + \left(\frac{1}{2} \pm \frac{\sqrt{3}}{6}\right)h$

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with step size h:

$$A_1 = A\left(\theta_n + \left(\frac{1}{2} - \frac{\sqrt{3}}{6}\right)h\right)$$
$$A_2 = A\left(\theta_n + \left(\frac{1}{2} + \frac{\sqrt{3}}{6}\right)h\right). \quad (7)$$

These are then used to calculate the Ω used to propagate the spinnor:

$$\Omega^{[4]}(h) = \frac{h}{2}(A_1 + A_2) - h^2\frac{\sqrt{3}}{12}[A_1, A_2]$$
$$\Psi_{n+1} = \exp(\Omega^{[4]}(h))\Psi_n. \quad (8)$$

He3 LIFETIME

Near store energy $G\gamma = -820$ the intrinsic spin resonances are shown in Fig. 1. Here we can see that spin resonances are between 0.1 and 0.2 magnitude for a 10 mm-mrad normalized particle. This is an order of magnitude or more than for protons. Including orbit errors yielding an RMS orbit of 0.5528 mm gives imperfection spin resonances shown in Fig. 2 are in a similar range.

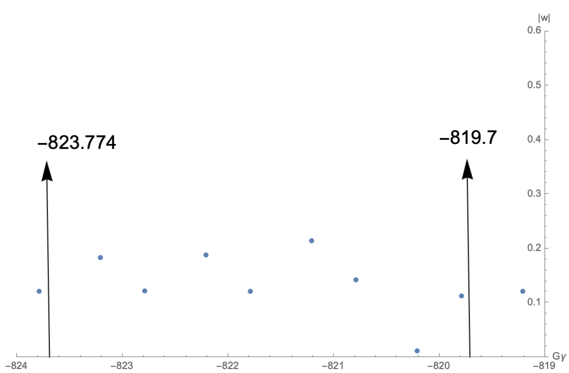

Figure 1: He3 intrinsic spin resonance strength for 10 mm-mrad normalized emittance near the proposed store energy of $G\gamma$ = -819.7. We also show our results for the $G\gamma$ = -823.774.

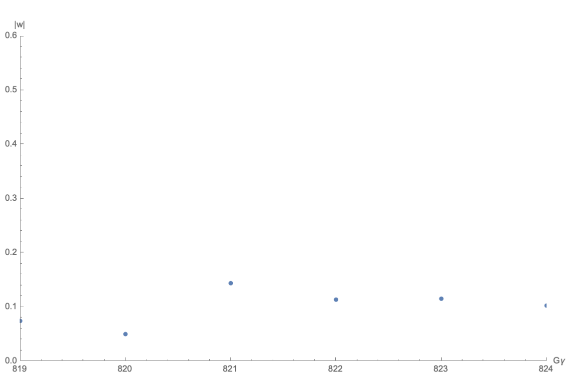

Figure 2: He3 imperfection spin resonance strength for RMS orbit of 0.5528 mm near the proposed store energy of $G\gamma$ = -819.7.

We considered two energy locations at $G\gamma = -823.774$ and another at $G\gamma = -819.7$. Figure 3 shows the -823.774 location which showed lifetimes around 10%/hr. In Fig. 4 the -819.7 location gave better lifetimes of about 5%/hr. Interestingly for the smallest emittances increasing from two to six snakes didn't universally improve this number.

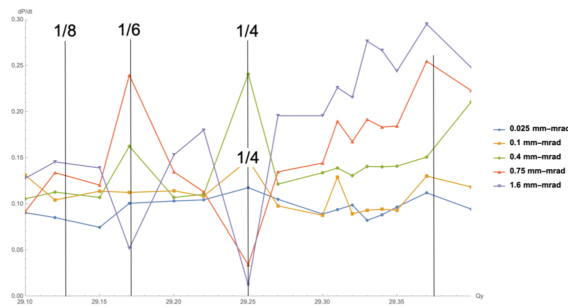

Figure 3: He3 Polarization lifetime at store energy of $G\gamma$ = -823.774, for various emittances with RMS orbit distortion of 0.5528 mm. The locations of the strong snake resonances are indicated at 1/8, 1/6, 1/4 and 3/8 fractional tunes.

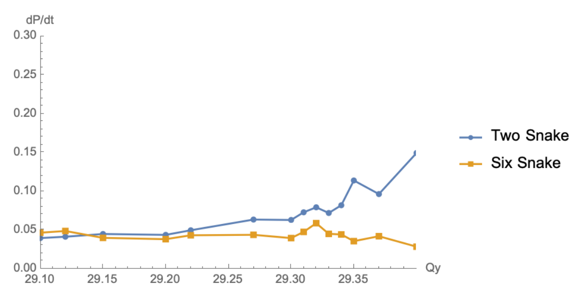

Figure 4: He3 Polarization lifetime at store energy of $G\gamma$ = -819.7, for 0.75 mm-mrad normalized emittance with RMS orbit distortion of 0.5528 mm.

DISCUSSION

In the current case we are modeling He3 polarization lifetimes using 6 snakes as opposed to only two snakes, however lifetime remained in the 5-10% per hour range. Lacking the ability to benchmark the simulations involving 6 snakes and in light of the issues for longer tracking time we could not be sure that the high polarization decay rate was not a numerical artifact. To understand this better we are in the process of re-evaluating the magnus integrator to better understand the numerical characteristics. In previous work checks of unitarity of the transported spinor found deviations of less than 1×10^{-8} after tracking over two hours. Also reverse tracking was performed for 128 particles tracked over 1 hour (2.5×10^{8} turns) reverse tracking recovered the starting spin values to within 5×10^{-6}.

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