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DECOUPLE TRANSVERSE COUPLED BEAM IN THE DTL WITH TILTED PMQs *

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Abstract

The coupling of the beam is widely studied in the accelerator physics field. Projected transverse emittances easily grow up if the beam is transversely-coupled. If we decouple the transverse coupled beam, the transverse emittance can be small. The matrix approach based on the symplectic transformation theory for decoupling the coupled beam is summarized. For a proton accelerator, the transverse coupled beam is introduced by an RFQ tilted by 45°. The beam is decoupled with the first five tilted quadrupoles mounted in the DTL section. A study on the gradient choice of the quadrupoles considering space charge effect is given in this paper.

INTRODUCTION

The transverse acceptance limits are strict in the aspect of synchrotron injection. Beam coupling can lead to projected transverse emittance growth. It is important to decouple the coupled beam for small projected transverse emittance. The theories of describing the coupled dynamics with matrix [1,2], Hamiltonian theory [3,4], Courant-Snyder theory [5,6] have been developed.

The beam coupled by a solenoid is decoupled with a normal triplet and skew triplet after the beamline [7]. Quadrupole-solenoid-quadrupole system is used to cancel the coupling in a solenoid [8]. A quadrupole corrector, consisting of normal and skew quadrupoles, eliminates the coupling in a solenoid [9].

In this paper, the summary of the matrix approach based on the symplectic transformation theory to decouple the coupled beam is given. The beam is uncoupled with five tilted fixed-gradient PMQs in the DTL cavity for a proton therapy system.

BASIC DEFINITIONS

The beam can be described in the transverse beam matrix, which is symmetric with ten independent variables,

$$\Sigma = \begin{pmatrix} \langle xx\rangle & \langle xx'\rangle & \langle xy\rangle & \langle xy'\rangle \\ \langle xx'\rangle & \langle x'x'\rangle & \langle x'y\rangle & \langle x'y'\rangle \\ \langle xy\rangle & \langle x'y\rangle & \langle yy\rangle & \langle yy'\rangle \\ \langle xy'\rangle & \langle x'y'\rangle & \langle yy'\rangle & \langle y'y'\rangle \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}. \tag{1}$$

The four-dimensional RMS emittance ε_{4D}, projected RMS emittances in x plane ε_x, and projected RMS emittances in y plane ε_y are the square root of the determinant of Σ, σ_{xx}, σ_{yy}, respectively. The transverse emittance is $\varepsilon_t = \sqrt{\varepsilon_{4D}}$. The elements in σ_{xy} or σ_{yx} represent the coupling in x and y planes. We can obtain $\varepsilon_{4D} \le \varepsilon_x \cdot \varepsilon_y$, which is illustrated in other papers [10].

The emittance growth can be used as a criterion to characterize the decoupling capacity [11]:

$$\Delta\varepsilon = \frac{\sqrt{\varepsilon_x \cdot \varepsilon_y} - \varepsilon_t}{\varepsilon_t}. \tag{2}$$

The beam matrix at s_2 ($\Sigma(s_2)$) can be calculated with transfer matrix from s_1 to s_2 (R) and the beam matrix at s_1 ($\Sigma(s_1)$),

$$\Sigma(s_2) = R\Sigma(s_1)R^T, \tag{3}$$

R obeys the symplecticity condition,

$$S = R^T S R, \tag{4}$$

which is a congruent transformation, with

$$S = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \tag{5}$$

DECOUPLING APPROACH

The beam is decoupled if the beam matrix is a diagonal matrix. To solve the problem of decoupling, the beam matrix is proved to be symplectically congruent to a diagonal matrix based on the symplectic transformation.

In particular, as the matrix Σ is symmetric and $\Sigma S^{-1}\Sigma S$ is diagonalizable [11], Σ can be simplectically congruent to a diagonal matrix D according to the theorem in Ref. [12]:

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$$D = R\Sigma R^T = \begin{pmatrix} \epsilon_1 & 0 & 0 & 0 \\ 0 & \epsilon_1 & 0 & 0 \\ 0 & 0 & \epsilon_2 & 0 \\ 0 & 0 & 0 & \epsilon_2 \end{pmatrix}, \tag{6}$$

where R is the transfer matrix, ϵ_1 and ϵ_2 are eigen-emittances [7], with

$$\epsilon_1 = \frac{1}{2}\sqrt{-\mathrm{tr}(\Sigma S \Sigma S) - \sqrt{(\mathrm{tr}(\Sigma S \Sigma S))^2 - 16\det(\Sigma)}}, \tag{7a}$$

$$\epsilon_2 = \frac{1}{2}\sqrt{-\mathrm{tr}(\Sigma S \Sigma S) + \sqrt{(\mathrm{tr}(\Sigma S \Sigma S))^2 - 16\det(\Sigma)}}. \tag{7b}$$

DECOUPLE IN THE DTL

Injector

The linac injector consists of an ECR source, a LEBT, an RFQ, and a DTL. The linac injector is a part of a proton therapy system [13]. The transverse coupling is introduced by the RFQ cavity, which is rotated 45° around the beam axis (see Fig. 1). The vacuum pumps are installed beneath the RFQ cavity. The output beam distribution is shown in Fig. 2. The projected normalized RMS emittance at the entrance of the RFQ is 0.2 mm·mrad, it grows to 0.37 mm·mrad at the end of the RFQ. The beam matrix at the end of the RFQ with SI unit is given as

$$\Sigma = 10^{-6} \times \begin{pmatrix} 0.727 & 0.169 & -0.014 & 3.979 \\ 0.170 & 29.774 & 3.964 & 2.373 \\ -0.014 & 3.964 & 0.725 & 0.158 \\ 3.979 & 2.373 & 0.158 & 29.845 \end{pmatrix}. \tag{8}$$

With the procedure in Ref. [11], the decouple matrix can be solved as

$$P = \begin{pmatrix} 3.21 & -0.091 & 0.87 & -0.53 \\ 0 & 0.082 & 1.30 & 0.056 \\ 1.32 & -0.060 & 0 & 0.084 \\ 0.57 & 0.53 & -3.15 & -0.10 \end{pmatrix}. \tag{9}$$

Thus, the diagonal matrix D can be obtained, $D = P\Sigma P^T$.

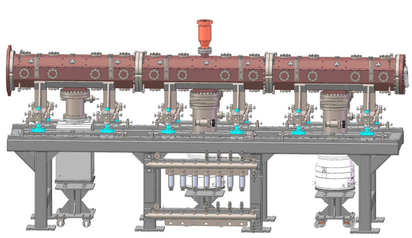

Figure 1: RFQ cavity.

To make the linac compact, no MEBT is between the RFQ and the DTL, which means the beam needs to be decoupled

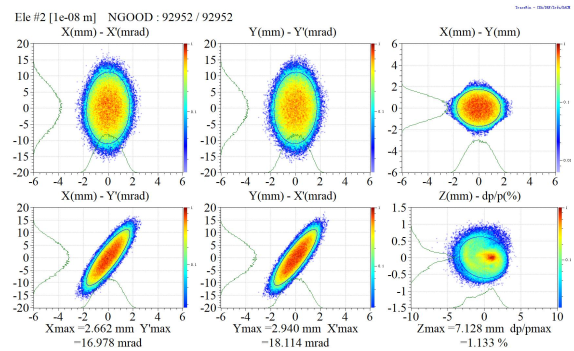

Figure 2: Beam distribution at the exit of the RFQ.

in the DTL cavity. Fixed-gradient PMQs are installed inside the Alvarez-type DTL with FD lattice. The bore radius of the drift tubes is 7 mm. Tilted PMQs can be adopted to decouple the beam.

The first five PMQs are rotated for the decoupling. The cut view of the DTL cavity is shown in Fig. 3. The rotation angles can be solved numerically [11].

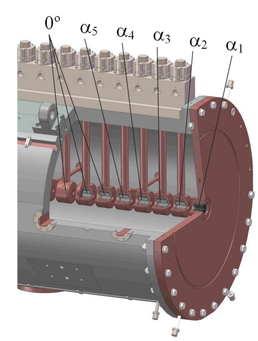

Figure 3: Cut view of the DTL cavity.

Gradient

With different gradients, numerical solutions of rotation angles are presented in Fig. 4 (zero current) and Fig. 5 (16 mA). If the gradient is more than 75 T/m, the solutions are different. Other solutions are similar. For other cases, the zero-current solutions of the rotation angles gradually change from around 45° to around 10 degrees, while the 16 mA solutions gradually change from around 45° to around 10°. 70 T/m is chosen for the gradient of the PMQs. One reason is the solution is similar in most cases, the other one is the beam size is not that large [11].

Space Charge Effect

With the different current of the beam, the matched beam parameters can be obtained, the numerical solution of the rotation angles with a gradient of 70 T/m is shown in Fig. 6. It shows the solutions are quite stable if the current is larger than 15 mA.

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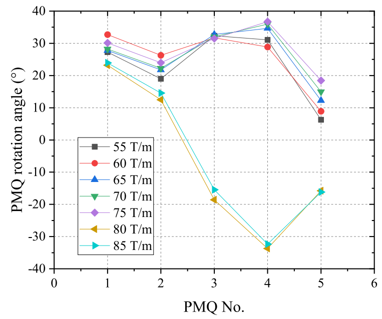

Figure 4: Solutions of rotation angles without space charge effect.

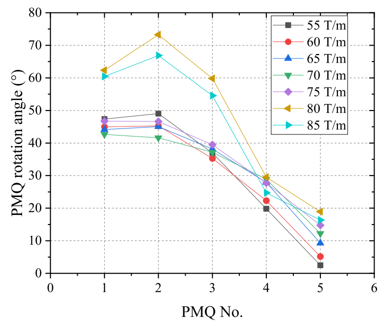

Figure 5: Solutions of rotation angles with space charge effect.

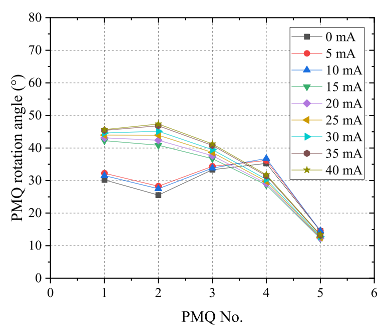

Figure 6: Solutions of rotation angles with a different current.

Final Design

The final decoupling section consists of 5 tilted PMQs with a gradient of 70 T/m. The rotation angles of the PMQs are 45°, 35°, 25°, 15°, 5° for easy assembly [11]. The multi-particle simulation is performed with TraceWin [14]. The beam distribution at the end of the DTL is shown in Fig. 7. The projected transverse emittance is 0.20 mm·mrad. The multi-particle simulation shows that the emittance growth can be decreased to 1.8% with the space charge effect.

Online-Rotatable PMQ Inside the Drift Tube

In most general cases, the transverse-coupling is not known in advance. To solve those cases, online-rotatable PMQ inside the drift tube is designed. With several online-rotatable PMQs, the beam can be decoupled in the accelerating cavity.

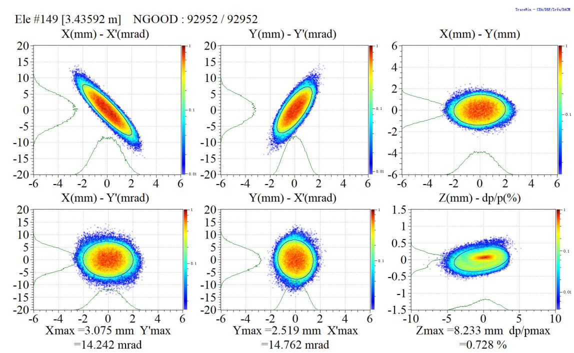

Figure 7: Beam distribution at the exit of the DTL.

The mechanical design of online-rotatable PMQs is a challenge as the size of the drift tube is small. The online-rotatable PMQs need to be compact. A possible mechanical design is shown in Fig. 8. The PMQ is supported by a thin sleeve, which can be rotating coaxially on the ball bearing. The sleeve is equipped with gear. The power for the rotation is provided by a servo motor placed at the one end and transmitted through a small gearbox system.

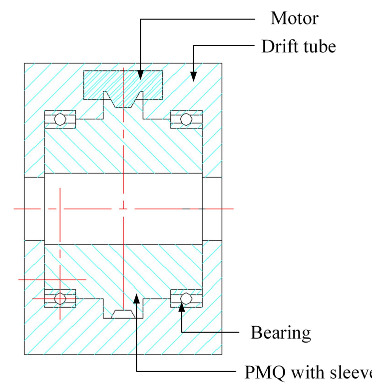

Figure 8: Mechanical design of the online-rotatable PMQ.

CONCLUSION

This paper gives a summary of the matrix approach based on the symplectic transformation theory to decouple the coupled beam. The beam is uncoupled with five tilted fixed-gradient PMQs in the DTL cavity for a proton therapy system. The emittance growth can be suppressed below 2% with multi-particle simulation. Also, a mechanical design for an online-rotatable PMQ is given.

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REFERENCES

[1] G. Parzen, "Linear parameters and the decoupling matrix for linearly coupled motion in 6 dimensional phase space", BNL, Upton, NY, USA, Rep. BNL-61552, Oct. 1995.

[2] A. Wolski, "Alternative approach to general coupled linear optics", *Phys. Rev. ST Accel. Beams*, vol. 9, no. 2, p. 024001, Feb. 2006. doi:10.1103/physrevstab.9.024001

[3] C. Baumgarten, "Geometrical method of decoupling", *Phys. Rev. ST Accel. Beams*, vol. 15 no. 12, p. 124001, Dec. 2012. doi:10.1103/physrevstab.15.124001

[4] R. Calaga, R. Tomás, and A. Franchi, "Betatron coupling: Merging Hamiltonian and matrix approaches", *Phys. Rev. ST Accel. Beams*, vol. 8, no. 3, p. 034001, Mar. 2005. doi:10.1103/physrevstab.8.034001

[5] M. Chung and H. Qin, "Generalized parametrization methods for centroid and envelope dynamics of charged particle beams in coupled lattices", *Physics of Plasmas*, vol. 25, no. 1, p. 011605, 2018. doi:10.1063/1.5018426

[6] M. Chung, H. Qin, R. C. Davidson, L. Groening, and C. Xiao, "Generalized Kapchinskij-Vladimirskij distribution and beam matrix for phase-space manipulations of high-intensity beams", *Phys. Rev. Lett.*, vol. 117, no. 22, p. 224801, 2016. doi:10.1103/physrevlett.117.224801

[7] C. Xiao *et al.*, "Single-knob beam line for transverse emittance partitioning", *Phys. Rev. ST Accel. Beams*, vol. 16, no. 4, p. 044201, 2013. doi:10.1103/physrevstab.16.044201

[8] D. H. Dowell, F. Zhou, and J. Schmerge, "Exact cancellation of emittance growth due to coupled transverse dynamics in solenoids and rf couplers", *Phys. Rev. Accel. Beams*, vol. 21, no. 1, p. 010101, 2018. doi:10.1103/physrevaccelbeams.21.010101

[9] L. Zheng *et al.*, "Overestimation of thermal emittance in solenoid scans due to coupled transverse motion", *Phys. Rev. Accel. Beams*, vol. 21, no. 12, p. 122803, Dec. 2018. doi:10.1103/physrevaccelbeams.21.122803

[10] P. F. Ma *et al.*, "Matrix Approach to Decouple Transverse-Coupled Beams", in *Proc. 10th Int. Particle Accelerator Conf. (IPAC'19)*, Melbourne, Australia, May 2019, pp. 1144–1147. doi:10.18429/JACoW-IPAC2019-MOPTS112

[11] P. F. Ma *et al.*, "Decoupling a transversely-coupled beam based on symplectic transformation theory and its application", *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 968, p. 163925, Jul. 2020. doi:10.1016/j.nima.2020.163925

[12] R. J. de la Cruz and H. Faßbender, "On the diagonalizability of a matrix by a symplectic equivalence, similarity or congruence transformation", *Linear Algebra and Its Applications*, vol. 496, pp. 288-306, May 2016. doi:10.1016/j.laa.2016.01.030

[13] P. F. Ma *et al.*, "Physical design of a single-amplifier-driven proton linac injector for a synchrotron-based proton-therapy system in China", *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 900, pp. 32-39, Aug. 2018. doi:10.1016/j.nima.2018.05.047

[14] Tracewin documentation, http://irfu.cea.fr/dacm/logiciels/codesdacm/tracewin/tracewin.pdf