SPATIAL AUTORESONANCE ACCELERATION OF ELECTRONS BY AN AXISYMMETRIC TRANSVERSE ELECTRIC FIELD

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Electron Dynamics

Classical Cyclotron Motion

$$\frac{d\vec{v}}{dt} = -\frac{e}{m_e}\,\vec{v}\times\vec{B} \quad\Rightarrow\quad \Omega_{c0} = \frac{e\,B_0}{m_e}$$

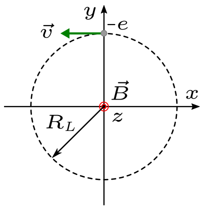
Resonant Interaction

$$\frac{d}{dt}\left(\gamma\,\vec{v}\right) = -\frac{e}{m_e}\left[\vec{E} + \vec{v}\times\vec{B}\right] \;\Rightarrow\; \Omega_c = \frac{e\,B_0}{m_e\,\gamma}$$

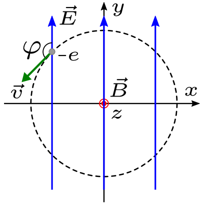
ECR Condition: $\Omega_c = \omega \;\Rightarrow\;$ **Acceleration Band:** $\frac{\pi}{2} \leq \varphi \leq \frac{3\,\pi}{2}$

Spatial Autoresonance

$$\omega = \Omega_c = \frac{e\,B(z)}{\gamma m_e}$$

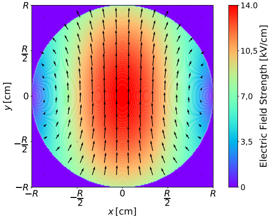


$\vec{B} = B_z\ \hat{k}$

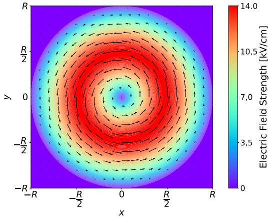


$\vec{B} = B_z\ \hat{k}$ and $\vec{E} = E_0\ \cos\left(\omega t\right)\ \hat{j}$

Spatial Autoresonance Acceleration (SARA)



Cylindrical Mode TE_{111}



Cylindrical Mode TE_{011}

SARA Model

Considering:

$$\vec{E} = \vec{E}^{\,\mathrm{hf}} \quad \text{and} \quad \vec{B} = \vec{B}^{\,\mathrm{hf}} + \vec{B}^{\,\mathrm{c}}$$

where $\vec{E}^{\,\mathrm{hf}}$ and $\vec{B}^{\,\mathrm{hf}}$ (Cylindrical Mode TE_{11p})

$$\vec{E}^{\,\mathrm{hf}} \approx E_0 \left[\sin(\varphi)\,\hat{\mathrm{r}} + \cos(\varphi)\,\hat{\theta}\right] \sin\left(\frac{p\pi z}{d}\right)$$

$$\vec{B}^{\,\mathrm{hf}} \approx -E_0 \left(\frac{p\pi z}{d\,\omega}\right) \left[\sin(\varphi)\,\hat{\mathrm{r}} + \cos(\varphi)\,\hat{\theta}\right] \cos\left(\frac{p\pi z}{d}\right) + B_z^{\mathrm{hf}}\,\hat{\mathrm{k}}$$

and the extern magnetic field: $B_z^c(z) = B_0\left[\gamma_0 + b(z)\right]$ where $B_0 = \dfrac{\omega m_e}{e}$

Cylindrical Mode TE_{011}

$$\vec{E}^{\,\mathrm{hf}}(\vec{r},t) = \frac{E_0}{J_1(p_{01})} J_1\left(\frac{q_{01}}{R} r\right) \sin\left(\frac{\pi}{L} z\right) \cos(\omega t)\,\hat{\theta}$$

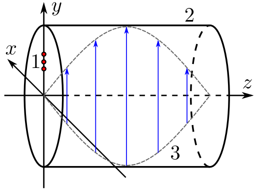
$$\vec{B}^{\,\mathrm{hf}}(\vec{r},t) = \frac{E_0}{J_1(p_{01})} \left[\frac{\pi}{L\,\omega} J_1\left(\frac{q_{01}}{R} r\right) \cos\left(\frac{\pi}{L} z\right) \sin(\omega t)\,\hat{\mathrm{r}} - \frac{q_{01}}{R\,\omega} J_0\left(\frac{q_{01}}{R} r\right) \sin\left(\frac{\pi}{L} z\right) \sin(\omega t)\,\hat{\mathrm{k}}\right]$$

where $q_{01} = 3{,}83171$, $p_{01} = 1{,}84118$, $R = 7{,}84$ cm, $L = 20$ cm, $E_0 = 14$ kV/cm and $f = 2{,}45$ GHz.

Physical Scheme and Simulation Model

Electromagnetic Field

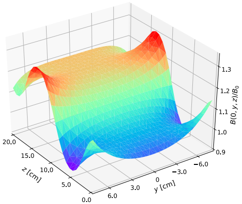
Cylindrical Mode TE_{011} $\Rightarrow$ $\vec{E} = \vec{E}^{\,\text{hf}}$ y $\vec{B} = \vec{B}^{\,\text{hf}} + \vec{B}^{\text{ext}}$



Physical scheme: (1) Electron injection points, (2) Cylindrical Cavity and (3) Longitudinal electric field profile.

Simulation Model

- Three Coil System: $\vec{B}^{\text{ext}}$ (Biot-Savart Law - Integral Form).
- Interpolation Bilinear: $\vec{B}^{\text{ext}}(\vec{r}_p)$.
- 3D Relativistic Newton-Lorentz equation: Boris integrator.



Numerical experiments

1. An electron injected longitudinally at points $P = \{R/2, 3R/8, 9R/16\}$ with an energy of 1 keV.
2. An electron injected longitudinally at point $P_1 = R/2$ with different energies (3 and 5 keV).

Results

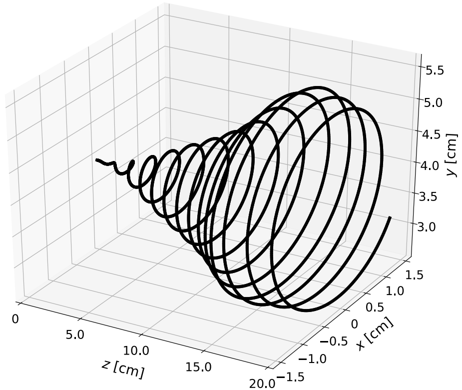


Fig 1: Helical trajectory of the electron injected at P_1 with an energy of 4 keV.

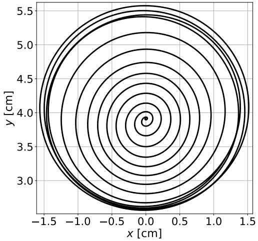


Fig 2: XY view of the trajectory.

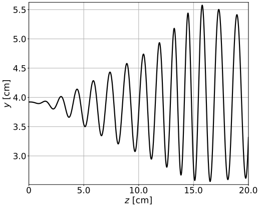


Fig 3: YZ view of the trajectory.

Results

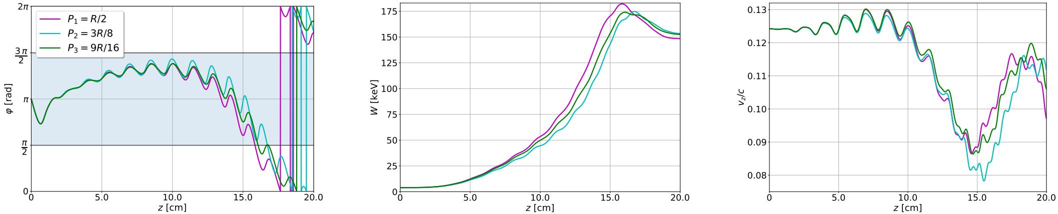


Fig 4: Longitudinal evolution of φ, γ and v_z/c for different injections points.

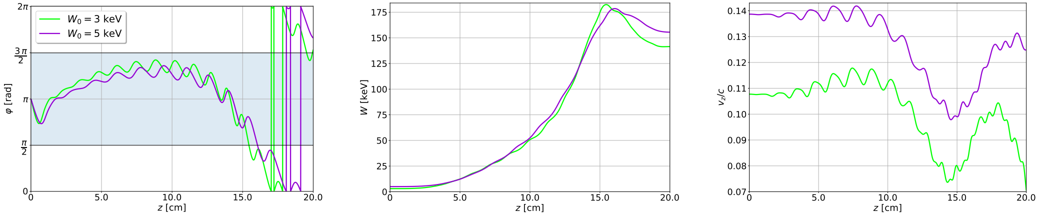


Fig 5: Longitudinal evolution of φ, γ and v_z/c for different injection energies.

Conclusions

- It was showed by numerical experiments that it is possible to accelerate electrons under electron cyclotron resonance conditions in inhomogeneus magnetostatic fields using the TE_{011} cylindrical mode.
- It was found an inhomogeneous magnetostatic field which maintains the electron acceleration regime close to the exact resonance condition along almost its entire trajectory.

Future Works

We will study this acceleration scheme by using other TE_{01p} cylindrical mode ($p = 2\,,3$).

References

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