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Tools for use of generalized gradient expansions in accelerator simulations

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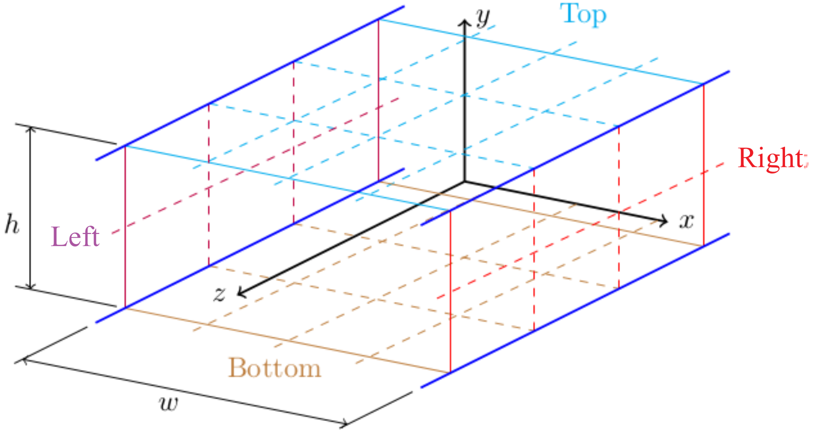
Introduction

- Most accelerator modeling uses a hard-edge approximation
    - This is often very good but ignores longitudinal variation of fields
    - Fringe fields are added in an impulse approximation but aren't easy to derive for complex magnets
- An alternative is to use generalized gradient expansions[1-4] (GGEs)
    - Provide z-dependent expansions for magnetic fields
    - Symplectic integration possible (e.g., **`elegant`** does it)
- We've developed tools to make creation and use of GGEs easy
- Applied to modeling of Advanced Photon Source Upgrade[5]

Extending GGEs to include a non-zero B_z on-axis

- Published algorithms[1-4] for computing GGEs do not accurately compute non-zero B_z along the axis
- This shortcoming can be fixed if we generalize the results to also use the longitudinal B_z on the surface
- For the rectangular boundary, we define the Fourier coefficients



$$b_n^T(k) = \int_{-w/2}^{w/2} dx \, \frac{\tilde{B}_z(x, y = +h/2, k)}{w/2} \sin(n\pi x/w + n\pi) \qquad b_n^L(k) = \int_{-h/2}^{h/2} dy \, \frac{\tilde{B}_z(x = +w/2, y, k)}{h/2} \sin(n\pi y/h + n\pi)$$

$$b_n^B(k) = \int_{-w/2}^{w/2} dx \, \frac{\tilde{B}_z(x, y = -h/2, k)}{w/2} \sin(n\pi x/w + n\pi) \qquad b_n^R(k) = \int_{-h/2}^{h/2} dy \, \frac{\tilde{B}_z(x = -w/2, y, k)}{h/2} \sin(n\pi y/h + n\pi)$$

- We then look for a solution for the magnetic potential that satisfies $\left(\nabla_\perp^2 - k^2\right)\psi = 0$ subject to the Neumann boundary condition $\psi(x, y, k)\big|_{\mathcal{S}} = \frac{1}{ik}\tilde{B}_z(x, y, k)\big|_{\mathcal{S}}$ on the rectangular surface
- The generalized gradient that gives the on-axis $B_z(k) = kC_{0,c}(k)$ is then given by

$$\tilde{C}_{0,c}(k) = \sum_{p=0}^{\infty} \left[\overset{\text{Top}}{\hat{\mathcal{T}}^c_{0,p} b_p^T(k)} + \underset{\text{Bottom}}{\hat{\mathcal{B}}^c_{0,p} b_p^B(k)} + \overset{\text{Right}}{\hat{\mathcal{R}}^c_{0,p} b_p^R(k)} + \underset{\text{Left}}{\hat{\mathcal{L}}^c_{0,p} b_p^L(k)} \right]$$

$$\hat{\mathcal{T}}^c_{0,p} = \hat{\mathcal{B}}^c_{0,p} = \frac{1}{ik} \frac{\sin(p\pi/2)}{2\cosh\left[h\sqrt{k^2 + (p\pi/2w)^2}/2\right]}$$

$$\hat{\mathcal{R}}^c_{0,p} = \hat{\mathcal{L}}^c_{0,p} = \frac{1}{ik} \frac{\sin(p\pi/2)}{2\cosh\left[w\sqrt{k^2 + (p\pi/2h)^2}/2\right]}$$

Tools available for computation of GGEs

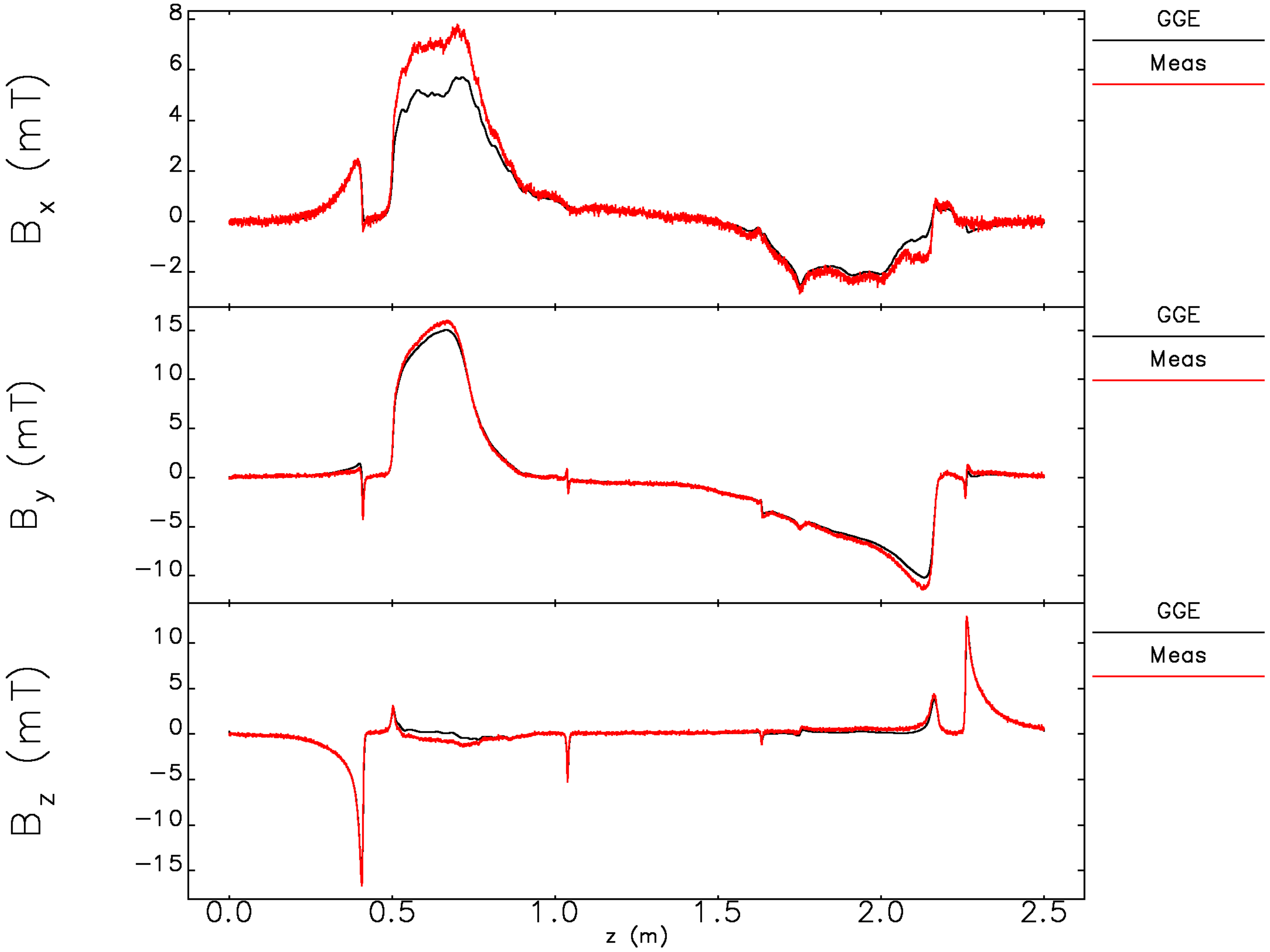
- **`computeCBGGE`** computes GGE from B_ρ data on a circular cylindrical boundary
  - Suitable for straight multipoles
- **`computeRBGGE`** computes GGE from (B_x, B_y, B_z) data on four rectangular planes forming a rectangular cylinder
  - Suitable for wigglers, undulators, small-angle dipoles, etc.
- Common features
  - Compiled C for good performance
  - SDDS file input of field data
  - Create normal and skew GGE files for use with **`elegant`**[6]
  - Auto-tune number of multipoles and gradients to minimize errors
  - Available with version 2021.1 of **`elegant`**

Lambertson septum is challenging to model

- The original APS-U vertical injection scheme[7] used a Lambertson septum
- Integrated leakage field fairly small, but only because designed to cancel between two ends[8]
  - In addition to dipole, significant normal and skew quadrupole
- Hard to mesh the stored beam chamber finely, giving coarse data
  - Insufficient data for a high-quality kickmap
  - Rapid z variation makes multipoles dubious
- Generated GGEs using **`computeRBGGE`** from both OPERA[9]-generated and measured data

GGE matches measured data fairly well

- Using boundary data, reproduce on-axis B_y and B_z data very well
- B_x data shows a curious discrepancy confined to one section
  - Could be issue In the measurement



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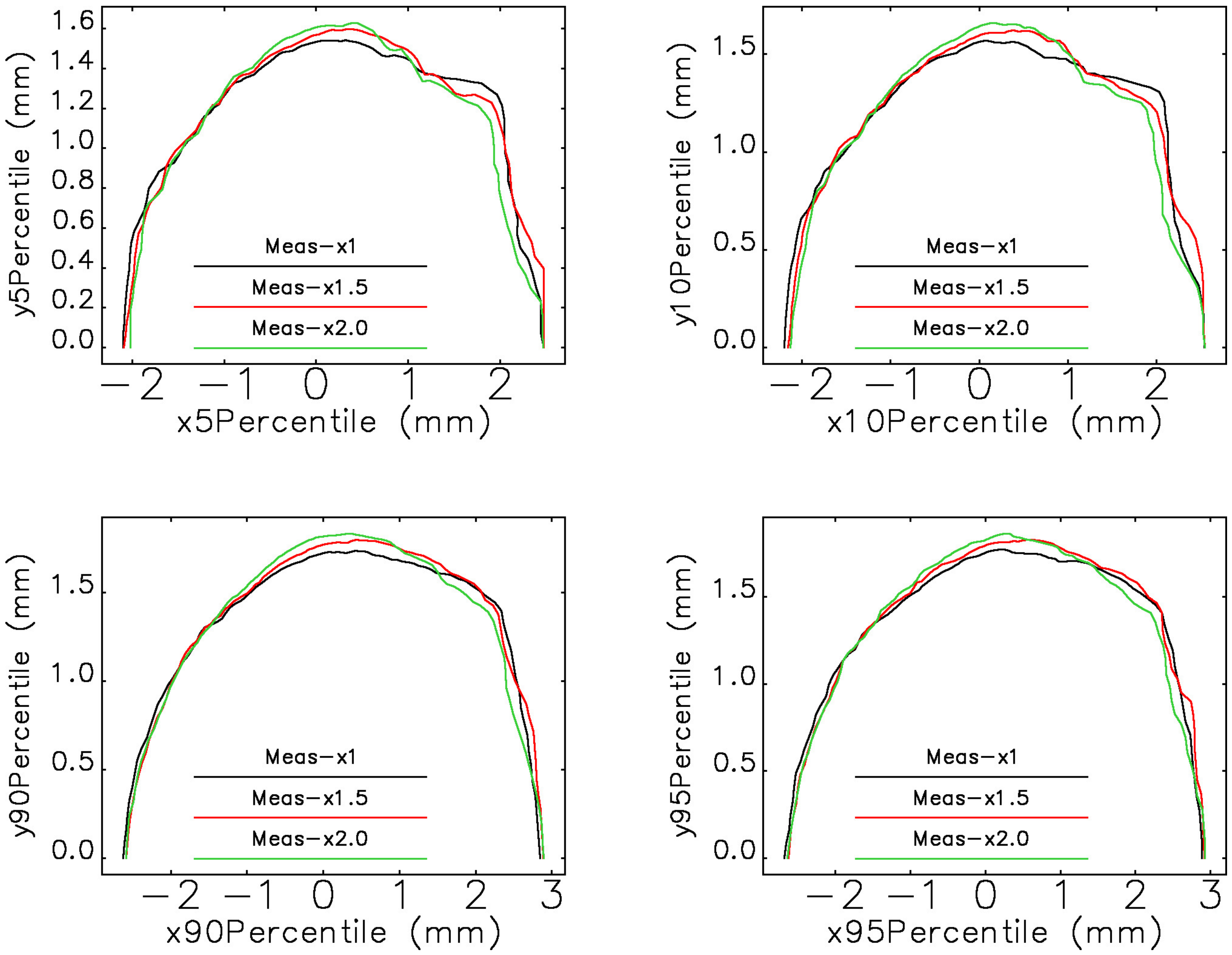
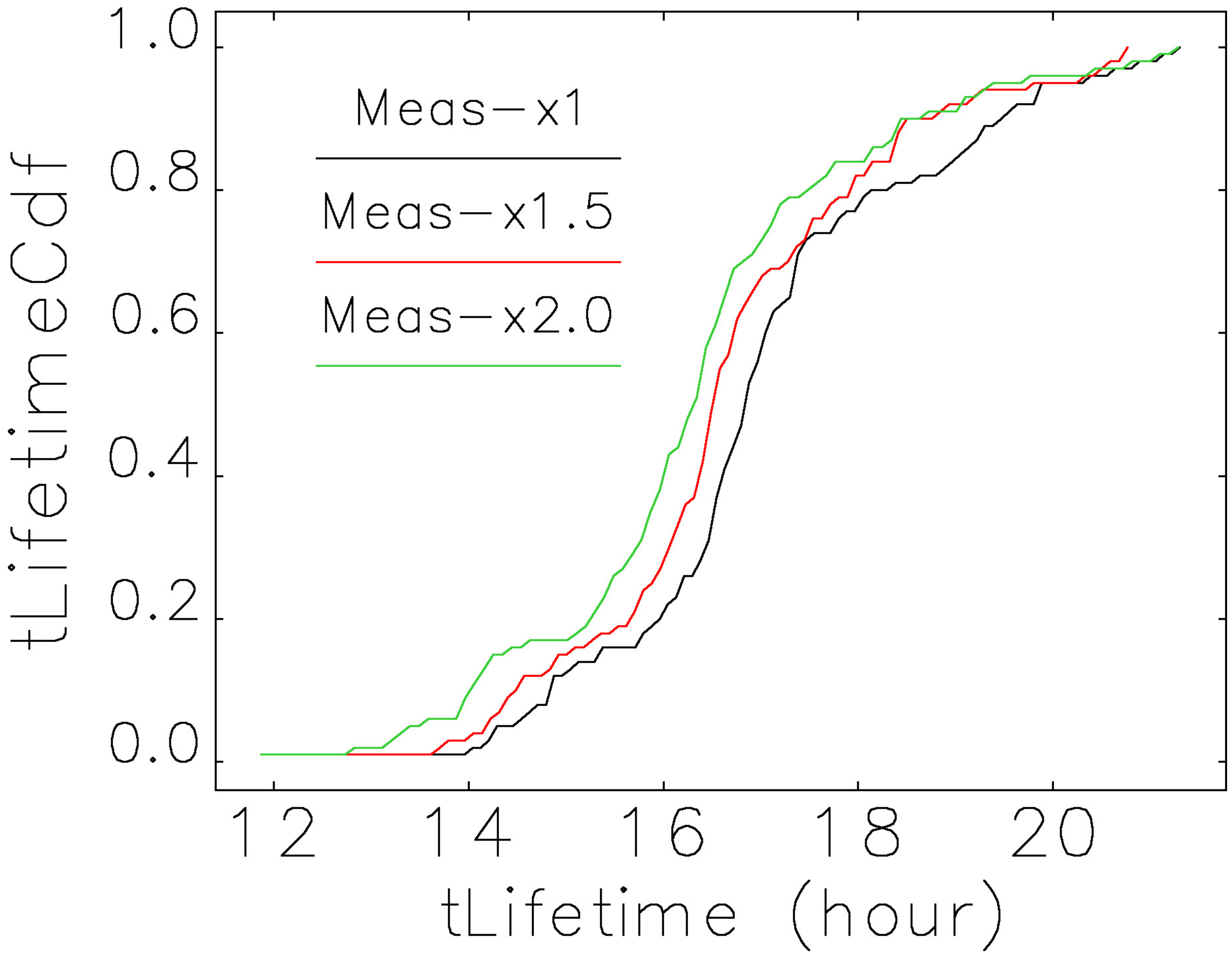
DA acceptable even if leakage 2-fold higher

- Use **`Pelegant`**[10] to compute DA for 100 post-commissioning ensembles[11] including GGE leakage model
- Even multiplying leakage by 2 doesn’t cause a problem



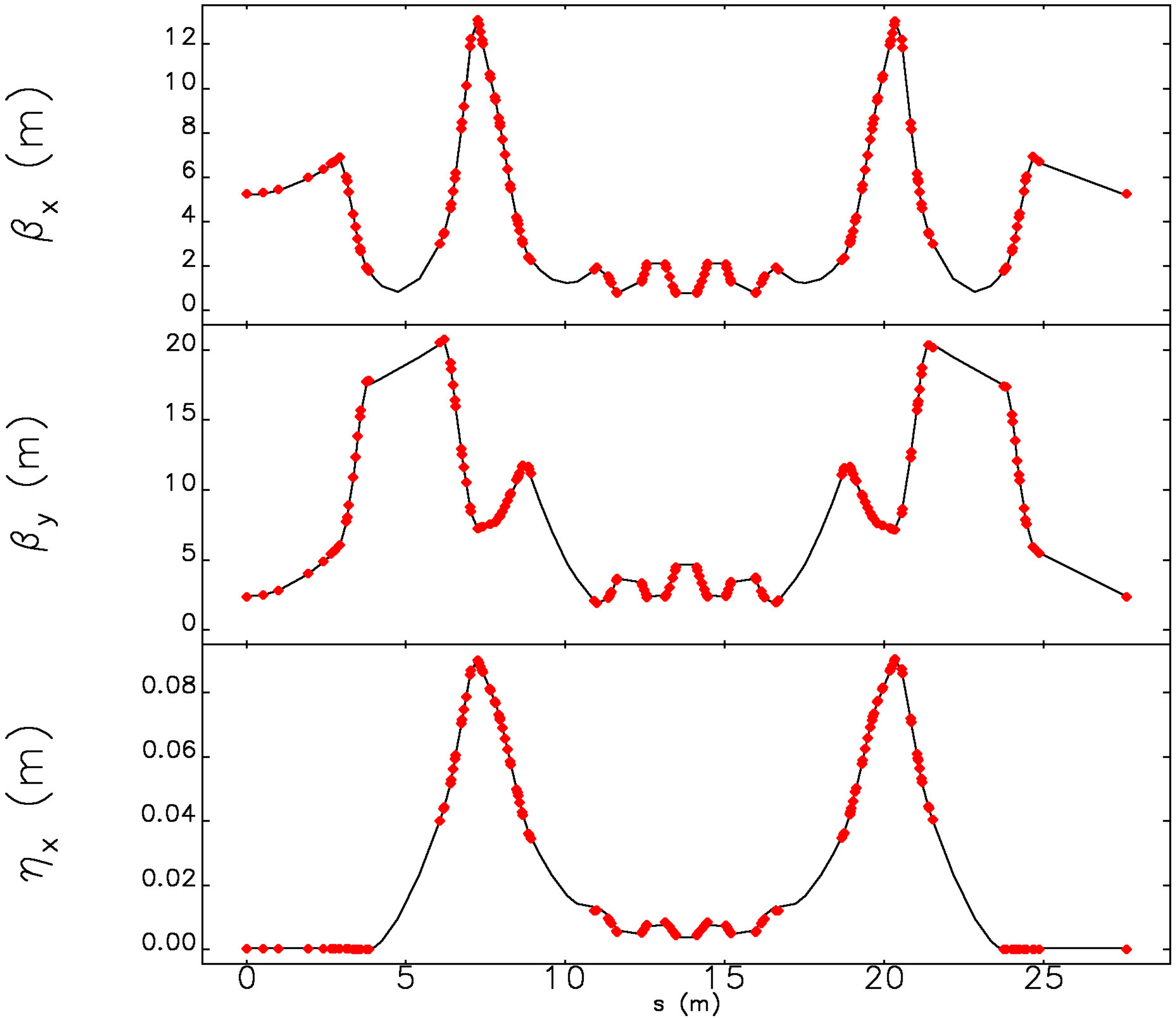
Touschek lifetime shows negligible effects

- Use `Pelegant` to compute LMA and then Touschek lifetime for 100 post-commissioning ensembles including GGE leakage model
- Even multiplying leakage by 2 doesn't cause a problem
- Conclusion: septum meets beam dynamics requirements



All-GGE lattice of APS-U tuned to match design

- We assembled an all-GGE lattice model for APS-U using OPERA data
- Unsurprisingly, we can return to the design lattice by tuning the GGE-based elements
- Plan is to do this ahead of time using magnetic measurements to generate GGEs



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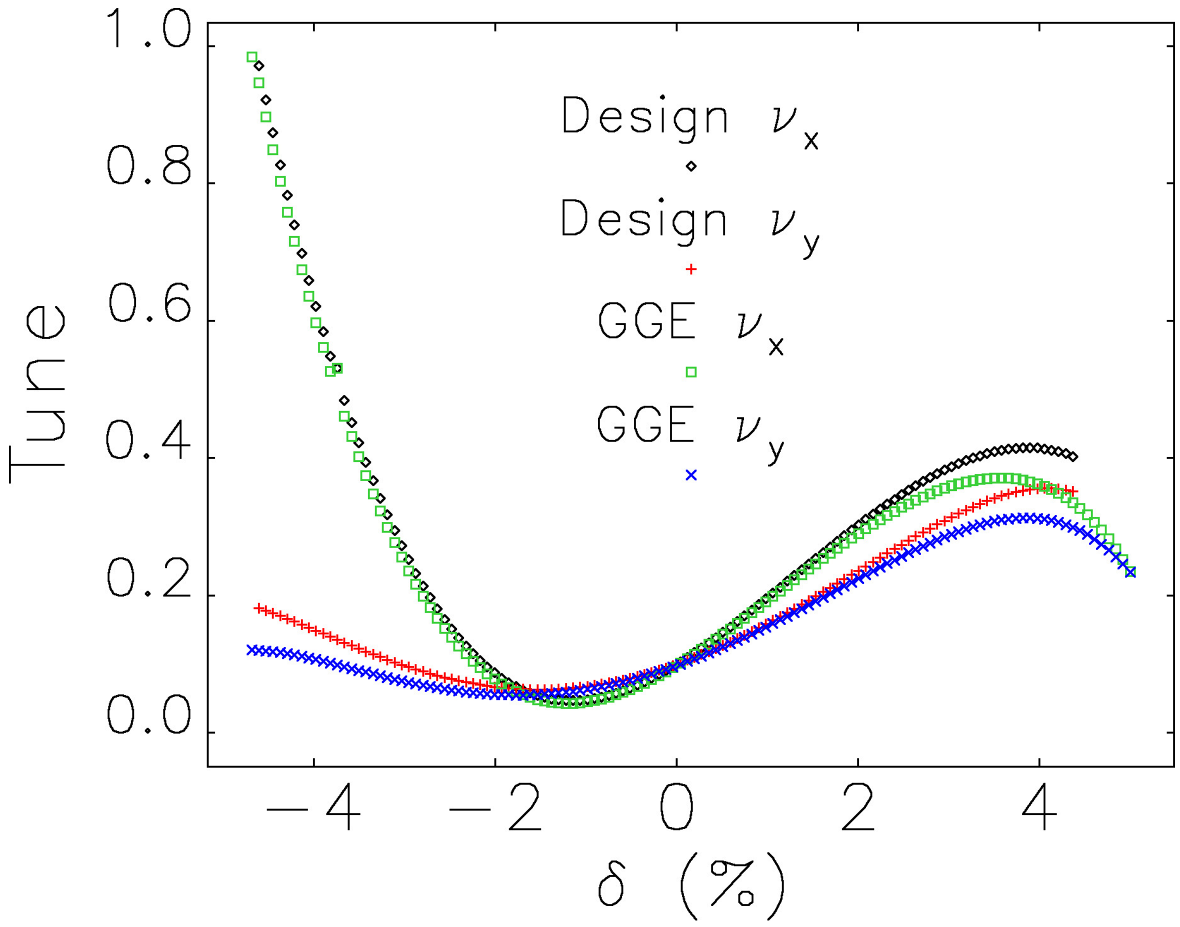
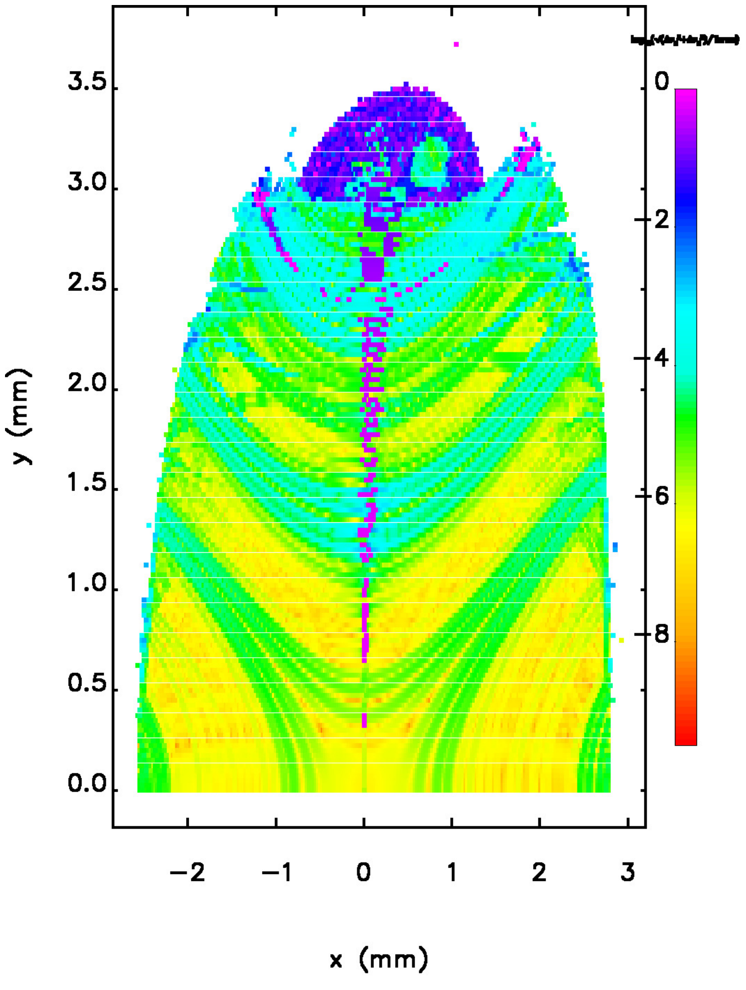
Chromatic tune footprint matches fairly well

- Tracking with **`Pelegant`** allows determining the chromatic tune footprint with conventional or GGE model
- Agreement is fairly good
- Note that “tuning” only matched the tunes and linear chromaticities

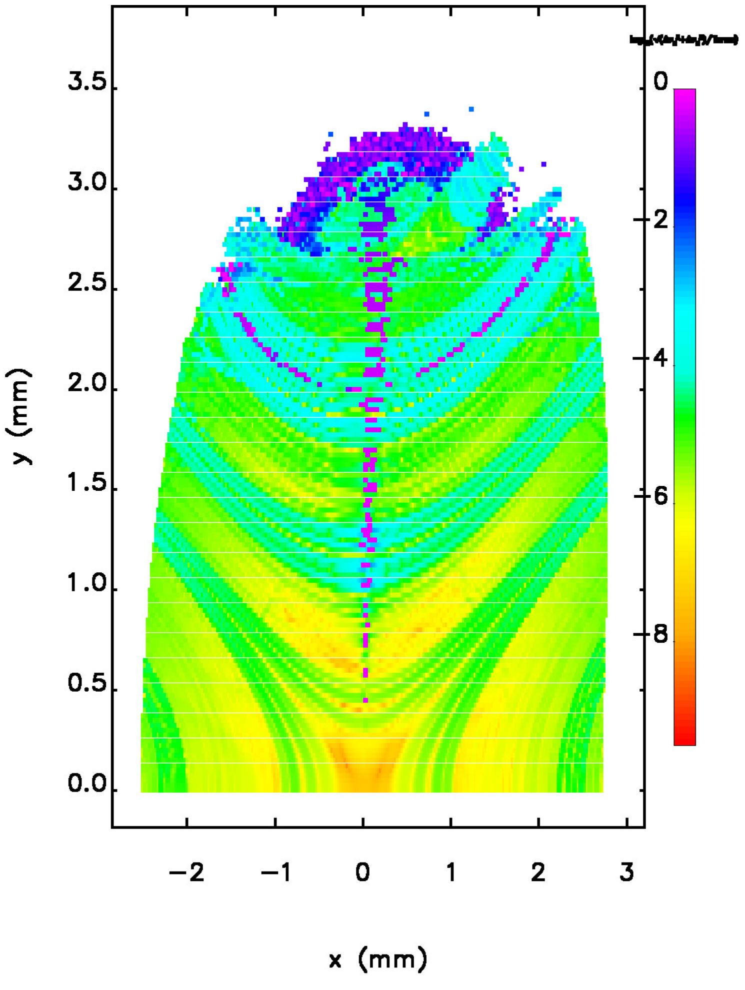


Frequency maps are quite similar

- Parallel tracking with **`Pelegant`** allows determining frequency map even for all-GGE model
- Takes about 180 times longer than for conventional model
- All-GGE model best used for reference analysis, understanding, refinement of conventional model



Conventional tracking



GGE−based tracking

Conclusions

- Have developed several tools to make use of GGEs in accelerator modeling relatively painless
- Allows symplectic tracking with 3D field distributions derived from magnetic modeling or measurements
- Applied to APS upgrade lattice
  - Modeled effects of leakage field from Lambertson septum
  - Composed an all-GGE lattice and showed significant agreement with conventional model
- Future
  - Use GGE models to better understand fringe effects in transverse and longitudinal gradient dipoles
  - Use with measured data for all APS-U magnets

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