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NUMERICAL NOISE ERROR OF PARTICLE-IN-CELL POISSON SOLVER FOR A FLAT GAUSSIAN BUNCH*

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Abstract

The Electron-Ion Collider (EIC) presently under construction at Brookhaven National Laboratory will collide polarized high energy electron beams with hadron beams with design luminosity up to $1 \times 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ in center mass energy range of 20-140 GeV. We simulated the planned electron-proton collision of flat beams with Particle-In-Cell (PIC) based Poisson solver in strong-strong beam-beam simulation. We observed a much larger proton emittance growth rate than that from weak-strong simulation. To better understand the emittance growth rate from the strong-strong simulation, we compare the beam-beam kicks between the PIC method and the analytical calculation and calculate the RMS variation in beam-beam kicks among 1000 sets of random Gaussian particle distributions. The impacts of macro-particle number, grid number, and bunch flatness are also studied.

INTRODUCTION

The Electron-Ion Collider (EIC) presently under construction at Brookhaven National Laboratory will collide polarized high energy electron beams with hadron beams with design luminosities up to $1 \times 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ in center mass energy range of 20-140 GeV [1]. We focus on the collision involving 275 GeV protons and 10 GeV electrons since both protons and electrons reach their highest beam-beam parameters for this collision mode in the EIC.

Both strong-strong and weak-strong models have been used for the EIC beam-beam simulation studies [2,3]. For weak-strong model, electron bunch is assumed rigid and is represent by a 6-d Gaussian charge distribution. The beam-beam kick to protons are analytically calculated. For strong-strong model, both bunches are represented with typically 0.5-1 million macro-particles. Particle-in-cell (PIC) method and Fast Fourier Transformation (FFT) are used to solve 2-d Poisson equation on rectangle grids.

In the EIC beam-beam simulation studies, we observed a much larger proton emittance growth rate in strong-strong simulation than in weak-strong simulation. Strong-strong simulation is subject to numerical noises due to limited macro-particles, transverse grids, longitudinal slices, and the algorithm itself [4]. To understand the sources of numerical noises in the strong-strong simulation, in the following we will calculate RMS variations in beam-beam kicks for 1000 sets of 4-d Gaussian distributions. As we know, the emittance growth due to numerical noises is proportional to square of the beam-beam force's variation. Two kinds of beam flatness are used for comparison: a round beam and a flat beam.

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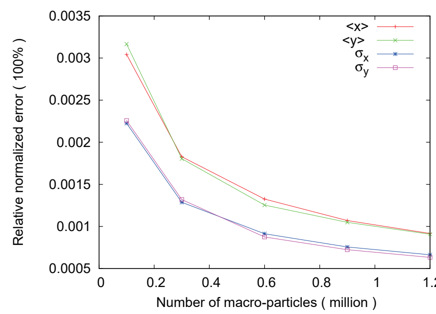

Figure 1: Relative RMS variations of <x>, <y>, and σ_x, σ_y as function of macro-particle number.

GAUSSIAN DISTRIBUTION ERRORS

First we study the statistical errors from a 4-d Gaussian particle distribution. We adopt the Gaussian distribution random number generator provided by GNU Scientific Library (GSL). Figure 1 shows the relative RMS variations in bunch center's position and beam sizes for 1000 sets of Gaussina distributions as function of the number of macro-particles. The relative error is normalized by the RMS beam size.

From the plot, the numerical error in the bunch center's position and RMS beam sizes decrease with increased macro-particles. For a typical 0.5 million macro-particles, the relative errors in the bunch center's position and beam sizes are about 0.1‰ - 0.2‰ of the RMS beam size. In our strong-strong simulation for the EIC, we observe about 0.2‰-0.3‰ relative variations in turn-by-turn bunch center's position and transverse beam sizes.

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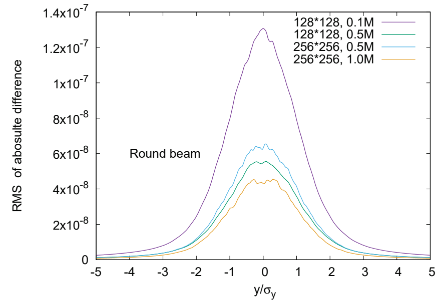

Figure 2: RMS of beam-beam kick difference for a round beam.

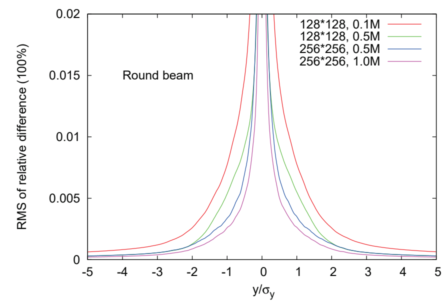

Figure 3: RMS of relative beam-beam kick difference for a round beam.

A ROUND BEAM CASE

In the following, we generate 1000 sets of random Gaussian particle distributions and calculate the variations of beam-beam kicks with PIC Poisson solver. For the round beam case, both horizontal and vertical beam sizes are 77 μm, which are similar to the 250 GeV proton beam sizes at the interaction points in RHIC.

Figure 2 shows the RMS variation of the beam-beam kick difference of 1000 sets of random Gaussian particle distributions. The beam-beam kick is calculated in the 45 degrees in the $(x/\sigma_x, y/\sigma_y)$ plane. The beam-beam kick difference means between the PIC based Poisson solver and the analytical calculation using Bassetti-Erskine formula. Here we scanned the number of macro-particles and the number of grids for the PIC based Poisson solver.

From the plot, the RMS variation in beam-beam kick difference is larger for small amplitudes less than 2σ. It goes down with increase in transverse amplitude. For the same number of grids, the beam-beam kick variation decreases with increased macro-particle number. Also the variation in beam-beam kick is reduced with more grids. With more grids, we need to increase the total number of macro-particles at the same time to have a same amount of macro-particles in one rectangle grid.

Figure 3 shows the relative RMS variation of beam-beam kick difference for the round beam. The relative RMS variation of beam-beam kick difference is normalized by the local beam-beam kick from analytical calculation. The relative error in beam-beam kick is much larger for amplitudes less than 0.5 σ since the beam-beam kick from analytical calculation is small in that region.

In Fig. 3, the relative RMS variation in beam-beam kick difference gradually goes down with increase in amplitude. At 3σs from the bunch center, the RMS relative variation in beam-beam kick difference falls below 0.15%.

A FLAT BEAM CASE

Next we go through the same calculations for a flat beam. For it, the horizontal and vertical beam sizes are 77 μm and 7.7 μm, which are close to the proton beam sizes at 275 GeV in EIC. For the 275 GeV proton in the EIC, its transverse beam sizes at IP are (95 μm, 8.5 μm). The flatness σ_y^*/σ_x^* is 0.09. In our simulation, the round beam's flatness is 0.1.

Figure 4 shows the RMS variation of vertical beam-beam kick difference for 1000 sets of random 4-d Gaussian particle distributions. Again, we observed a bigger variation for transverse amplitude less than 2σ. As we know, the artificial emittance growth is proportional to square of the force's variation. Therefore, macro-particles in bunch core will have larger beam size growth rate than those with larger transverse amplitudes. This is confirmed by the dependence study of macro-particles' beam size growth rate as function of their transverse actions [5].

Compared to Fig. 1, the amplitude of RMS variation in beam-beam kick is greater for the flat beam than the round beam. The peak variation at zero-amplitude is about 2.5 times greater than the round beam case. From strong-strong beam-beam simulation, we observed more times 10 times faster growth rate for the EIC's flat beam than the RHIC's round beam. The beam-beam parameters for both cases are almost the same.

Figure 5 shows the RMS variation of relative beam-beam kick difference for a flat beam. Compared to the round beam case shown in Fig. 3, we observed much larger relative variation for the flat beam than the round beam. The relative variation stays at a high level even when the transverse amplitude reaches 4σ. For example, at 4σ, with 128×128 grids and 1 million macro-particles, the relative variation is about 0.5%.

Figure 6 compares the RMS variations of beam-beam kick difference on the horizontal axis and on the vertical axis for the flat beam. The RMS variation in beam-beam kick difference is always higher in the vertical direction than that in the horizontal direction. And the variation level goes down much slower in the vertical direction than in the horizontal direction. In our strong-strong beam-beam simulation for the EIC, the vertical beam size growth rate is 2-3 times higher than the horizontal one. While for the RHIC beam-beam simulation, the beam size growth rates are comparable in both horizontal and vertical planes.

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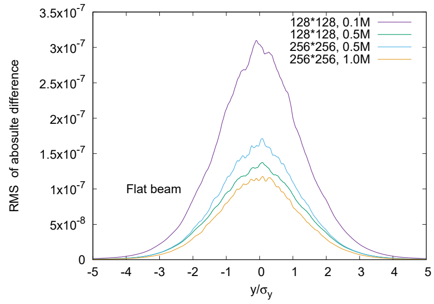

Figure 4: RMS variations in beam-beam kick difference for a flat beam.

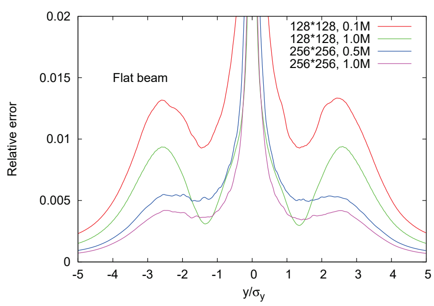

Figure 5: RMS of relative variation in beam-beam kick difference for a flat beam.

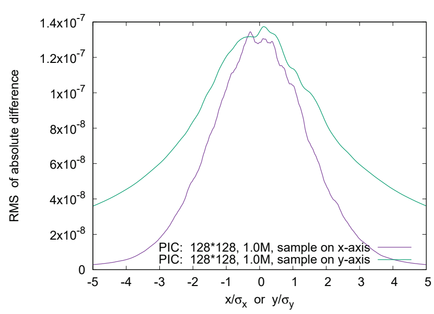

Figure 6: RMS variation of beam-beam kick difference on horizontal and vertical axes for a falt beam.

COMPARED TO SOFT-GAUSSIAN MODEL

Next we compare the variation in beam-beam kick for PIC based strong-strong simulation and soft-Gaussian model based strong-strong simulation. The latter still uses analytical formula to calculate the beam-beam kick. Different from weak-strong model, the opposite bunch's center position and RMS beam sizes are updated turn-by-turn. Soft-Gaussian model assumes Gaussian distribution for each bunch. This may not be true for the EIC with crossing angle collision.

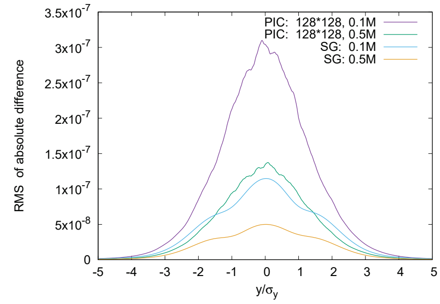

Figure 7: RMS variation in beam-beam kick with PIC method and soft-Gaussian model for a flat beam.

Figure 7 shows the RMS variation of beam-beam kick difference with PIC method and soft-Gaussian model. From the plot, soft-Gaussian model give mores than 3 times smaller variation in beam-beam kick than the PIC method. From beam-beam simulation for the EIC, we observed more than 10 times smaller proton emittance growth rate with soft-Gaussian model than with the PIC method.

SUMMARY

In the beam-beam simulation study for EIC, we observed a much larger proton emittance growth rate in strong-strong simulation than in weak-strong simulation. In this articles, through calculating the beam-beam kick variation for 1000 sets of Gaussian particle distributions, we found that the RMS variation in beam-beam kick is larger in bunch core than with larger transverse amplitudes. The flat beam gives much larger variation than the round beam. For the flat beam, the beam-beam kick variation is larger in vertical direction than in the horizontal direction.

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