NUMERICAL CALCULATION OF COUPLING IMPEDANCES IN KICKER MODULES FOR NON-RELATIVISTIC PARTICLE BEAMS

B. Doliwa, T. Weiland, TEMF, Technische Universität Darmstadt, Germany

Abstract

In the context of heavy-ion synchrotrons, coupling impedances in ferrite-loaded structures (e.g. fast kicker modules) are known to have a significant influence on beam stability. While bench measurements are feasible today, it is desirable to have the coupling impedances in hands already during the design process of the respective components. To achieve this goal, as a first step, we have carried out numerical analyses of simple ferrite-containing test systems within the framework of the Finite Integration Technique. This amounts to solving the full set of Maxwell's equations in frequency domain, the particle beam being represented by an appropriate excitation current. With the resulting electromagnetic fields, one may then readily compute the corresponding coupling impedances. Despite the complicated material properties of ferrites, our results show that their numerical treatment is possible, thus opening up a way to determine a crucial parameter of kicker devices before construction.

INTRODUCTION

Within the design work of the planned heavy-ion synchrotron at the GSI accelerator facility, detailed impedance studies are required. Due to the target vacuum quality of 10^{-12} mbar and particle currents of up to 1 A, beam instabilities would have tremendous effects on the operability of the synchrotron. One unknown is the beam response to the ferrite-loaded kickers. Vice versa, heating of the kicker components may be a problem.

Caspers has addressed the measurement of coupling impedances via the so-called coaxial-wire technique [1], which is most accurate for ultra-relativistic particle beams. The obvious drawback of this approach is that a prototype component has to be at hand. During the design process of new components, therefore, simulations may be helpful.

In this paper we consider the numerical determination of the longitudinal coupling impedance [2]

$$Z_{||}(\omega) = \frac{1}{q^2}\int \mathrm{d}x\mathrm{d}y\rho(x,y)\int \mathrm{d}z E_z(x,y,z;\omega)e^{i\omega z/\beta c} \tag{1}$$

where $\rho(x,y)$ is the transverse charge distribution of the particle beam and $q=\int \mathrm{d}x\mathrm{d}y\rho(x,y)$. The electric field in the above expression is generated by the current density

$$\boldsymbol{J}_{\text{ext}}(x,y,z;t) = \beta c\hat{z}\rho(x,y)\delta(z-\beta ct), \tag{2}$$

which corresponds to an infinitesimally short bunch of particles travelling with velocity βc along the positive z direction. The equivalent expression in frequency domain is

$$\boldsymbol{J}_{\text{ext}}(x,y,z;\omega) = \hat{z}\rho(x,y)e^{-\mathrm{i}\omega z/\beta c}. \tag{3}$$

In the case of kicker impedances, we will restrict ourselves to frequencies below 100 MHz, which are of primary interest for the planned heavy-ion synchrotron.

COMPUTATIONAL APPROACH

In order to determine $Z_{||}$ for a given geometry and excitation current we need to calculate the electric field. Our starting point is

$$\boldsymbol{\partial}\times\nu\boldsymbol{\partial}\times\boldsymbol{E} - \omega^2\epsilon\boldsymbol{E} = -\mathrm{i}\omega\boldsymbol{J}_{\text{ext}},$$

a descendant of Maxwell's equations in frequency domain. Here $\nu\equiv 1/\mu$, with possibly complex permeability μ and ϵ denoting permittivity.

Within the Finite Integration Technique[3], we carry out an appropriate discretization, which leads to a matrix counterpart of the former equation,

$$\left(\tilde{\boldsymbol{C}}M_\nu\boldsymbol{C} - \omega^2 M_\epsilon\right)\boldsymbol{e} = -\mathrm{i}\omega\boldsymbol{j}_{\text{ext}}. \tag{4}$$

In the presence of ferrites, this system of linear equations may become highly ill-conditioned due to the large jumps in permeability. We therefore do not attempt to solve the matrix equation as a whole but proceed as follows: Firstly, an electrostatic problem is solved yielding a divergence-free source term $\boldsymbol{j}'_{\text{ext}}$ and the 'static' part of the electric field solution. We then note that, at low enough frequencies, the term $\omega^2 M_\epsilon \equiv b$ is small compared with $\tilde{\boldsymbol{C}}M_\nu\boldsymbol{C}\equiv B$ (in the sense of some matrix norm). One may then expand the solution in terms of bB^{-1} (symbolically). It has turned out in our simulations that keeping up to six terms of this series expansion is sufficient (below 100 MHz). The inversion of the matrix B in each expansion term formally corresponds to solving a standard magnetostatic problem, which will therefore not be discussed here.

We finally remark that simulations are carried out using the software tools CST MICROWAVE STUDIO®[4] and MATLAB [5].

BOUNDARY CONDITIONS

One problem in modelling an elementary particle beam traversing an accelerator component is the question of appropriately chosen boundary conditions. This is crucial since the fourier transform of a short bunch, Eq. 3, extends from $z=-\infty$ to $+\infty$. Since our computational domain is finite we have to take one of the following options:

- open boundary conditions, e.g. by using perfectly-matched layers[6]
- periodic boundary conditions

- no special prerequisite despite long enough pieces of beam pipe leading to the accelerator component

For reasons of simplicity we have chosen the last option. This is possible since the frequency range of interest (< 100 MHz) is well below the cutoff frequency of the beam pipe (radius 10 cm). We will now shed some light on whether this approach is practical in numerical simulations.

To this end let us consider the z dependent imaginary part of the integrand of Eq. 1 on the beam axis, i.e.

$$\operatorname{Im}\left(E_z(0,0,z;\omega)e^{\mathrm{i}\omega z/\beta c}\right), \tag{5}$$

which will help us to display the boundary effects. Consider the test system consisting of two pieces of a perfectly conducting beam pipe (20 cm $\times$ 20 cm quadratic cross section, variable length) connected to a cubic cavity (60 cm edge length, again perfectly conducting), as sketched in Fig. 1, top. Figure 1, bottom, shows a comparison of

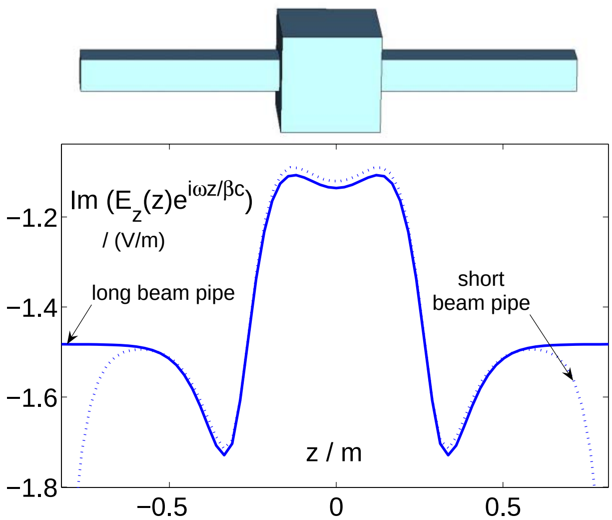

Figure 1: Top: sketch of the test system (length 300 cm). Bottom: integrand, as given by Eq. 5, for short beam pipes (length 120 cm each, dashed line) and long beam pipes (length 270 cm each, solid line), at $\omega/2\pi = 1$ MHz and $\beta = 0.85$.

the integrand, Eq. 5, at $\omega/2\pi = 1$ MHz, between two versions of the test system, one having overall length 300 cm, the other 600 cm. For $|z| > 50$ cm, the solid curve is a constant reflecting the space-charge impedance per length within the beam pipe (see next section). In the case of short beam pipes (dashed curve), boundary effects lead to a deviation from this expected behavior. For $|z| < 50$ cm, both curves are nearly identical. Thus, the impedance contribution of the cavity is correctly reflected by both simulations.

We have observed that boundary effects generally become larger towards lower frequencies (not shown here), which implies the need for longer beam pipe pieces in the simulation. At high frequencies, in contrast, boundary effects pose a less serious problem.

In summary, simulations with no other prerequisite than long enough adjacent beam pipes are sufficient for the calculation of coupling impedances. However, a further quantification of the reported effects would be useful. Moreover, one may imagine that more sophisticated kinds of boundary conditions (see above) would lead to more economic simulations. This will be one of the subjects of our further research.

SPACE-CHARGE EFFECTS

For $\beta \rightarrow 1$ the electric field of a point charge assumes a nearly 'plate-like' shape, meaning that its Coulomb interaction with preceding or following particles is negligible. For $\beta < 1$, however, these interactions (called space-charge effects) have a negative contribution to the imaginary part of the longitudinal coupling impedance.

As a test of our simulations, we have quantified this contribution for the case of an infinitely long beam pipe of either cylindrical or quadratic cross section. The walls are assumed to be perfectly conducting.

Due to the simple geometries of these examples, analytical expressions for the coupling impedances are available which we compare to the ones obtained from simulation.

Cylindrical beam pipe

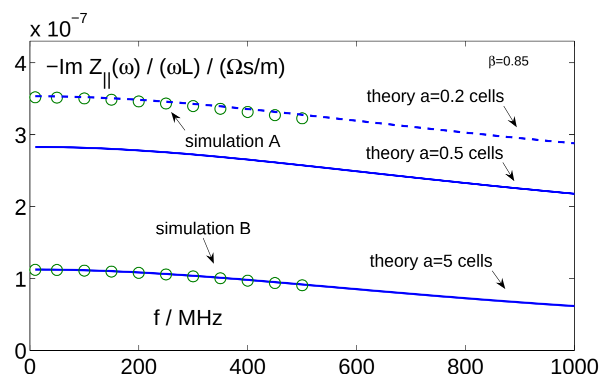

Figure 2: Negative imaginary part of the coupling impedance per unit length and frequency of a cylindrical beam pipe (radius 20cm, length 3m). Simulation A uses a current based on a single grid line along the z axis, whereas simulation B mimics a cylindrical current using grid lines within distance five from the origin of the transverse plane.

The expression for the longitudinal impedance for this case can be found in [7] and is not repeated here. We only remark that the impedance sensitively depends on the radius, a, of the particle beam. In Fig. 2, we see the comparison between theory and simulation in the case where the beam current has been imprinted on the central grid line along the z axis. Expressed differently, in each transverse plane, one 'dual' grid cell has been used to model the current. One may ask what radius should be assigned to this current. Firstly, Fig. 2 clearly shows that a radius of half the grid spacing is in contradiction to theory. Secondly, matching analytical results to simulation (dashed line), we find $a \approx 0.2$ times the grid spacing (only equidistant grids are used in this section). This finding agrees with the results of Waldschmidt and Taflove [8] who investigated the effective extension of current filaments. Thirdly, when modelling the beam current using several cells per transverse plane (i.e. a bundle of grid lines along the z axis) simulation data and theory coincide, as expected. This is also shown in Fig. 2 for the case of a bundle of grid lines with transverse diameter of 10 cells.

Rectangular beam pipe

The impedance for this case can be calculated exactly via textbook methods. A comparison with simulations then leads to a plot similar to Fig. 2, which we omit here due to the limited space. Again, assuming an effective radius of ca. 0.2 cell sizes for a single-line current leads to the agreement of theory and simulation.

FERRITE-LOADED COMPONENTS

We finally come to the main objective of our work, i.e. impedance calculations for kicker modules. Figure 3 shows the model under investigation here. It is similar to the SIS injection/extraction kicker operated at GSI, with respect to the key features, i.e. the design of ferrite modules (held by 20 mm-thick steel plates) and the dimensions of vacuum cavity and adjacent beam pipes. Simplifications have been introduced by assuming perfectly-conducting walls and by omitting finer geometric details. The material specifications (permittivity, complex frequency dependent permeability) for the used ferrite 8C11 have been obtained from the supplier's data sheet (*www.ferroxcube.com*).

The motivation behind treating an existing accelerator component is to be able to compare our numerical results with measurements.

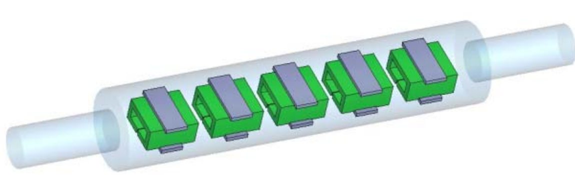

Figure 3: Model similar to GSI's existing SIS kicker, total length is 290 cm, cavity radius 20 cm, beam-pipe radius 10 cm. We put five ferrite modules here.

Figure 4 shows the coupling impedance below 100 MHz stemming from numerical simulation as described above. The z integration (see Eq. 1) extends over the whole cavity, the beam pipe parts being omitted.

Since the ferrite permeability possesses a considerable temperature dependence, we have to fix the simulation temperature (here to room temperature).

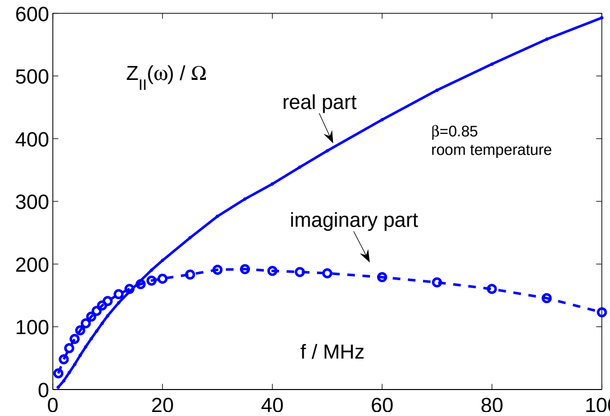

Figure 4: Longitudinal coupling impedance of the model shown in Fig. 3.

OUTLOOK

Further steps in our work will cover two main subjects: Firstly, we will further improve our numerical approach (use of different boundary conditions, multi-grid solvers). Secondly, comparison with measurements is needed. To this end, it would be desirable to measure the coupling impedance of the existing SIS kicker either within accelerator operation or on-bench, e.g. by the coaxial wire method.

ACKNOWLEDGEMENTS

We thank U. Blell and O. Boine-Frankenheim for helpful discussions and for the supply of the SIS-kicker geometry data, and W. Müller for proofreading the manuscript. B. Doliwa's work has been funded by the GSI and the 'Graduiertenkolleg Physik und Technik von Beschleunigern' at the Technische Universität Darmstadt.

REFERENCES

[1] Caspers F. *Handbook of Accelerator Physics (A.W. Chao, M.Tigner, Eds.)*, Chapter 7.5.1, 1999.

[2] U. van Rienen and T. Weiland. *IEEE Trans. Magn.*, 26(2):743, 1990.

[3] T. Weiland. *AEU-Int. J. Electron. Commun.*, 31(3):116, 1977.

[4] CST GmbH. *Bad Nauheimer Str. 19, 64289 Darmstadt, www.cst.com.*

[5] The Mathworks GmbH. *Adalperostr. 45, D-85737 Ismaning, Germany*.

[6] J. P. Berenger. *J. Comput. Phys.*, 114(2):185, 1994.

[7] A. M. Al-Khateeb, O. Boine-Frankenheim, I. Hofmann, and G. Rumolo. *Phys. Rev. E*, 6302(2):026503, 2001.

[8] G. Waldschmidt and A. Taflove. *IEEE Microw. Guided Wave Lett.*, 10(6):217, 2000.