SIMPLE THEORY OF THERMAL FATIGUE CAUSED BY RF PULSE HEATING

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Abstract

The electron-positron linear colliders projects imply that accelerating structures and other RF components will undergo action of extremely high RF fields. Except for electrical the breakdown threat there is an effect of the copper surface damage due to multi-pulse mechanical stress caused by Ohmic losses in the skin layer [1].

A new theory of the thermal fatigue is presented. The theory is based on the consideration of the quasi-elastic interaction between neighborhood grains in the metal due to the thermal expansion of the skin-layer. With a proposed method one can estimate a total number of the RF pulses needed for surface to fracture depending on temperature rise, pulse duration, and average temperature. The parameters necessary for the final equation were found, using experimental data points obtained at 11.4 GHz for the copper [2]. Experimental studies of the pulsed heating fatigue of the copper surface at 30 GHz are also under way [3].

FATIGUE MODEL

The copper crystal has a cubic structure. Ideally, each atom has 6 links with neighbors (Fig. 1). Probability to break any link is given by next equation:

$$p_{j} = \exp\left(-\frac{U_{c}}{k_{B}T_{c}}\right), \qquad (1)$$

which shows that the higher the temperature the higher the probability. Here U_c – energy of the coupling of the atoms, T_c is a stationary temperature, and k_B is a Boltzman's constant. If $T_c>0$, the probability is not zero and there are some broken links. Should mention that because of (2), the value of p_j is rather small for the temperatures less than metal melting point.

$$\frac{U_c}{k_B T_c} >> 1, \tag{2}$$



Figure 1: Structure of copper's crystal.

However, real copper has many defects. In particular, any bulk copper consists of grains (Fig. 2). Energy of

coupling between grains is weak compared to the ideal crystal, and there is certain expansion of the grains if surface is hot.

During RF pulse, the Ohmic skin layer δ_{ohm} represents a thermal source. A thin layer of copper δ_T , which is exposed to the temperature increase, is a thermal skin layer. Next we suggest that the thermal skin layer is significantly smaller than the typical size of the individual copper grain. In this case the expansion in the thermal skin layer brings to situation when grains start to push each other (Fig. 3). As a result, additional elastic forces come to scene, thus increasing the probability to break links between atoms.



Figure 2: Schematic view of copper's grain structure.



Figure 3: Interaction of grains under pulse RF heating.

High Temperature Region Approach

In the presence of the elastic forces the equation (1) has to be modified:

$$p_{j} = \exp\left(-\frac{U_{c} - U}{k_{B}T_{c}}\right), \qquad (3)$$

where \overline{U} is an energy of the external forces breaking the given link. In assumption that individual grain has n_s links with the other grains, the probability to break all these links can be expressed as:

$$p = \prod_{j=1}^{j=n_s} p_j.$$
 (4)

The probability to break all n_s links after N thermal cycles is:

$$P = 1 - (1 - p)^{N}, (5)$$

Taking into account (2), the above equation (5) can be rewritten in a simple way:

$$P = N \cdot p. \tag{6}$$

Obviously, large enough critical value of P exists, when the copper surface start to fracture. See [2] for example. Using (3), (4), (6) one can express the total number of pulses needed to develop the cracks on a surface as:

$$N_f = P_c \cdot \exp\left(\frac{U_c n_s}{k_B T}\right) \cdot \exp\left(-\frac{\overline{U} \cdot n_s}{k_B T}\right), \quad (7)$$

where P_c is the critical value.

Next we will look at how the elastic forces energy depends on the temperature rise ΔT and pulse duration τ . Taking into account that these forces have elastic nature, for the given grain with n_s links we can write now:

$$\overline{U} = \frac{kx^2}{2} \cdot \frac{1}{n_s},\tag{8}$$

Following equation (9) expresses the linear thermal expansion of the individual grain with typical size l_0 :

$$\mathbf{x} = \boldsymbol{\alpha} \cdot \boldsymbol{l}_0 \cdot \Delta \boldsymbol{T},\tag{9}$$

where α is a coefficient of thermal expansion. Defining *k*, as $k=E \cdot S/l_0$, where $S=\delta_T \cdot l_0$ and *E* is Yung's module we finally get:

$$\overline{U} = \frac{E \cdot \alpha^2 \cdot \delta_T \cdot l_0^2 \cdot \Delta T^2}{2 \cdot n_s}, \qquad (10)$$

Taking into account the δ_T dependence on the pulse duration:

$$\delta_T = \beta \cdot \sqrt{\tau}, \qquad (11)$$

where β is a constant, we end up with a general equation for the number of pulses necessary to fracture the copper surface:

$$N_{f} = P_{c} \cdot \exp\left(\frac{U_{c}n_{s}}{k_{B}T}\right) \cdot \exp\left(-\frac{E \cdot \beta \cdot \alpha^{2} \cdot l_{0}^{2} \cdot \sqrt{\tau} \cdot \Delta T^{2}}{2k_{B}T}\right),$$
(12)

As an important consequence one can see that the bigger the size of the copper grains, the less number of the pulses are needed to fracture the surface. Another conclusion is that this number is a strong function of the bulk steady temperature.

In equation (12) there are several coefficients, which could not be defined exactly. That is why we rewrite this equation in the most general form, which contains two principal unknown coefficients only:

$$N_f = B \cdot \exp\left(-\zeta \cdot \sqrt{\tau} \cdot \Delta T^2\right)$$
(13)

As a guess, these coefficients (*B* and ξ) we suggest to extract from the experimental data taken from [2] for example.

Theory, Codes, Simulations Theory, Codes, Simulations, Other In Fig. 4 the experimental data points in coordinate system N_{f^-} - ΔT are represented by boxes. They correspond to τ equal to 1000 ns. Following equation (13), the number of pulses to failure as a function of the temperature rise of the copper surface is shown in Fig. 4 (blue line).



Figure 4: Total number of pulses needed for degradation (τ =1000 ns): lilac boxes taken from [2], blue curve – high temperature rise approach, red curve – the modified theory.

Extrapolation of the High Temperature Approach to the Low Temperature Region

As it follows from Fig. 4 the equation (13) predicts finite number of pulses even under condition when $\Delta T=0$, which certainly does not respect the reality. This means that the proposed model is rather simple and is valid only for the high temperature region. That is why, next we studied the processes in more details, see Fig. 5.



Figure 5: Scheme of the balance between concentrations of broken (n_b) and unbroken (n_u) links.

In general, the heating increases the probability of the atomic links breaks. Nevertheless, after the end of the thermal cycle, this probability recovers its initial value and most of the broken links are recovered too. However, there is certain number of the links which are not recovered because of the irreversible nature of the crystal defects. It will be naturally to suggest that the probability to create the irreversibly broken link is proportional to the difference of probabilities during temperature rise and before (or after) RF pulse. This make it possible to modify equation (13) in a following way:

$$N_f = \frac{C}{\exp(\zeta \cdot \sqrt{\tau} \cdot \Delta T^2) - 1}.$$
 (14)

The constants *C* and ξ are defined now using the same experimental data points as before. The results are shown in Figs 6-7, where the plots of N_f versus ΔT and τ are presented.

Now one can see that following the modified equation, the number of pulses to failure goes to infinity, when ΔT goes to zero. If ΔT is high enough, the formula (14) and formula (13) do agree well.



Figure 6: Total number of pulses needed for degradation Vs temperature rise for different pulse durations.



Figure 7: Total number of pulses needed for degradation Vs pulse durations for different temperature rise values.

CONCLUSION

The proposed method make it possible to find a simple way to estimate the number of the thermal cycles

to fracture the metal surface using few (two at least) experimental data points.

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REFERENCES

- [1] V F.. Kovalenko. Introduction to microwave electronics. Sov. Radio, M, 1971. (in Russian).
- [2] D.P. Pritzkau et al. Experimental Study of RF Pulsed Heating on Oxygen Free Electronic Copper, Physical Review Special Topics – Accelerators and Beams, Vol. 5, 2002, pp. 112002-1 - 112002-21.
- [3] S.V.Kuzikov et al. Experiment to Study Effects of Multi-Pulse Heating in a 30 GHz Resonant Cavity, Proc. of the Strong Microwaves in Plasmas Conf., Vol. 1, 2003, pp. 195-198.