CALCULATION OF RF PROPERTIES OF THE THIRD HARMONIC CAVITY

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Abstract

Recently a third harmonic structure has been proposed for the injector of the TTF-FEL to avoid nonlinear distortions in the longitudinal phase space. This structure consists of four nine cell TESLA-like cavities. For the use of this structure in combination with the TTF-FEL it might be interesting to investigate higher order modes (HOM) in the structure and their effect on the beam dynamics. In the 5th dipole passband one mode with a frequency around 9.05 Ghz was found to be almost trapped in the cavity with very small fields in the end cells and the beam pipes. CST MicrowaveStudio® (MWS) and Coupled S-Parameter Calculation (CSC) have been applied to investigate this frequency range. The CSC method [1] is based on the scattering parameter description of the rf components found with field solving codes or analytically for components of special symmetry. This paper presents the results of the calculation of frequencies and field distributions of dipole modes in the frequency range around 9.05 GHz.

INTRODUCTION

A 3.9 GHz 3rd harmonic superconducting section for the photoinjector of the TTF 2 has been proposed [2] and designed [3] recently. This section consists of four nine cell cavities. Its purpose is to correct for nonlinearities of the longitudinal phase space to produce highly charged bunches. The basic rf parameters for many monopole, dipole and quadrupole modes for one nine cell cavity of the 3rd harmonic section have been calculated [4] using MAFIA eigenmode solver [5]. As a result of these calculations a dipole mode in the frequency range around 9.05 GHz was found which has almost no electric field in the beam pipes and even in the cavity end cells. Therefor this mode might be difficult to be damped by the HOM-couplers.

This paper presents the first results of CSC calculations of frequencies and field distributions for dipole modes in this frequency range and compares the results with the results of calculations done with CST Microwave-Studio® [5] for the same frequency range.

CSC THEORY

The reflection and transmission of waves between the ports of any rf system can be described by scattering-parameters (S-parameters). In general, each S-parameter is a complex function of the frequency containing information about amplitude and phase. S-parameters can be represented by the scattering matrix $\mathbf{S}$:

$$\mathbf{b_k} = \mathbf{S_k}\,\mathbf{a_k} \tag{1}$$

with the vectors $\mathbf{a_k}$ and $\mathbf{b_k}$ describing all input- and output-signals of the k-th object, respectively.

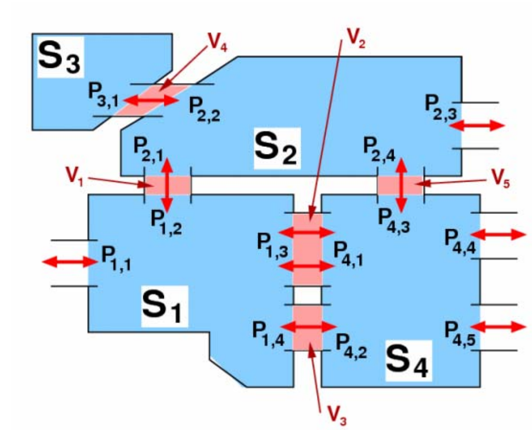

Figure 1: Complex rf-structure consisting of four substructures S_1, S_2, S_3, S_4, connected to each other by the connections V_1, V_2, V_3, V_4, with external ports $P_{1,1}$, $P_{2,3}$, $P_{4,4}$, $P_{4,5}$ and internal ports $P_{1,2}$, $P_{1,3}$, $P_{1,4}$, $P_{2,1}$, $P_{2,2}$, $P_{2,4}$, $P_{3,1}$, $P_{4,1}$, $P_{4,2}$, $P_{4,3}$.

For a complex structure consisting of several subsections the S-parameters of all subsections are arranged in a block diagonal matrix $\mathbf{S}$:

$$\mathbf{b} = \mathbf{S}\,\mathbf{a} = \begin{pmatrix} \mathbf{S_1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{S_N} \end{pmatrix} \begin{pmatrix} \mathbf{a}_1 \\ \vdots \\ \mathbf{a_N} \end{pmatrix} = \begin{pmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b_N} \end{pmatrix} \tag{2}$$

Next all incident signals are collected in a vector $\mathbf{a_{inc}}$ and all signals travelling from one subsection into a neighbouring subsection are grouped together in a coupling vector $\mathbf{a_{cop}}$. These rearrangements are performed by two permutation matrices $\mathbf{P}$ and $\mathbf{F}$:

$$\mathbf{a} = \mathbf{P}\begin{pmatrix} \mathbf{a}_{\mathrm{cop}} \\ \mathbf{a}_{\mathrm{inc}} \end{pmatrix} \quad ; \quad \begin{pmatrix} \mathbf{a}_{\mathrm{cop}} \\ \mathbf{a}_{\mathrm{sct}} \end{pmatrix} = \mathbf{P}^{-1}\,\mathbf{F}\,\mathbf{b} \tag{3}$$

$\mathbf{a_{sct}}$ is the vector of all signals leaving the structure at the external ports. These signals are kept untouched. The combination of the above equations yields:

$$\begin{pmatrix} \mathbf{a}_{\mathrm{cop}} \\ \mathbf{a}_{\mathrm{sct}} \end{pmatrix} = \underbrace{\mathbf{P}^{-1}\,\mathbf{F}\,\mathbf{S}\,\mathbf{P}}_{\mathbf{G}} \begin{pmatrix} \mathbf{a}_{\mathrm{cop}} \\ \mathbf{a}_{\mathrm{inc}} \end{pmatrix} \tag{4}$$

where the matrix $\mathbf{G} = \mathbf{P}^{-1}\mathbf{FSP}$ describes the structure of the whole system. According to the dimensions of vectors $\mathbf{a_{cop}}$, $\mathbf{a_{sct}}$ and $\mathbf{a_{inc}}$ this system matrix $\mathbf{G}$ can be split into submatrices:

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$$\mathbf{G} = \begin{pmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{pmatrix} \tag{5}$$

which allows to solve eq. (4) for $\mathbf{a_{sct}}$:

$$\mathbf{a_{sct}} = \left(\mathbf{G_{21}} \left(\mathbf{I} - \mathbf{G_{11}} \right)^{-1} \mathbf{G_{12}} + \mathbf{G_{22}} \right) \mathbf{a_{inc}} \, . \tag{6}$$

From eq. (6) the overall scattering matrix of the whole structure, denoted as $\mathbf{S^{(T)}}$ can thus be written as $\mathbf{S^{(T)}} = \mathbf{G_{21}} (\mathbf{I} - \mathbf{G_{11}})^{-1} \mathbf{G_{12}} + \mathbf{G_{22}}$. It describes scattering at all open ports including all possibly existing multi mode scattering.

A resonator has no open (external) ports and $\dim(\mathbf{a_{inc}}) = \dim(\mathbf{a_{sct}}) = 0$ holds, only the coupling between the internal ports remains. For this case the block matrices **G12**, **G21** and **G22** vanish and eq. (6) reduces to:

$$\left(\mathbf{I} - \mathbf{G_{11}} \left(\omega_0 \right) \right) \mathbf{a}_{\mathrm{cop}} = 0 \tag{7}$$

Eq. (7) is fullfilled for discrete frequencies only, the resonant frequencies aimed for. In order to find these frequencies ω_0 the eigenvalues of $(\mathbf{I} - \mathbf{G_{11}}(\omega))$ have to be calculated. A resonant frequency is found, if at least one eigenvalue equals zero. In that case the vector $\mathbf{a_{cop}}$ holds the amplitudes of the waveguide modes at the positions of the internal ports.

APPLICATION

The CSC method was applied to the investigation of higher order modes in the third harmonic section which is planned to be installed in the TTF-FEL. Especially in the 5th dipole passband one mode with a frequency in the range of about 9.05 GHz was found to be almost trapped in the cavity. This mode was found to have very small fields in the end cells and in the beam pipes.

For all calculations the geometry of the Fermilab 3rd harmonic 3.9 GHz cavity was used, see figure 2.

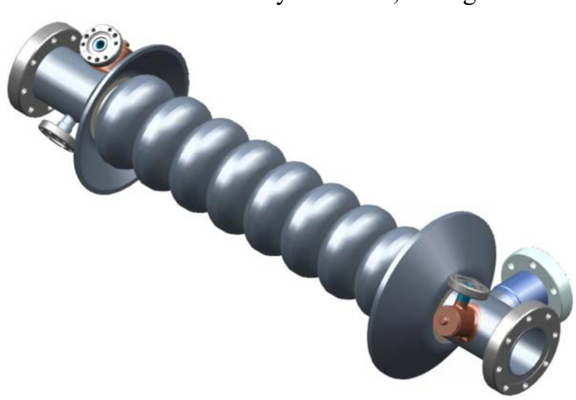

Figure 2: Design of 3rd harmonic 3.9 GHz niobium cavity (geometry provided by N. Solyak, Fermilab).

RESULTS OF CALCULATIONS

For the investigation of the dipole modes in the 5th dipole passband the field distribution of some modes in the frequency range around 9.05 GHz was computed using CST MicrowaveStudio®. Utilising the symmetry of the 9 cell cavity, field distributions were only calculated for the structure shown in figure 3.

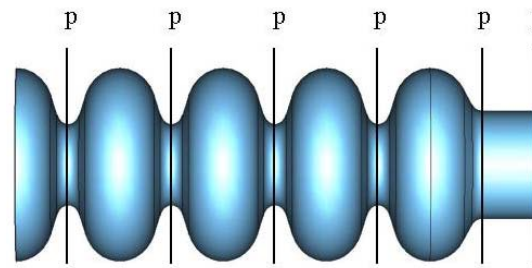

Figure 3: Geometry used for CST MicrowaveStudio® calculations, p indicates the positions of the ports used for CSC calculations of single cells.

The boundary conditions at the left and right boundary of the structure were choosen to electric – electric, electric – magnetic, magnetic – magnetic and magnetic – electric. The dipole modes in the 5th dipole passband in the frequency range around 9.05 GHz have nearly no field in the beam pipe, therefor no significant change occurred in the calculated field distributions with the same boundary conditions at the right boundary (at the beam pipe). The results of the MWS calculations are presented in figures 4 and 6.

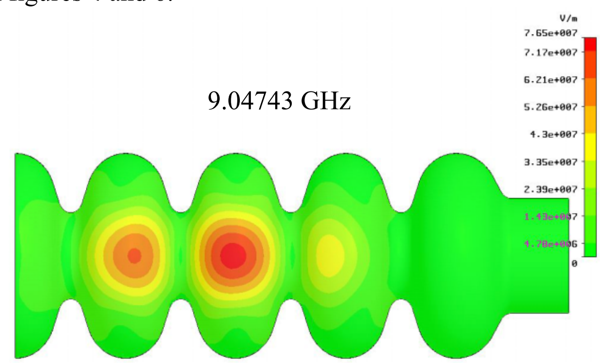

Figure 4: Contour plot of the absolute value of the electrical field of a dipole mode, calculated with MWS.

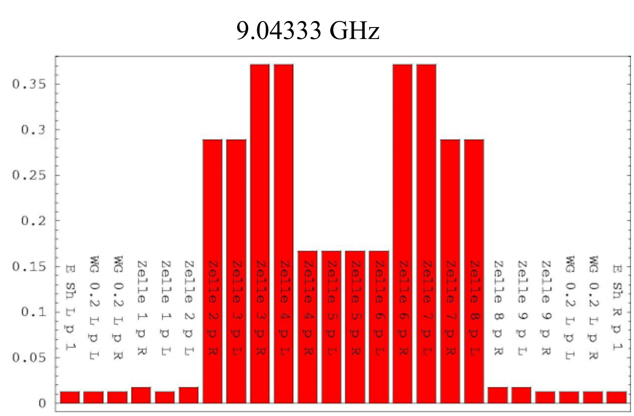

Figure 5: Absolute value of wave amplitudes of electrical field of a dipole mode at the ports of the single cells, calculated with CSC (L, R … left, right; E Sh … electrical short; WG 0.2 … waveguide with 0.2 m length).

CSC calculations were performed for a 9 cell cavity with the beam pipes shorted. The results, presented in the figures 5 and 7, were composed of the calculation of each single cell taking into account the identical geometry of the inner cells. Within a first step of these calculations the frequencies and the absolute value of the wave amplitudes of the electric field of dipole modes in the 5th dipole passband at the ports of the single cells were calculated. For the coupling between the ports of the single cells only TE 11 mode was taken into account. This was sufficient to reproduce the frequencies calculated with MWS. S-parameters of the beam pipe and of the shorts were calculated analytically.

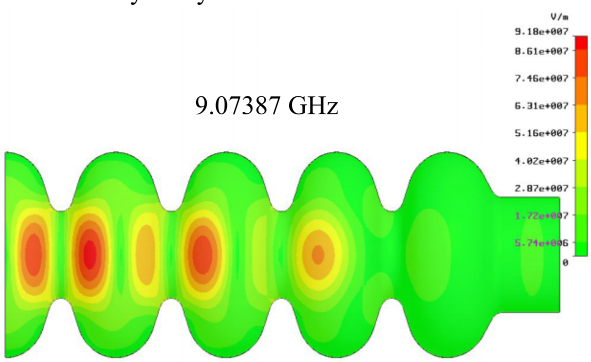

Figure 6: Contour plot of the absolute value of the electrical field of a dipole mode, calculated with MWS.

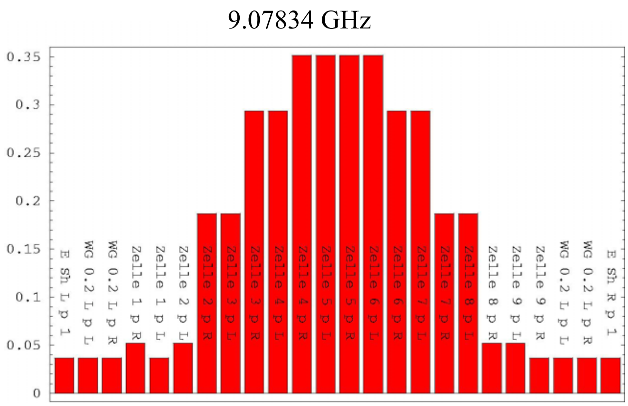

Figure 7: Absolute value of wave amplitudes of electrical field of a dipole mode at the ports of the single cells, calculated with CSC (L, R … left, right; E Sh … electrical short; WG 0.2 … waveguide with 0.2 m length).

For the evaluation of the results, obtained with CSC, these results have to be compared with the results, obtained with MWS (figure 5 – figure 4; figure 7 – figure 6). The difference of the frequencies is lower than 5 MHz. The field distributions, calculated with MWS, have to be compared with the field distributions, calculated with CSC at the location of the ports of the correspondding cells of the cavity. This comparison also shows, that the results of the CSC calculations fit well to the results of MWS calculations.

CONCLUSIONS

CST MicrowaveStudio® and CSC calculations were performed for the investigation of dipole modes in the 3rd harmonic 3.9 GHz cavity. Results of the calculations with both methods yield evidence for the existence of trapped dipole modes in the frequency range around 9.05 GHz. Frequencies and field distributions, calculated with CSC fit well to the results of MWS calculations and may be used for further calculations of beam relevant parameters of modes in the structure, completely equipped with HOM- and input-couplers.

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