RIBBON ION BEAM DYNAMICS IN UNDULATOR LINEAR ACCELERATOR[*]

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Abstract

The possibility of non-synchronous radio frequency field harmonics using for ribbon ion beam focusing and acceleration in linac is discussed. In periodical resonant structure the accelerating force is produced by the combination of two spatial harmonics of RF field (two RF undulators). The examples illustrating the efficiency of the proposed method of acceleration are given for longitudinal and transverse undulators.

INTRODUCTION

The space-charge influence is the main factor limiting the beam intensity in low energy RF linac. The ion ribbon beam can be used in order to increase current in linac. The stable motion of ribbon ion beam can be provided by using an external focusing devices or a special configuration of RF fields (RF focusing). The latter approach seems to be more promising for low energy ions. In a conventional RF linac ion beams are accelerated by a synchronous wave. Non-synchronous wave will be used for focusing a charged particles only.

The values of synchronous and nonsynchronous harmonic amplitudes must be chosen to provide both longitudinal and transverse stability. For two wave approach (synchronous and one nonsynchronous harmonics) RF focusing conditions were founded for axisymmetric (ARF) and ribbon (RRF) beams in Ref. [1-2].

Another method to accelerate ions in the RF periodical structure without synchronous wave was suggested in Ref. [3-4]. In this case the acceleration force is to be driven by a combination of two non-synchronous waves (two undulators) and transverse focusing is realized by means of each RF undulator. The peculiarity of a ribbon ion beam focusing and acceleration in RF undulator linear accelerator (UNDULAC-RF) is discussed in this paper.

RF FIELD IN PERIODICAL RESONATOR

The UNDULAC-RF can be realized using an interdigital H-type resonator. The especial form of electrodes must be used in order to provide the transverse focusing along the ribbon width (see Fig.1, [4]). The field excited in a periodical resonator can be found as a periodic solution of the Maxwell equations. Under the assumption that the cross-section size of the accelerator channel is much smaller than the wavelength of the RF field, the Fourier series coefficients for field can be calculated with quasi-electrostatic approximation. The

potential of RF field in periodic resonator can be represented as the sum of the spatial harmonics:

$$U = \sum_{n=0}^{\infty} U_n(x,y) \cdot \sin\left(\int h_n \, dz + \alpha\right) \cos(\omega t) \quad (1)$$

where $h_n = \mu / D + 2\pi n / D$ is a longitudinal wave number of field harmonic, μ is a phase advance of the field per period of RF structure, n is the harmonic number. The n-th harmonic amplitude U_n can be found from the equation

$$\Delta_\perp U_n = h_n^2 U_n \quad (2)$$

Two solutions of this equation are existent: $U_n(x,y) \sim \cosh(h_{n,x} x) \cosh(h_{n,y} y)$ and $\alpha = 0$ for longitudinal radio frequency undulator and $U_n(x,y) \sim \cosh(h_{n,x} x) \sinh(h_{n,y} y)$, $\alpha = \pi / 2$ for transverse one. Here $h_{n,x}$ and $h_{n,y}$ are transverse wave number, $h_{n,x}^2 + h_{n,y}^2 = h_n^2$. The ratio $h_{n,x} / h_{n,y}$ is defined by the electrodes form and it's value is small for ribbon beams with large aspect ratio.
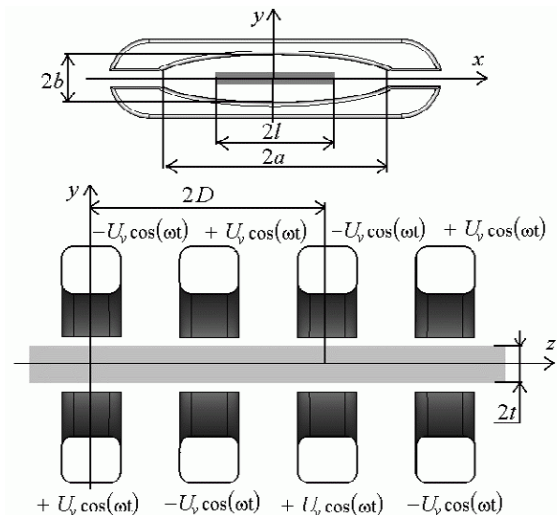

Figure 1: The structure of transverse UNDULAC-RF.

EQUATION OF MOTION

Let us consider the equation of motion for a non-relativistic ion beam in RF field (1) assuming that the particle velocity v differs from the phase velocity of all harmonics: $v_{ph,n} = \omega / h_n$, $n=0,1,2...$. In general, the interaction of the particles with the non-synchronous harmonic of the RF field does not change the average

energy of the beam but causes the fast oscillations in the longitudinal and transverse directions. The synchronism between the beam particles and the spatial harmonics of the RF field is absent in UNDULAC-RF. The effective beam-field interaction takes place if the beam velocity is close to the combined wave phase velocity $v_c = \omega / h_c$ in this case [3]. The value $h_c = (h_n + h_p)/2$ defines the wave number of the combined wave resulting when fields of the n-th and p-th harmonics are added. Introducing a slowly varying coordinate $\bar{r}$ and phase $\varphi = \int h_c d\bar{z} - \omega t$ and averaging over the fast oscillations as it is done in Ref. [3-4], one can obtain the equation in the Hamilton's form:

$$\frac{d^2 \bar{r}}{dt^2} = -\frac{d}{d\bar{r}} U_{eff} , \qquad (3)$$

where the function U_{eff} depends on the amplitudes of the RF field harmonics $E_{n,p}$. The transverse and the longitudinal electric RF field components in periodical resonant structure can be found from (1): $E_{\perp,n} = -\nabla_\perp U_n(x,y)$; $E_{z,n} = h_n U_n(x,y)$, if $h_n \neq 0$, and $E_{z,0} = const$, if $h_0 = 0$. The function U_{eff} can be considered as the effective potential function that specifies the Hamiltonian of the beam-wave system. The study the 3D beam dynamics in the smooth approximation can be provided by means of this function. The existence of the absolute minimum of U_{eff} is necessary condition for effective beam focusing and acceleration, as it follows from Eq. (3).

It can be introduced the reference particle for analysis of the beam stability in polyharmonic RF system. The reference particle velocity is equal to the combined wave velocity, v_c, and it's phase in this wave, φ_c, remains constant. The longitudinal bunching and acceleration of the beam are possible if v_c and φ_c are slow varying with the longitudinal coordinate. From the equation

$$\frac{dv_c}{dt} = -\frac{d}{dz} U_{eff} \bigg|_{x=0, y=0} \qquad (4)$$

one can find the law of v_c variation, the structure period D and the range for the reference particle phase, φ_c, when the bunching of the ion beam is the most efficient.

The beam acceleration and focusing in UNDULAC can be realized using longitudinal or transverse undulator, where RF field is generated at $\mu = \pi$ and $\mu = 0$ modes. Let us consider the plane UNDULAC-RF structure with two fundamental harmonics $n=0$, $p=1$. Introducing the dimensionless values of the field harmonic amplitude $e_{\perp,z} = e\lambda E_{\perp,z} / 2\pi mc^2$, the particle velocity β, and the slowly varying coordinates $R = [\rho, \eta, \varphi]$, $\rho = 2\pi \bar{x} / \lambda$, $\eta = 2\pi \bar{y} / \lambda$, $\xi = 2\pi \bar{z} / \lambda$, where λ is a wavelength; $\tau = \omega t$, one can write the dimensionless equation of motion in the form

$$\frac{d^2 R}{d\tau^2} = -\frac{d}{dR} \bar{U}_{eff} \qquad (3a)$$

The dimensionless function $\bar{U}_{eff}$ is equal

$$U_{eff} = \frac{\nu}{4} \big[k_0 e_0^2 + k_1 e_1^2 + \\ + 2(e_{0,z} \cdot e_{1,z} - e_{0,\perp} \cdot e_{1,\perp}) \cos(2\varphi + 2\alpha) \big] \qquad (5)$$

Here amplitudes $e_0(x,y)$ and $e_1(x,y)$ are the functions of transverse coordinates. The coefficients are equal $k_0 = 10/9$, $k_1 = 26/25$, $\nu = 1$ for $\mu = \pi$ mode, and $k_0 = 1$, $k_1 = 5/9$, $\nu = 1/2$ for $\mu = 0$ mode.

The equation (4) for the reference particle velocity can be written using the effective potential function (5):

$$\frac{d\beta_c}{d\tau} = e_{eff} \sin 2\varphi_c , \qquad (4a)$$

where $e_{eff} = \nu \cdot e_0 \cdot e_1 / \beta_c$ is the effective amplitude of combined wave. The longitudinal beam stability is possible if the reference particle phase in the combined wave lies in the intervals of $[\pi/4, \pi/2]$ and $[5\pi/4, 3\pi/2]$ and the particle velocity v is close to the reference particle velocity v_c. In this case two bunches per one RF field period will be configured.

BEAM DYNAMICS IN LONGITUDINAL UNDULATOR

We will consider the beam dynamics in UNDULAC-RF linac for deuterium ions. This accelerator consists of two sub-section: the gentle buncher section of length L_b and acceleration sub-section of length L. In the buncher the field harmonic amplitude smoothly increase as $E(z) = E_m \cdot \sin(\pi z / 2L_b)$ and the reference particle phase in combined wave φ_c linearly decreases from $\pi/2$ to $3\pi/8$ or from $3\pi/2$ to $11\pi/8$. The harmonic amplitude, E_m, and the reference particle phase, φ_c, remains unchanged at the acceleration sub-section. In the simplest case one can account the ratio of RF field harmonics amplitudes $\chi = E_1 / E_0$ is constant.

It is interesting to compare the beam focusing and acceleration conditions in longitudinal UNDULATOR where the field is generated at the modes π and 0. Let us suppose that the particle velocity differs significantly from the phase velocities of zero, β_{s0}, and first, β_{s1}, RF field harmonics. In our case $\beta_{s0} = 2\beta_c$, $\beta_{s1} = 2\beta_c / 3$, $\beta_c = D / \lambda$ for $\mu = \pi$ mode and $\beta_{s0} = \infty$, $\beta_{s1} = \beta_c / 2$, $\beta_c = 2D / \lambda$ for $\mu = 0$ mode. The vertical size of the separatrix for the combined wave is shown in Fig. 2, curve 1. This figure is plotted for the next parameters: $E_0 = 150$ kV/cm, $\chi = 0.9$, the buncher length $L_b = 1.2$ m, accelerator length $L = 1.3$ m, $\lambda = 1.5$ m, the initial energy of the deuterium ions $W_{in} = 100$ keV. The variation of longitudinal velocity of the reference particle (with initial phase $\varphi(0) = \pi/2$ in bunch) is shown for the smooth

approximation (curve 2) and for polyharmonic field (curve 3).

The separatix sizes for the zero (curve 4) and the first RF field harmonics (curve 5) are shown in the same figure also in assumption that the beam velocity is close to β_{s0} or to β_{s1} accordingly. One can show that for small value of RF field amplitudes ratio ($\chi = 0.1$-0.3) the combined wave separatix does not overlap first harmonic separatix, but of the longitudinal beam velocity in polyharmonic field can be outside of separatix for combined wave. For large χ ($\chi \geq 0.5$) the oscillation amplitude is small, but these separatrixes are overlapping. In both cases, the particle velocity value can attain the first harmonic separatrix and the ions can be recatched by first RF field harmonic or loused. The analysis of a equation solution for π mode shows that the choice of the parameter χ influences on the beam dynamics and optimal χ value must be found for realization of large current transmission coefficient K_t. In our case the optimal value χ is equal 0.3-0.4. In the smooth approximation coefficient K_t for UNDULAC-RF with $\mu = \pi$ mode is equal 90-95 %.

In the UNDULAC-RF for $\mu = 0$ mode the distance between the separatrixes of combined wave and first RF field harmonic is larger and the influence of χ is smaller comparatively $\mu = \pi$ mode. The current transmission coefficient K_t is equal 85-90 % for $\mu = 0$ mode in the smooth approximation.

The transverse beam dynamics can be analyzed by means of the effective potential function too. In the longitudinal undulator the amplitude of a rapid transverse oscillations is small, and the beam envelop in smooth approximation is closely to real size. One can be shown that the transverse focusing condition is satisfied for the all χ value in UNDULAC-RF when $\mu = \pi$ mode of the RF field is used. In the UNDULAC-RF for $\mu = 0$ mode the transverse focusing is realized when the parameter $\chi > 1$ only for paraxial particles. Detailed study of the transverse beam focusing for $\mu = 0$ mode is shown that it is no effective because this focusing is provided by means of the first RF field harmonic only.

BEAM DYNAMICS IN TRANSVERSE UNDULATOR

The focusing and acceleration of particles is possible both in the longitudinal and the transverse RF field. In the last case the beam acceleration can be realized when electric field has only transverse components on the axis. In this case the amplitude of the rapid transverse oscillation is the largest then longitudinal one. The averaged equation of motion can be derived by means of the effective potential function (4) and all above results are valid for transverse undulator. The Hamiltonian of the beam-wave system determines the connection between the longitudinal and transverse dynamics. For analysis of

the longitudinal motion it is sufficiently the smooth approximation. In the longitudinal phase space there is only one separatix and the current transmission coefficient K_t is equal 95 % in this case.

The rapid transverse oscillations influence on the effective transverse emittans. The value of emittans increases in size and the particles can be lost due to the bounded aperture of the channel. As in the longitudinal undulator, the transverse beam focusing for $\mu = 0$ mode is less effective than for $\mu = \pi$ mode. The current transmission coefficient is equal 30 % for this case. It is clear that this type of undulator linac is not interesting.
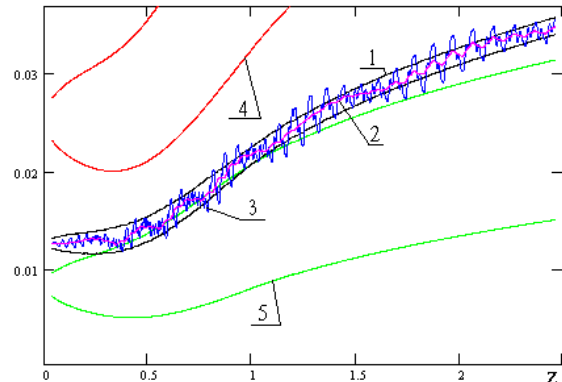

Figure 2: The separatrixes of combined wave and RF field harmonics.

CONCLUSION

The results of analytical investigation of the ribbon beam dynamics in UNDULAC-RF accelerator are discussed. It was shown that the undulator with the longitudinal RF field for $\mu = \pi$ mode is more preferable system for ion ribbon beam acceleration. The RF periodical structure for UNDULAC-RF is simpler for realization, than RRF accelerator where parameter $\chi = 10$ [2]. The zero RF field harmonic amplitude is smaller than in RRF linac for the same acceleration gradient. In the consider range of the energy, the UNDULAC-RF can be used for acceleration of high current ion beam.

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