REDUCTION OF RF POWER LOSS CAUSED BY SKIN EFFECT

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Abstract

Skin effect on a metal foil that is thinner than a skin depth is investigated starting from general derivation of skin depth on a bulk conductor. The reduction of the power loss due to the skin effect with multi-layered conductors is reported and discussed. A simple application on a dielectric cavity is presented.

INTRODUCTION

RF current flows on a metal surface with only very thin skin depth, which decreases with RF frequency. Thus the surface resistance increases with the frequency. Because the skin depth also decreases when the metal conductivity increases, the improvement of the conductivity does not contribute much; it is only an inverse proportion to the square root of the conductivity. Recently, it is shown that such a power loss can be reduced on a dielectric cavity with thin conductor layers on the surface, where the layers are thinner than the skin depth [1]. The thin foil case is analyzed after a review of the well-known theory of the skin effect. Then an application to a dielectric cavity with TM0 mode is discussed [2].

SKIN DEPTH OF BULK MATERIAL

In materials with conductance σ ($\gg i\omega\varepsilon$), Ampere's law becomes:

$$\nabla \times \mathbf{H} = (\sigma + i\omega\varepsilon)\mathbf{E}. \qquad (1)$$

Suppose a system as shown in Fig. 1, Eq.1 finally becomes:

$$\partial_x^2 j = i\omega\mu\sigma j, \quad j = \sigma E_y. \qquad (2)$$

Under the condition of $\sigma \gg i\omega\varepsilon$, the solution of Eq.2 is:

$$j(x) = j_0 e^{-(1+i)x/\delta}, \quad \delta = \sqrt{2/\omega\mu\sigma}, \qquad (3)$$

where δ is the skin depth. Considering that $\nabla \times \mathbf{E}$ has only one nonzero component of $\partial_x E_y = \partial_x j/\sigma$, the magnetic field in the conductor is derived from Faraday's law $\partial_x E_y = -i\omega\mu H_z$ as

$$H_z(x) = \frac{\partial_x E_y}{-i\omega\mu} = \frac{\partial_x j}{-i\sigma\omega\mu} = \frac{\delta^2 \partial_x j}{-2i} = \frac{\delta}{2}(1-i)j(x). \qquad (4)$$

Thus $j(x)$ is expressed by the magnetic field on the conductor surface $H_z(0)$:

$$j(x) = \frac{1+i}{\delta} H_z(0)e^{-(1+i)x/\delta}. \qquad (5)$$

A typical value of δ in copper at the frequency of 3GHz is 1μm. Figure 2 shows the current j as a function of x/δ. By integrating j, we obtain total current in the conductor:

$$J = \int_0^\infty j\, dx = H_z(0). \qquad (6)$$

The power loss in the conductor can be calculated as

$$P_{bulk} = \int_0^\infty |j|^2 / \sigma\, dx = \frac{H_z(0)^2}{\sigma\delta} = \sqrt{\frac{\omega\mu}{2\sigma}} H_z(0)^2, \qquad (7)$$

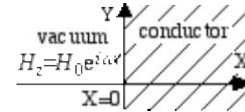where $1/\sigma\delta$ is often written as surface resistance Rs.

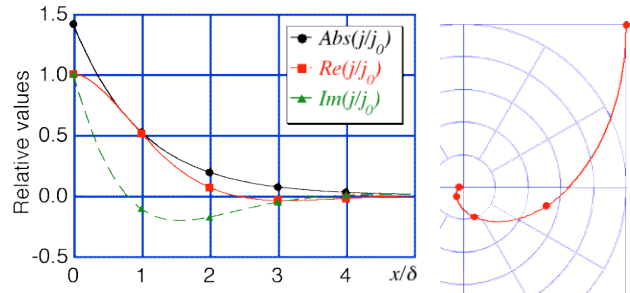Figure 1: Half of the space is filled by conductor.

Figure 2: Left: j as a function of x/δ. Right: polar plot. Markers are put in every unit.

Although it is necessary to increase the conductance for lower RF power loss, the higher conductance leads to higher current density and thus the thinner skin depth reduces the improvement by the good conductance. It may be possible to cure this situation if we can control the current density independent of the conductance.

Typical value of σ is 5.8×10^7 [S] for copper metal and that of $\omega\varepsilon$ is 1 in vacuum at 18 GHz. Reminding the Eq.1, if low loss dielectric material with relative permittivity of more than 10^7 at such a high frequency were available, the RF power loss could be determined by the dielectric loss other than the conductor loss. Composite material such as seen in multilayered capacitors may have such characteristics. This will be investigated in future.

SKIN EFFECT ON THIN FOIL

Let us consider a case that the thickness of the conductor is thinner than the skin depth δ (see Fig. 3) and the both sides have electromagnetic fields with different amplitudes. The solution of Eq.2 becomes:

$$j(x) = H_z(0)\left(j_f e^{-(1+i)x/\delta} + j_b e^{-(1+i)(\alpha\delta-x)/\delta}\right), \qquad (8)$$

$$j_f = \frac{(1+i)e^{(1+i)\alpha}\left(e^{(1+i)\alpha} - \xi\right)}{\delta\left(e^{2(1+i)\alpha} - 1\right)}, j_b = \frac{(1+i)e^{(1+i)\alpha}\left(\xi e^{(1+i)\alpha} - 1\right)}{\delta\left(e^{2(1+i)\alpha} - 1\right)}.$$

Figure 4 shows polar plots for $j(x)$ for the case when the magnetic fields of both sides have the same amplitude(ξ=1). They are symmetrical with respect to the origin because the currents from both sides cancel each other. The linear behaviour can be seen when the
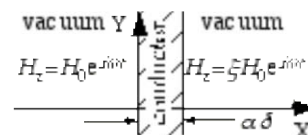

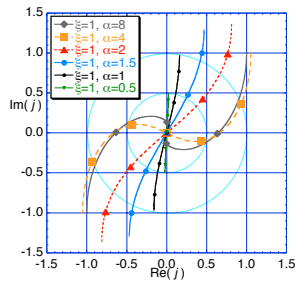Figure 3: The thickness is thinner than the skin depth.

Figure 4: Polar plots of complex current densities. The markers are at centers of every 1/5 layers in depth: $x=0.1\delta$, $0.3\delta, \ldots 0.9\delta$. (currents are normalized by H_0/δ).

thickness is less than the skin depth (markers are at the centers of even intervals).

Let us consider a simple example as shown in Fig. 5, where two layers of thin foils with thickness of just the skin depth δ are immersed in equally stepped RF fields ($\xi=0.5$). The current densities in the foils are shown in Fig. 6. The total current in each layer is just the magnetic field difference between the front and backsides: the currents of both the layers are the same. The total power loss is calculated by the similar way as Eq.7:

$$P_2 = \int_0^\delta \left(|j_1|^2 + |j_2|^2 \right) / \sigma \, dx , \qquad (9)$$

which is 70% of the bulk case. When we optimize the thickness of the first layer α and the current ratio ξ, the minimum power loss becomes 67.9% at $\alpha=0.826$ and $\xi=0.498$. Further, when the thickness of the last layer (second layer in this case) is thick enough compared to the skin depth ($\alpha=4$ in this case), the minimum power loss becomes 65.6% at $\alpha=0.785$ and $\xi=0.534$.

When n-layers of equal thickness are immersed in n-equally-stepped RF magnetic fields, the power loss can be given by:

$$P_n = \sum_{i=1}^n \int_0^d |j_i|^2 / \sigma \, dx . \qquad (10)$$

Such a configuration should be more practical than a configuration with non-uniform thicknesses. Figure 7 shows the relative power losses as functions of the normalized thickness $\alpha=d/\delta$. The thicknesses that show minimum power loss decrease as the numbers of layers increase. The minimum power loss is shown in Fig. 8 as a function of the number of layers. It shows $n^{-0.5}$ dependence, which can be also derived from an expansion of the power loss formula. Therefore, the RF power loss may be reduced by this geometry, if the current on each layer is well controlled. The improvement, however, is limited by the absolute thickness of each layer: it should be enough thicker than the inter-atomic distance. On the other hand, the power loss can be more than the bulk case, which may be useful for absorber applications such as EMI shields.

Suppose a plane wave comes from the left in Fig. 3. The transmission T through the foil is shown in Fig. 9 as a function of α., where $\alpha\delta$ is the thickness of the copper
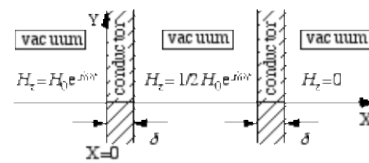

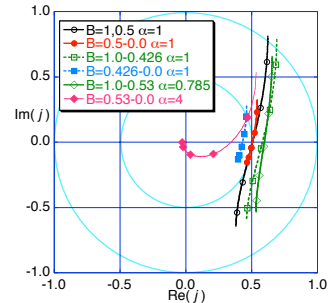Figure 5: Two conductor layers are immersed in stepped RF fields: 100%, 50%, and 0%.

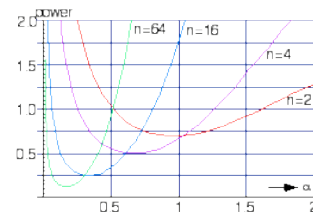Figure 6: Polar plots of pairs of complex currents in Fig. 5.

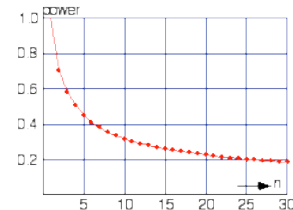Figure 7: Relative power losses as functions of thickness, (normalized by the bulk case)

Figure 8: Minimum power loss as a function of the number of layers.
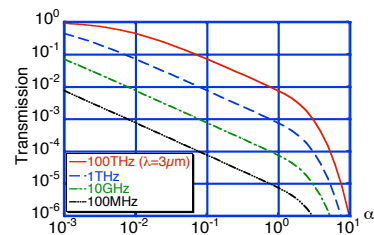

Figure 9: Transmission through a thin conductor foil with thickness of $\alpha\delta$.

foil. Because the transmission is less than 1% at 10GHz even if the thickness is one-hundredth of the skin depth, the two regions, the front side and the backside, are isolated by the foil: the left hand side field can hardly penetrate the foil and no power can be delivered to the other side. A plane wave with an angle other than normal makes a different situation from this and a solution to distribute the current to each layer is reported

in Ref[1]. This method, however, requires narrow choice in the material inserted between the conductor layers.

DIELECTRIC CAVITY WITH THIN FOIL

Let us consider a flat dielectric cylindrical cavity with TM_{010} mode as shown in Figure 10. Suppose a high dielectric constant ε_r, the boundary condition at the perimeter ($r=R$) is almost magnetic, where the magnetic field is normal to or zero on the boundary. The electric field E_z can be described by the Bessel function as:

$$E_z(r) = E_z(0) J_0(kr), \quad k = x_1'/R, \tag{11}$$

where x_1' is the first root of $J_0'(x)=0$. The displacement current in the dielectric material are collected on the conductor surface around the center and leaves from the conductor around the perimeter, so that the surface current density has a peak at the radius r_p (the first root of $J_0(kr_p)=0$). When we put extra thin washer foils on the peak as shown in Fig. 11, part of the displace currents go into the extra thin foils and flow on them, which redistributes the current flows. The power reduction can happen when the thickness of the conductor is not much more than the skin depth: the currents on both sides of the extra electrode surface flow in the opposite directions without the cancellation and the total net current on the surface is zero and the power loss just increases, otherwise. In order to isolate the layers, the in-and-out current on the extra electrode should balance:

$$\int_{r_1}^{r_2} \dot{D}\, 2\pi r dr = 0. \tag{12}$$

Another constraint minimizing the power loss P_1 will give the radii r_1 and r_2 together with eq.15:

$$P_1 = \int_0^R \int_0^\infty \frac{|j_1|^2}{\sigma}\, dz\, 2\pi r dr + \int_{r_1}^{r_2} \int_0^d \frac{|j_2|^2}{\sigma}\, dz\, 2\pi r dr. \tag{13}$$

The minimum P_1 is 70.5% of the bulk case at $r_1 =0.261R$ and $r_2=0.921R$. In order to simplify the constraint, we may use the following instead:

$$\int_0^{r_p} \dot{D}\, 2\pi r dr = 2\int_0^{r_1} \dot{D}\, 2\pi r dr \quad \left(= 2\int_{r_1}^{r_p} \dot{D}\, 2\pi r dr\right), \tag{14}$$

because the power loss density is proportional to the square of the current density and thus the peak value of the current is dominant in the power loss. The left side term is the total current at the peak (node) and is equally shared by the two conductors. The P_1 given by eq.17 is 75.4% of bulk case at $r_1=0.322R$ and $r_2=0.884R$. Although this simplified constraint does not give optimum condition for minimum power loss, it is useful for multi-layered application.
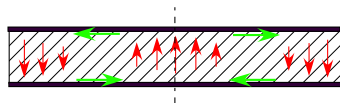
Figure 10: Flat dielectric cylindrical cavity where the dielectric constant ε_r is much larger than unity and the top and the bottom surfaces are covered by conductors.
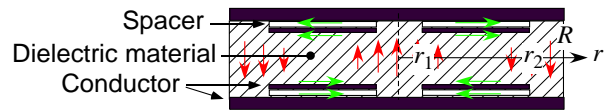
Figure 11: Dielectric cylindrical cavity with extra electrodes that have washer shape. The thicknesses are exaggerated: inside of the each conventional disc electrode, one smaller washer shape electrode is located with a small distance from the disc.

Figure 12 shows an axisymmetric simulation code result of the Q enhancement as a function of r_1/R. Although the used parameters that reduce the computing time may be unrealistic, the effectiveness can be seen from this simulation result. The used parameters are listed in the caption.

It should be noted that the space between the disc and the extra electrodes are essential; it should have lower dielectric constant than the body material. As can be seen, the width of the extra electrode is comparable to the half wavelength in the main body. Therefore the narrow space between two electrodes (not the wider one that forms the cavity itself) becomes another resonator to suck the power unless the space has enough lower dielectric constant than that of main body.

Although this example cannot be directly applicable to vacuum accelerating tubes, similar technique should be applicable to many cases. Needless to say, the number of layers can be increase to enhance the effect further.
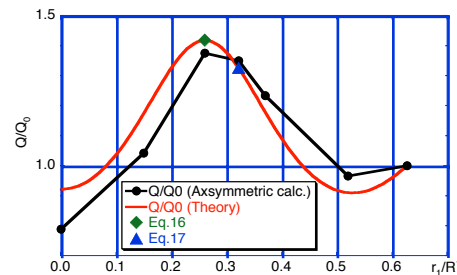
Figure 12: Q enhancement as a function of r_1/R.

Dielectric constant of the body material:	10000
Tan δ of the dielectric material:	0
Thickness of the dielectric body:	400(200)μm
(Symmetry is used to reduce the CPU time)	
Radius of the dielectric body:	600μm
Conductance:	58MS/m
Base conductor thickness:	8μm
Thinner conductor thickness:	1μm
Space between the conductors:	1μm
Eigenfrequency:	~3HGz

REFERENCES

[1] J. Hattori, et al.: Low Profile Dielectric Band Elimination Filter using Thin Film Layered Electrode for 2GHz Band Cellular Base Station, IEEE MTT-S Digest 1999

[2] Japanese Patent Laid -Open No.2004-120516A