F. Mills (MURA) Computed drift tube shapes

Present interest at MURA is directed toward a 10 Bev, high current, particle accelerator, of the FFAG type. In connection with this the problem of optimum injection energy has been considered regarding overall cost of the accelerator. The results suggest that with injection energies lower than 200 Mev costs rise rapidly, above 200 Mev these figures do not change as fast. Therefore a design goal has been set for a 200 Mev linear accelerator and at present a drift tube linac is being considered. In addition to high current output, good beam quality is desired ($\frac{\Delta E}{E}$ = 0.1 °/o or better), especially in connection with beam stacking in the FFAG machine. The linac emittance needs to be no better than presently achieved at BNL and CERN in order to facilitate multiple turn injection. At present one is not primarily concerned with gaining the utmost in the R_{sh} value for the accelerating structure, because it is expected that suitable rf power sources will be available; however R_{sh} is certainly being considered in connection with ease of construction and cost.

Because of the availability of rf power sources for low energy linacs some latitude in drift tube shapes is available. On the other hand the energy region of 100 Mev and over has not been explored to any extent yet and rf power may be a problem. Therefore a program was set up to study drift tube shapes for higher particle energies. Drawing on the computational experience at MURA with mesh techniques this type of attack was employed.

An outline of the mesh technique follows.

Maxwell's equations for ϕ independent TM modes yield:

$$F_{rr} - \frac{F_r}{r} + F_{zz} + k^2 F = 0$$

or

$$O\,F + k^2 F = 0$$

where $F = r H \phi$

and $k^2 = \frac{\omega^2}{c^2}$

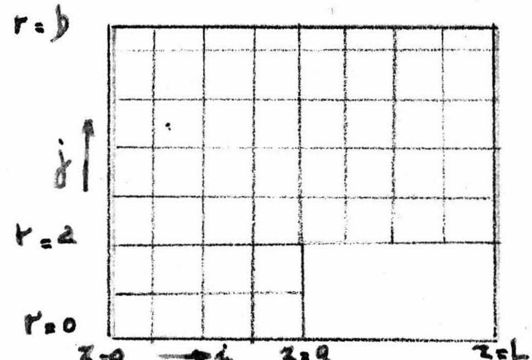
with the boundary condition $\left(\frac{\partial F}{\partial n}\right)_s = 0$

For the lowest mode

$$k_o^2 \le \frac{\langle FOF \rangle}{\langle FF \rangle} = k^2 \qquad \text{if } O \text{ is Hermitian.}$$

The next step is to assume reasonable trial functions for F and try to satisfy the above equations. Then calculate λ, resolve for a new F and recalculate λ. This iteration process is convergent; in fact the improvement in λ is better than the improvement in F values at each succeeding stage. (errors in eigenvalues due to field errors are of the order of $(\delta F)^2$).

To facilitate calculations the continuous problem is replaced with a mesh problem



The original differential equation yields now the difference relationship.

$$F_{i,j+1}\left(1-\frac{h}{2r_j}\right) + F_{i,j-1}\left(1+\frac{h}{2r_j}\right) + F_{i+1,j} + F_{i-1,j} + (k^2h^2-4)F_{i,j} + O(h^4) = 0$$

From the above formula one can derive relationships for successive improvement of mesh values. Inherent in each relationship is a certain convergence property. Therefore the following relationship was used.

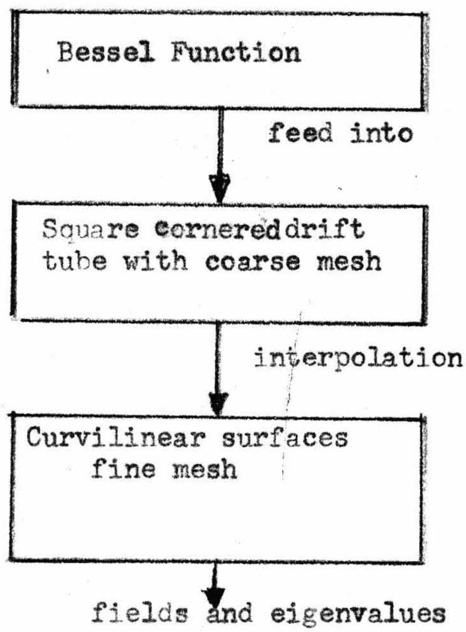
$$F_{i,j}^{n+1} - F_{i,j}^{n} = \alpha\left\{F_{ij}^{n}\left(\frac{h^2k^2}{4}-1\right) + \frac{1}{4}\left[F_{i,j+1}^{n}\left(1-\frac{h}{2r}\right) + F_{i,j-1}^{n+1}\left(1+\frac{h}{2r}\right) + F_{i+1,j}^{n} + F_{i-1,j}^{n+1}\right]\right.$$

where α is a parameter ≥ 1 which can be chosen to optimize the convergence rate.

At the drift tube surface the boundary condition is applied by using a suitable form of the iteration equation.

Similar iteration relations are used in the toroidal and ellipsoidal coordinate systems. In a problem where drift tube surfaces are toroidal or ellipsoidal a separate curvilinear mesh is used interlockingly with the original polar mesh.

The mode of operation for machine calculation used up to the present is illustrated below.



Remarks

15 x 18 mesh; 65 iterations
time $\cong$ 5 minutes

30 x 36 mesh; 14 iterations
time $\cong$ 5 minutes

From these data it is a simple matter with the present program to obtain losses, stored energy, $E_{r = o}$, E_{max} on walls. All other interesting quantities are computed from the ones mentioned. Field plots can be obtained from the mesh values with an x-y plotter. A typical field plot is shown on the next page.

At present some thoughts are being given to self adaptive computational programs to optimise accelerating structure designs.

This work was done by J. van Bladel, R. Christian, T. Edwards, F. Mills and D. Young.

Field Plot For a Drift Tube in The Region of 100 Mev