PHASE OSCILLATION AND TRANSVERSE FOCUSING*

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As part of the Yale Design Study of Linear Accelerators, a preliminary study has been made of phase oscillation and transverse motion of protons for the proton linear accelerator, as described by G. W. Wheeler in the preceding report. The coupling effects of longitudinal and radial motions are partially included. Relevant parameters used are those given in the previous report by G. W. Wheeler. The "effective" field gradient $\mathcal{E}_o$, (energy gain per one gap)/eLcos φ, where φ is the phase of each particle and L is equal to $\beta_s\lambda$, is given in Table 1.

Table 1

Tank No.	$\mathcal{E}_o$ (Mv/meter)
1	1.546 - 2.454
2	1.381 - 1.682
3	1.756 - 1.680
4	1.706 - 1.617
5	1.633 - 1.565
6	1.458 - 1.403
7	1.165 - 1.127

So far, numerical calculations have been done only for Tank No. 1 (751 kev - 31.22 Mev) and Tank No. 2 (31.22 Mev - 66.77 Mev), but the program used up until now applies equally well to the other tanks, up to about 200 Mev, where the iris-loaded sections begin. The I.B.M. 709 computer has been used for most of the numerical calculations.

*Report of work done by S. Ohnuma and J. N. Vitale

Phase Oscillation

Fourteen representative points (including the synchronous point) of the "fish" diagram as shown in Fig. 1 have been chosen as the starting values of phase and energy. Maximum phase spread is about $3|\varphi_s| = 1.353$ and the maximum energy deviation from the synchronous energy is $|\gamma-\gamma_s| = .554\text{x}10^{-4}$, or 52 kev. According to the linear (small amplitude) approximation, the phase spread and the energy deviation at the end of the first tank will be

$$1.353 \times \left[(\gamma_s^3\beta_s^3\mathcal{E}_o)i/(\gamma_s^3\beta_s^3\mathcal{E}_o)f\right]^{1/4} \simeq .237$$

and

$$.554 \times \left[(\gamma_s^3\beta_s^3\mathcal{E}_o)i/(\gamma_s^3\beta_s^3\mathcal{E}_o)f\right]^{-1/4} \simeq 3.17\text{x}10^{-4}$$

respectively, Numerical results for these quantities are, phase spread = .235, energy spread = $2.46\text{x}10^{-4}$. Applying the linear approximation up to 200 Mev, the maximum phase spread will be about .114. Changing the frequency from 200 Mc/s to 1,200 Mc/s will increase this phase spread to .114x6 = .684, well within the phase stability region. Phase oscillations vs. drift tube number, in the first tank, are shown in Fig. 2.

Several approximations have been made in this calculation. These are enumerated below:

1. For tank No. 1, the field is assumed to be constant at r=a (bore radius), $E_z(r=a)$=const. This gives the longitudinal transit time factor $\sin(\pi g/L)/(\pi g/L)$ where g is the gap distance and $L=\beta_s\lambda$. For tank No. 2, the transit time factor is calculated numerically from the field distribution of shaped drift tubes. Taking the field of the somewhat more realistic shape, $E_z(r=a,z) \propto (1-z^2/g^2)^{-1/2}$, the transit time factor is given by $J_o(\pi g/L)$. For a small value of g/L,

$$\sin(\pi g/L)/(\pi g/L) = 1 - \frac{\pi^2 g^2}{6L^2}, \quad J_o\left(\frac{\pi g}{L}\right) = 1 - \frac{\pi^2 g^2}{4L^2} \quad .$$

2. The effect of the bore on the transit time factor is assumed to be T (with bore) = T (without bore) x $\left[I_o(\frac{2\pi a}{L})\right]^{-1}$ for both tanks.

3. Energy gain is calculated only for particles on the axis. That is, the effect of transverse position r on the longitudinal transit time factor is neglected, i.e., $I_o(\frac{2\pi r}{L}) \simeq 1$.

4. To find the energy gain at the Nth gap from $\Delta W_N = \mathcal{E}_o \cos\varphi_N$, the phase φ_N of each particle is computed at $z_N = \sum_{n=o}^{N} \beta_s^n \lambda$, ; the synchronous velocity changes stepwise at this point.

These approximations are believed to be adequate at this stage of the design study. However, it is intended to include some of the neglected effects in the future analysis, particularly for tank No. 1 where these are most serious.

Transverse Motion

Transverse motion and its focusing by quadrupole magnets have been calculated for the above mentioned representative points. Since particles coming from the buncher are generally concentrated along an S-shaped region of the "fish" diagram instead of spreading uniformly, focusing requirements for some of these representative points would be more stringent than necessary in practice. However, it was decided to be rather conservative, at least for the preliminary design study, in view of several approximations. The method used for estimating the quadrupole strength giving transverse focusing is essentially that of L. Smith and R.L. Gluckstern.* Approximations made here are:

1. The radial transit time factor is taken to be the same as the longitudinal transit time factor. The r dependence of the transit time

*L. Smith and R.L. Gluckstern, Rev. Sci. Instr., 26, 220, (1955).

factor, which is different for this case, is again neglected, $I_1(\frac{2\pi r}{L})/(\frac{\pi r}{L}) \simeq 1$.

2. The defocusing (or focusing) action of the rf field is approximated by an impulse which changes (dr/dt) but not the transverse distance r itself. The largest change of r through the gaps is of the order of .1 cm. Therefore, this approximation is not too serious.

3. In all matrices representing the effects of the quadrupole magnets, drift spaces, and rf gaps, the synchronous velocity is used throughout, instead of different velocities for each particle. The correction is again largest at the beginning of the first tank where it is about 3.5%.

The transverse stability regions for +-+-, ++-- and +0-0 (magnets in every other drift tube) groupings of quadrupole magnets have been calculated to determine the suitable strength of magnets. These are shown in Fig. 3 and Fig. 4; θ^2 and $\Delta\theta$ are given by:

$$\frac{dH}{dx} = \frac{m\gamma}{e\beta}\frac{\nu^2}{\mathcal{E}^2}\theta^2 \quad \text{when the magnet length is } \mathcal{E}\beta\lambda.$$

For $\mathcal{E}=.5$, $\frac{dH}{dx}$ = 13.9 (kG/cm) for 750 kev (tank No. 1)

= 2.27 (kG/cm) for 31.22 Mev (tank No. 2).

$$\Delta\theta = \frac{\pi e \mathcal{E}_o \lambda}{2mc^2}\frac{1}{\gamma^3\beta}(-\sin\varphi).$$

The maximum excursion in the initially focusing (x) plane is given by $\left[x_o^2 + \hat{\gamma}^2(\dot{x}/\nu)^2\right]^{1/2}$ and in the initially defocusing (y) plane by $\left[\Psi^2 y_o^2 + (\hat{\gamma}/\Psi)^2(\dot{y}/\nu)^2\right]^{1/2}$. For constant θ, the operating point will oscillate between $A(\varphi=2\varphi_s)$ and $B(\varphi= -\varphi_s)$, gradually converging to near the point $C(\varphi = \varphi_s)$.

For tank No. 1, both +-+- (N=1) and ++-- (N=2) polarity groupings are applicable. For N=1 with θ^2=.62, the acceptance for the synchronous particle is 25π cm-mrad. If the beam is focused within 18.5π cm-mrad at the entrance of the first tank, all particles are accepted (regardless of their initial energy or phase provided they are inside the "fish" diagram) and the corresponding emittance is about 3.6π cm-mrad. It is not yet sure if one can install the very strong quadrupole magnets required for this grouping (8.6 kG/cm for $\mathcal{E}$=.5, 7.2 kG/cm for $\mathcal{E}$=.65 both at the beginning) inside the thin drift tubes. On the other hand, the beam focusing between ion source and the first tank will be much easier (because of the larger acceptance) than for the ++-- grouping. The values of θ^2 for the N=2 grouping which give a stable radial motion are between .30 and .35, a rather narrow range. With θ^2=.335 (4.7 kG/cm for $\mathcal{E}$=.5) the acceptance of the first tank is 8.6 π cm-mrad. This is a small but still acceptable figure. (See the preceding report "The New Bevatron Injector" by L. Smith.) The value of $\hat{\gamma}$ between point A and point B is 6 - 12 compared to 3 - 5 for the N=1 grouping, making the tolerance requirements for magnet misalignments somewhat severer. Also, the resonance effect due to the coupling between longitudinal and transverse motion, which is serious when the phase oscillation frequency is twice the radial motion frequency, has to be studied more carefully since point A is quite close to the $\cos\mu$ = + 1 boundary.

For tank No. 2, it is possible to use +(no magnet)-(no magnet) system, thereby reducing the number of magnets by a factor of two. With θ^2=.36 (800 G/cm, $\mathcal{E}$=.5), this gives an acceptance of 5.9π cm-mrad., about 50% larger than the emittance of the first tank. As one goes to the successive tanks, it would be possible to reduce the number of magnets further. This has not yet been studied quantitatively.

If the N = 1 grouping is used, the tolerance requirements for tank No. 1 are as follows (amplitude = .6 cm, change of amplitude = .3 cm):

1) Misalignment of the zero field line of magnets.

$$(\Delta x)_{max} = 0.006 - 0.009 \text{ inches.}$$

2) Misalignment of the transverse magnetic axes.

$$(\Delta\varphi)_{max} = .5^{\circ} - .7^{\circ}.$$

3) Misalignment of the magnetic field magnitudes.

$$(\Delta H/H) = 3.3 - 5.1\%.$$

Between the first and the second tank, there will be a connecting drift space of about one meter or so with a beam measuring device. It is then necessary to find a suitable set of quadrupole magnets (a triplet combination, for example) to be placed along this connecting pipe such that the shape of the beam in the transverse phase space at the entrance of the second tank would be similar to the acceptance shape of that tank. Otherwise, there will be a transverse mismatching which causes a loss of the beam quality even when the emittance of the first tank is smaller than the second tank acceptance. We have been able to find such a set which is not contradictory to engineering requirements.

Discussion

K. Johnsen (CERN): The linac at CERN is operated without any trouble beyond this so-called transverse stability boundary. The amplitude build-up you have observed might be due to the longitudinal-transverse coupling effect which could be important even when the operating point is not too close to $\cos\mu=1$ boundary.

S. Ohnuma (Yale): According to the numerical results, the amplitude build-up occurs when the operating point moves from right to left in the stability region, that is, when $\hat{\gamma}$ increases. This is in agreement with the theoretical prediction, that is, the amplitude build-up is proportional to $(\hat{\gamma})^{1/2}$. I do not think the coupling is the major cause in this case.

R. L. Gluckstern (Yale): If the transverse motion could be regarded as adiabatic, the stability region should indicate the characteristics of the motion. Also, if ++ - - is used for the first tank, there will be less mismatching when one moves to the +0-0 systems because the magnet repeat length is the same, whereas the repeat length for + - + - is half of that.

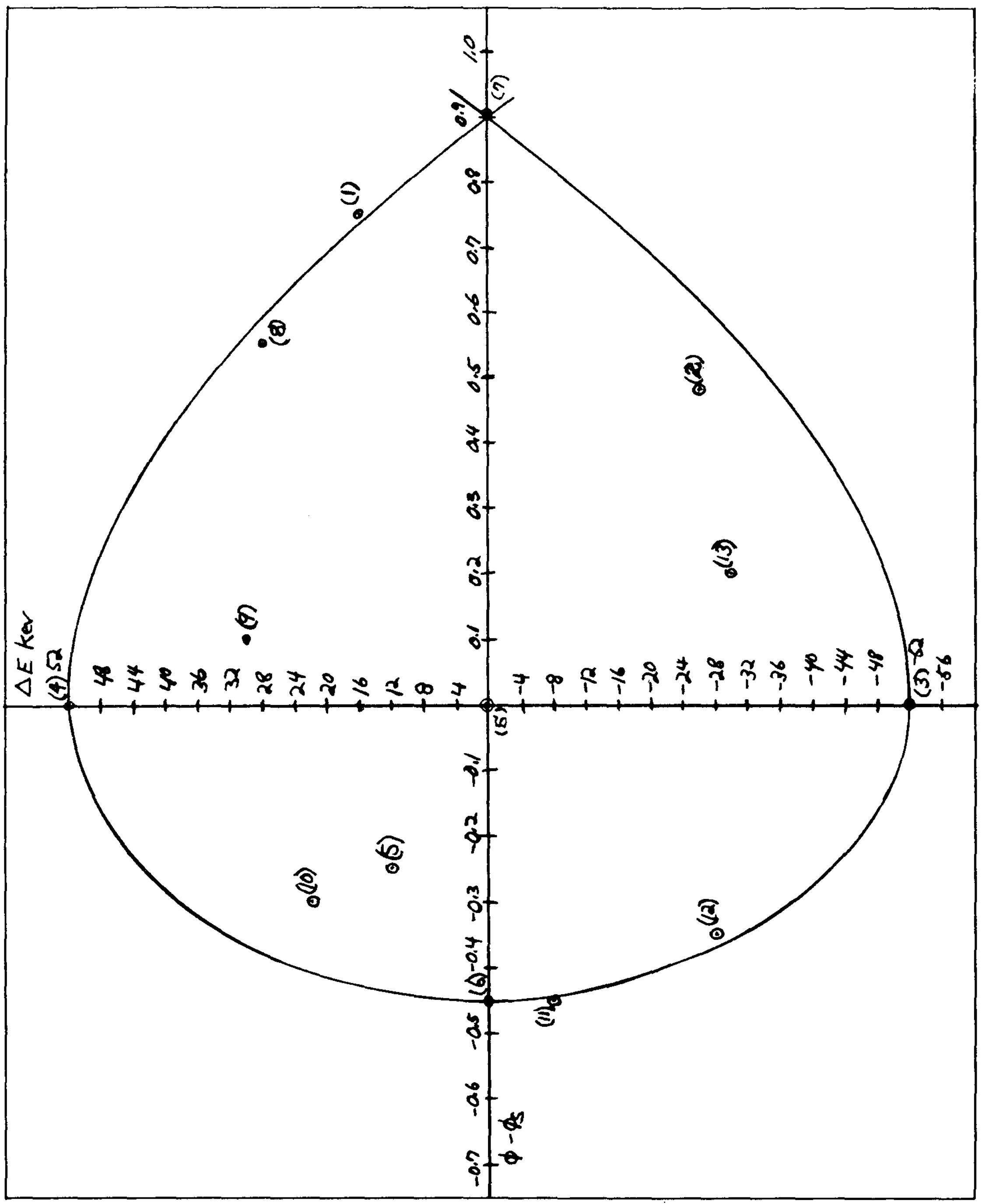

Fig. 1

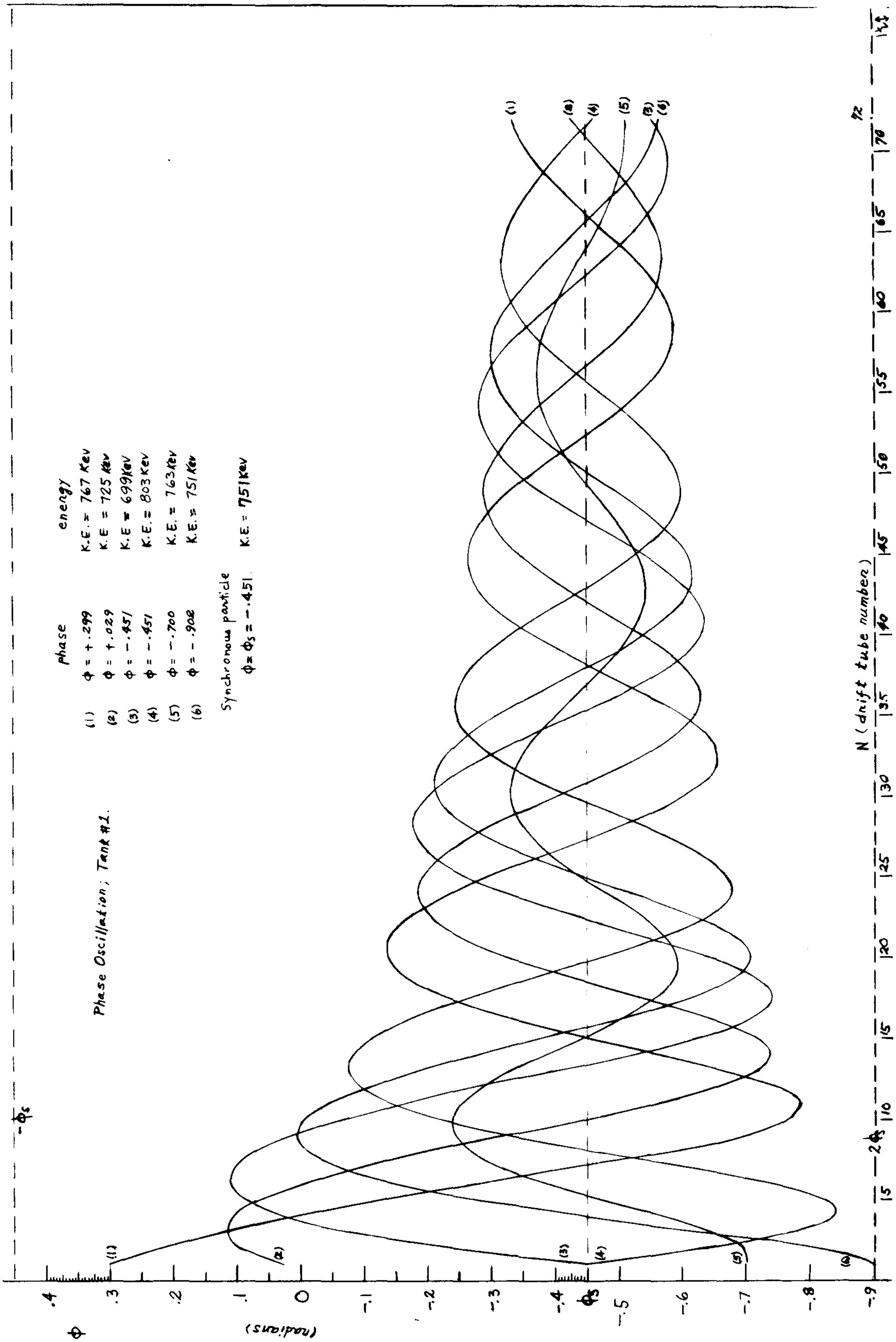
Phase Oscillation; Tank #1.
phase
energy
(1) ϕ = +.299 K.E. = 767 Kev
(2) ϕ = +.029 K.E = 725 Kev
(3) ϕ = −.451 K.E = 699 Kev
(4) ϕ = −.451 K.E. = 803 Kev
(5) ϕ = −.700 K.E. = 763 Kev
(6) ϕ = −.902 K.E. = 751 Kev
Synchronous particle
ϕ = ϕs = −.451 K.E. = 751 Kev
N (drift tube number)
ϕ
(radians)

Fig. 2

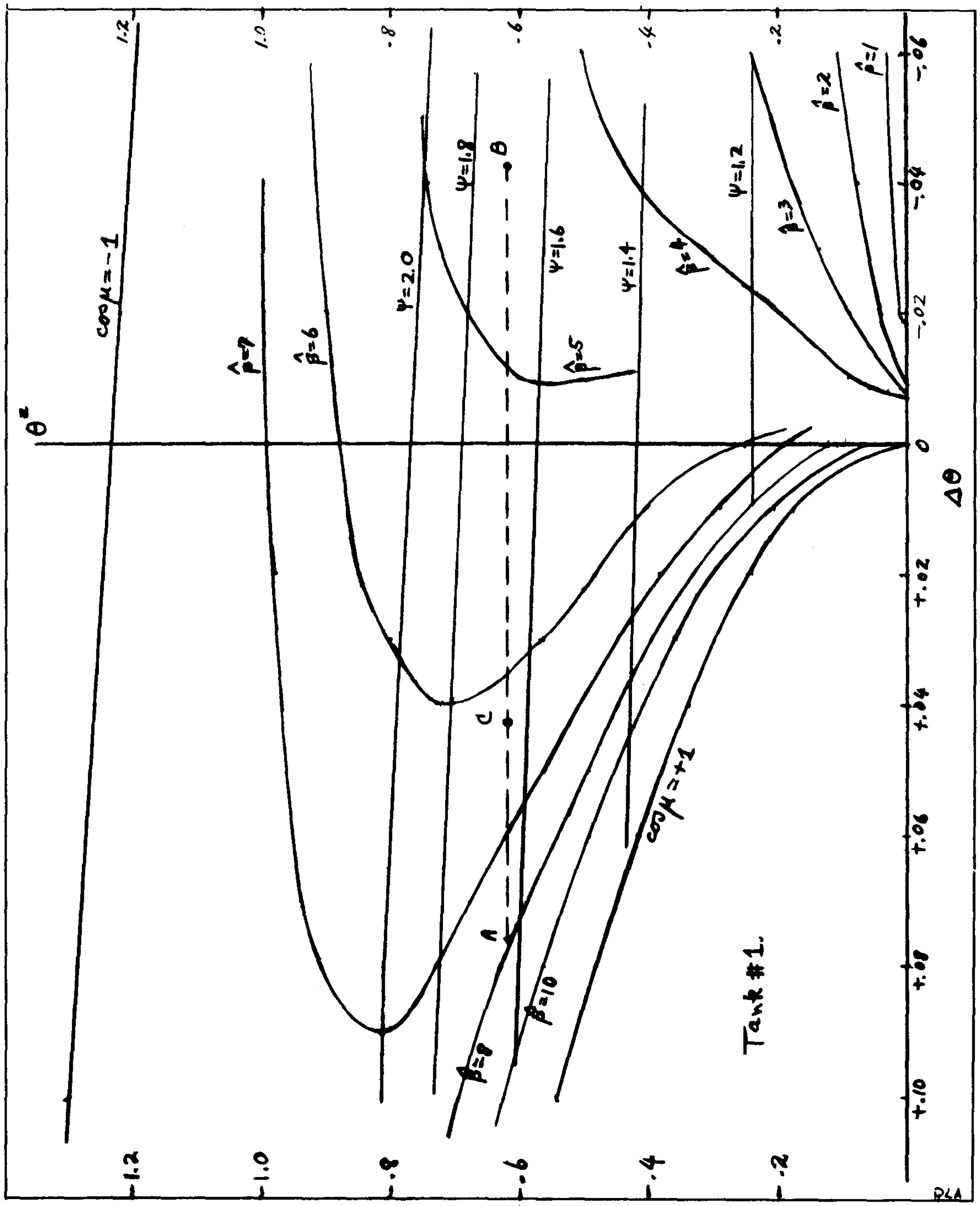

Fig. 3

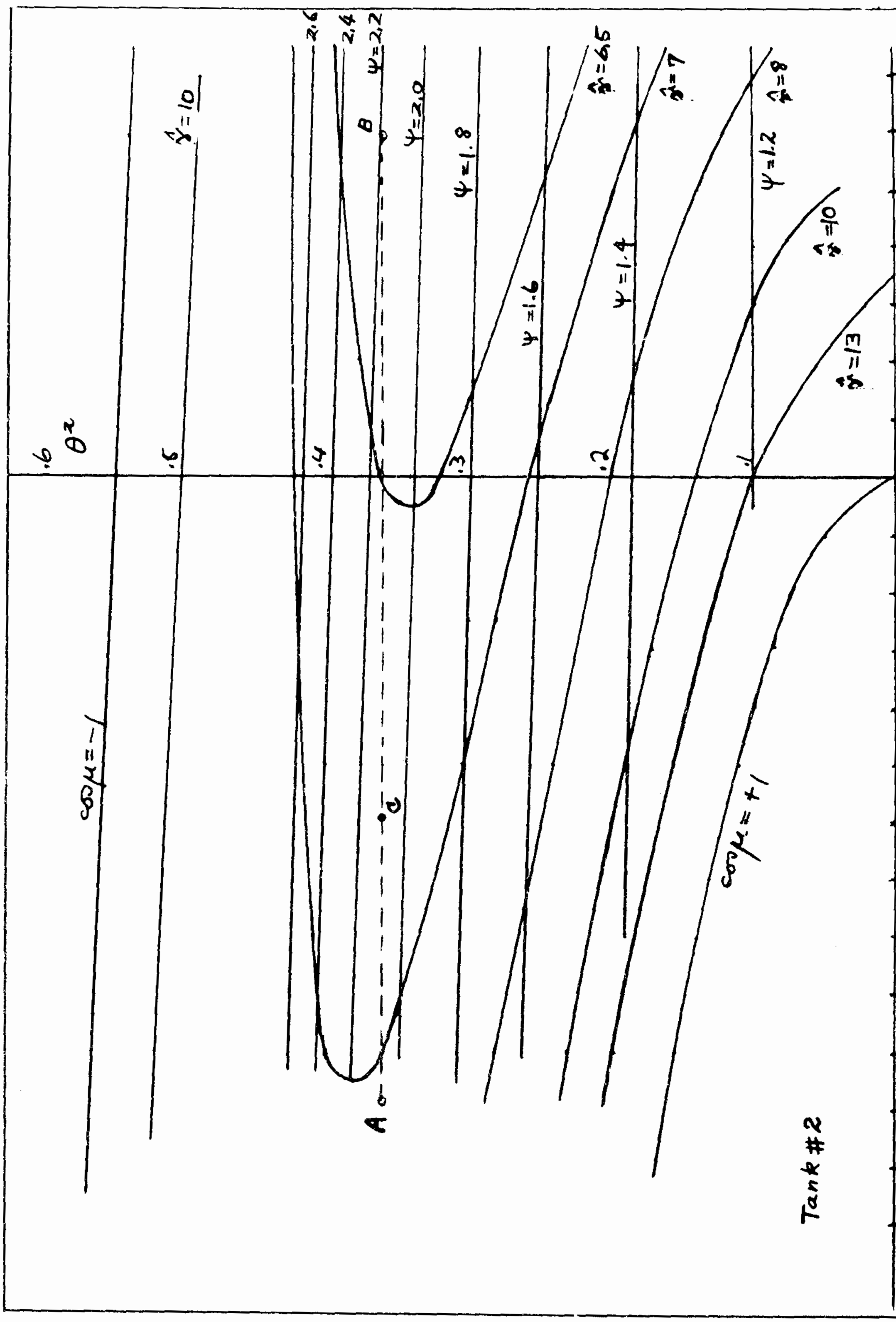
θ^2
$\Delta\theta$
$\cos\mu = -1$
$\cos\mu = +1$
Tank #2
.005
-.005
-.010
.6
.5
.4
.3
.2
.1
0
A
B
C

Fig. 4