PHASE AND RADIAL MOTION IN LINACS*

R. B. R-Shersby-Harvie
CERN

Orbit calculations for linear accelerators operating in the energy range from 200 Mev to 2 Gev have been carried out. The purpose of these calculations was to investigate the validity of the linear theory and to study the effects of errors in machine manufacture.

Two accelerators have been considered. In the first case a machine was studied quite similar to the design of L. Smith operating at 1000 Mc/s with a gradient of 12 Mev/meter. This gradient is undoubtedly higher than one would want in practice for economic reasons. In the second case a machine was considered operating at 400 Mc/s with a gradient of 6 Mev/meter. This gradient is again high for the frequency considered. In terms of the calculations, however, it is a much simpler case to treat. Both machines considered have 3 meter long, constant acceleration disk loaded tanks, with radial focusing provided by quadrupole doublets in 75 cm drift spaces separating the tanks. The synchronous phase angle is 30°.

Initially, the phase motion for the ideal case of no machine errors was calculated, as a check on the linear theory.

For the 1000 Mc/s machine, injection phase space ellipses with phase angle spreads of ±5°, ±10°, and ±20°, are considered. These correspond to energy spreads in the matched ellipses of 1.29 Mev, 2.58 Mev, and 5.12 Mev respectively.

*Report of work done by J. Gardner, Rutherford Laboratory.

Transforming through the linac it is found that for the $\pm 5^{\circ}$ and $\pm 10^{\circ}$ cases, the ellipses remain ellipses, just moving around in the ordinary fashion, indicating reasonable validity of the linear approximation. The resultant phase space areas, after 10, 20,40, and 60 tanks, for the $\pm 20^{\circ}$ case are shown in Fig. 1. Although the areas have become very distorted, clearly indicating a breakdown in the linear theory, they still remain bounded, and thus may still be usable. For the 400 Mc/s machine, the results are quite similar, with the linear approximation again breaking down at about $\pm 20^{\circ}$ phase spread. This is not surprising, since the synchronous phase angle is 30°.

The conclusions to be drawn here are that the linear theory forms quite a good guide, and that for the cases considered, a phase angle spread of about $\pm 20^{\circ}$ may be usable.

Radial motion was then considered for the perfect machine. For this calculation two focusing cases are considered. One considers a constant radial oscillation length per tank $\mu = \pi/2$, constant throughout the length of the machine up to 2 Gev. This gives considerable radial damping, at the expense of very high quadrupole strengths at the high energy end of the accelerator. The second case computed uses a μ value of $\pi/2$ up to 500 Mev, with constant strength quadrupoles thereafter. For this case, the radial damping is very small, but it is achieved with much lower quadrupole strengths.

The particles are assumed to fill that transverse phase space ellipse which is invariant to transformation through the first focusing period for the phase synchronous particle. For the non-synchronous particle, the same initial radial phase space ellipse was assumed.

Some results for the radial calculations are shown in Fig. 2. The envelope of the radial excursion is plotted versus machine length. The phase stable particle is shown by the solid line, the non-synchronous particle by the dotted line. Coupling of the longitudinal and radial motions causes the variation from smooth motion for the phase stable particle, as well as the motion of the non phase stable particle about the phase stable one. These calculations were done for the 1000 Mc/s machine. Similar work was also done for the 400 Mc/s machine, resulting in slightly smaller radial excursions. Little difference was noted for the weaker focusing case mentioned, so it was concluded that this simpler system would be acceptable.

Next, errors in the field amplitude, gaussian in form with half widths of 0.5, 1,2, and 5 percent were distributed randomly along the length of the machine. The results of some of these calculations are shown in Fig. 3. The ellipse shown is the phase space occupied by the beam in the perfect machine, which had an initial phase angle spread of $\pm 10^{\circ}$, shown after 60 tanks. The synchronous particle may be seen at the center. The other points shown are for phase synchronous particles taken through a machine with field errors. Several points are shown for the same percentage half-width error in the field. Each of these is for a different random distribution of field error along the tank. As can be seen, all of the 1% points are contained within the ellipse representing the ideal machine, while some of the 2% and 5% points lie outside this ellipse.

The results of some further calculations with field errors are shown in Fig. 4. This shows the maximum phase excursion as a function of machine length, for different random distributions. The amplitude builds up to a large value in the first one-third of the machine and thereafter seems to stay at values of the same order of magnitude. Evidently, the first one-third of the machine is the most critical. In conclusion, it seems that if one wants to keep the phase errors to within about $\pm 7^{\circ}$, the field errors have to be within about 2%.

A final point considered by J. Gardner was a method of setting up the tank fields to the requisite accuracy, without making the rather difficult 1% field measurements. According to K. Batchelor, it is possible to measure particle energies to 0.1% by a flight time technique. Thus the machine might be set up by measuring the particle energies at the exit of each tank, adjusting each tank until its theoretical energy is reached. Then, putting in a gaussian distribution of error on the energy measurements, one obtains a corresponding distribution of field error along the machine. At the low energy end, this is a rather small error, but at the high energies, this grows much larger than the 0.1% to which the energy is measured.

An example of the drift of phase angle of an initially synchronous particle as a function of machine length is shown in Fig. 5. This is for different distributions of measurement error, assuming the measurements to be made with a 0.05% standard deviation. Again, it is the first one-third of the machine that seems critical.

It might be possible to do better than this by adjusting the tanks in groups of quarter phase oscillation wavelengths, particularly in the later sections, where the phase oscillations are fairly slow.

To summarize, for the 1000 Mc/s machine, energy measurement of 0.1% does not contain the orbits to within a $\pm 10^{\circ}$ phase angle spread, while for the 400 Mc/s machine, measurement to 0.2% seems to contain most orbits to within $\pm 10^{\circ}$ phase angle spread. This makes a strong case for finding more precise methods of setting up the tank fields. The above-mentioned method calls for extreme accuracy; energy measurements to 0.1% are not quite good enough in the 1000 Mc/s case.

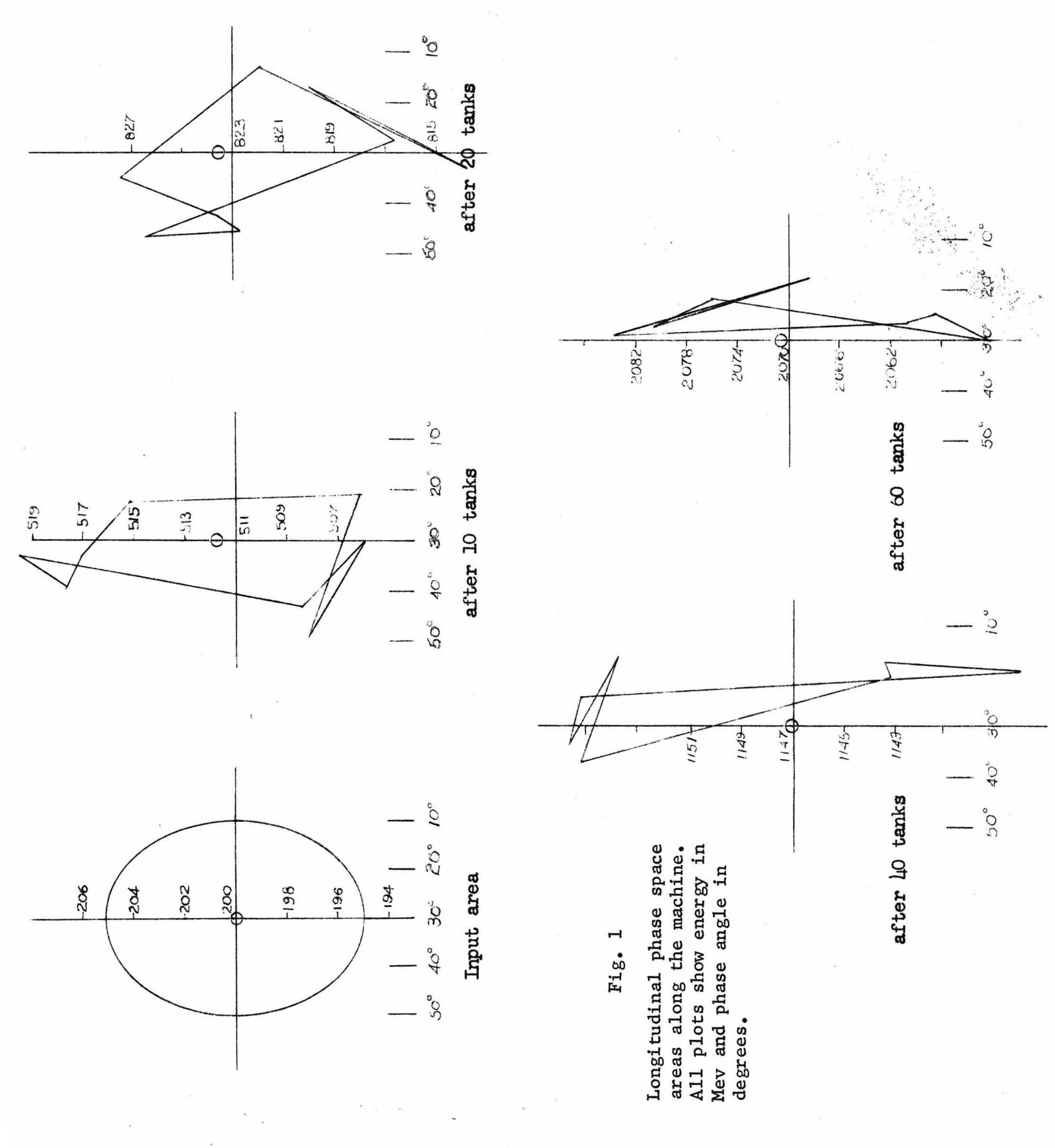

Fig. 1

Longitudinal phase space areas along the machine. All plots show energy in Mev and phase angle in degrees.

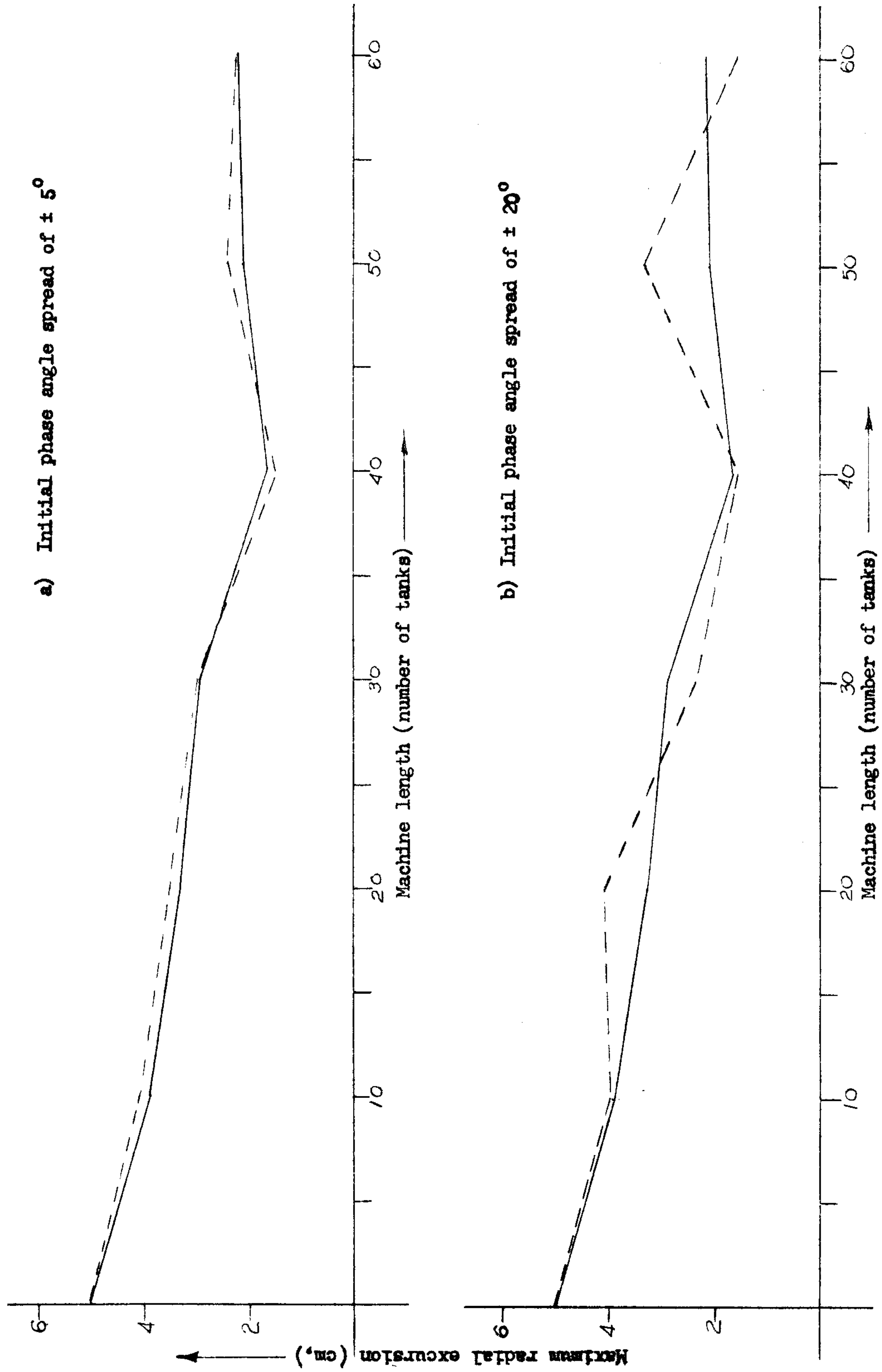

Fig. 2 Maximum radial excursion versus machine length, for the 1000 Mc/s machine

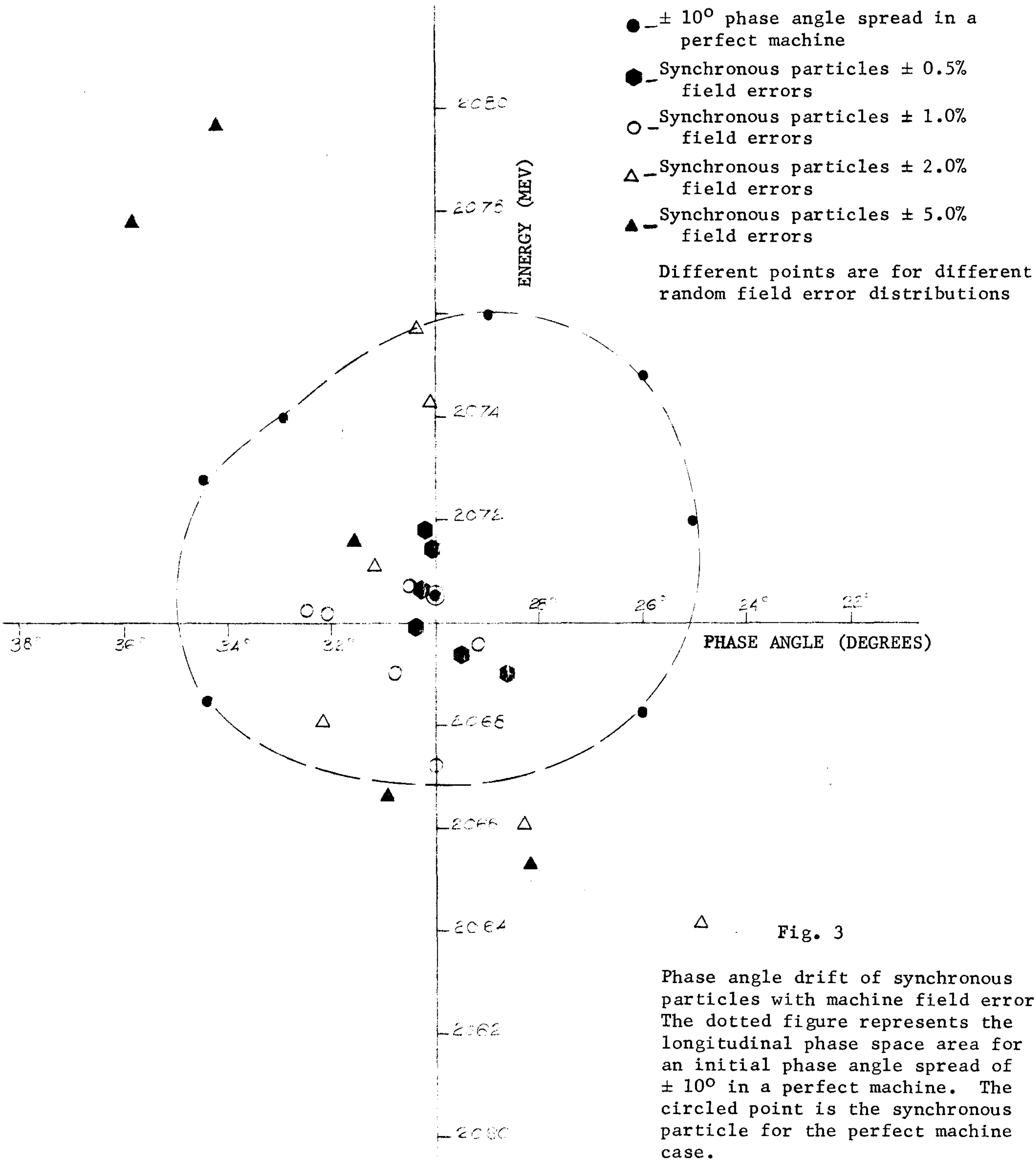

Fig. 3

Phase angle drift of synchronous particles with machine field errors. The dotted figure represents the longitudinal phase space area for an initial phase angle spread of ± 10° in a perfect machine. The circled point is the synchronous particle for the perfect machine case.

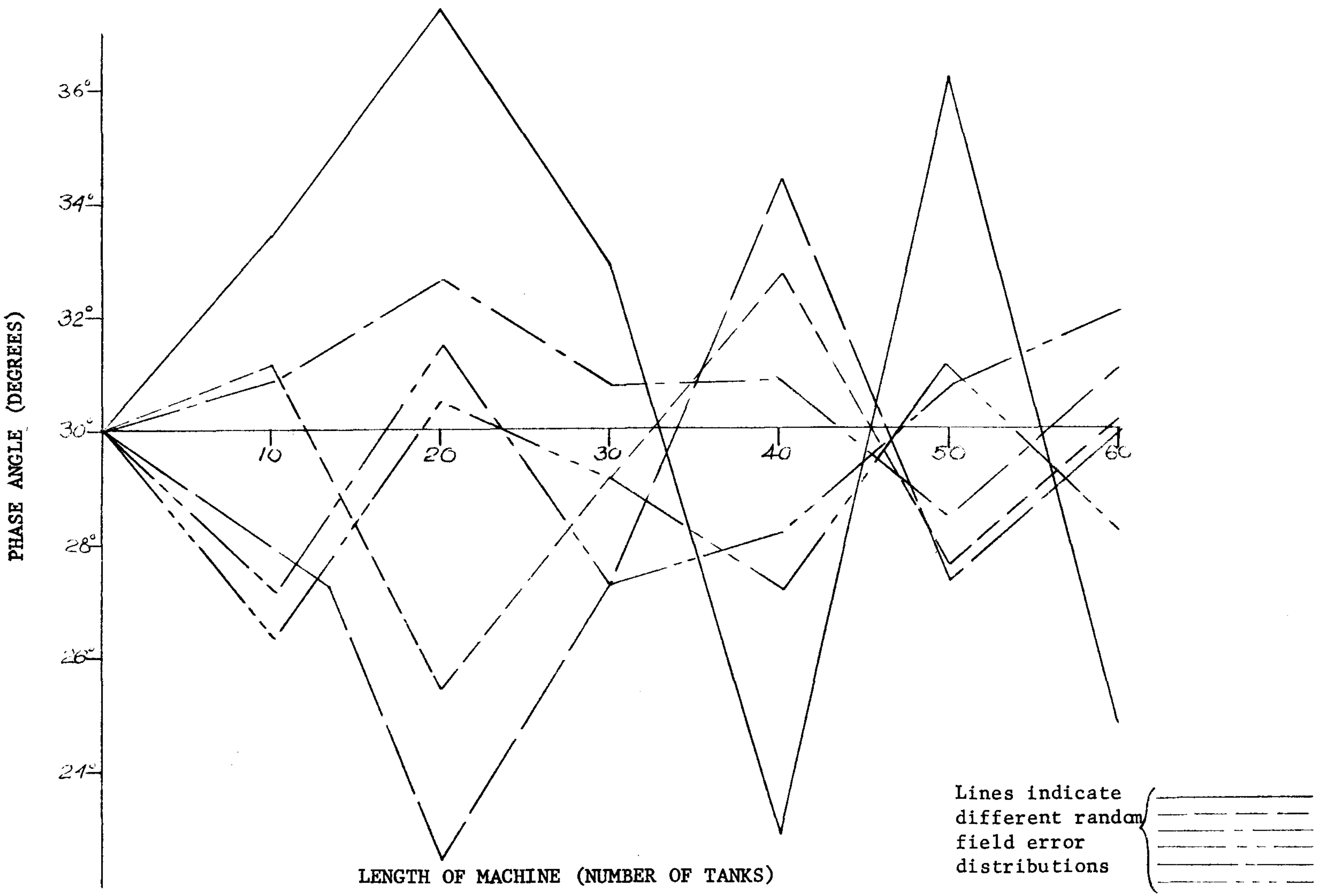

Fig. 4 Drift in phase of the synchronous particle for randomly distributed errors of half-width 2%

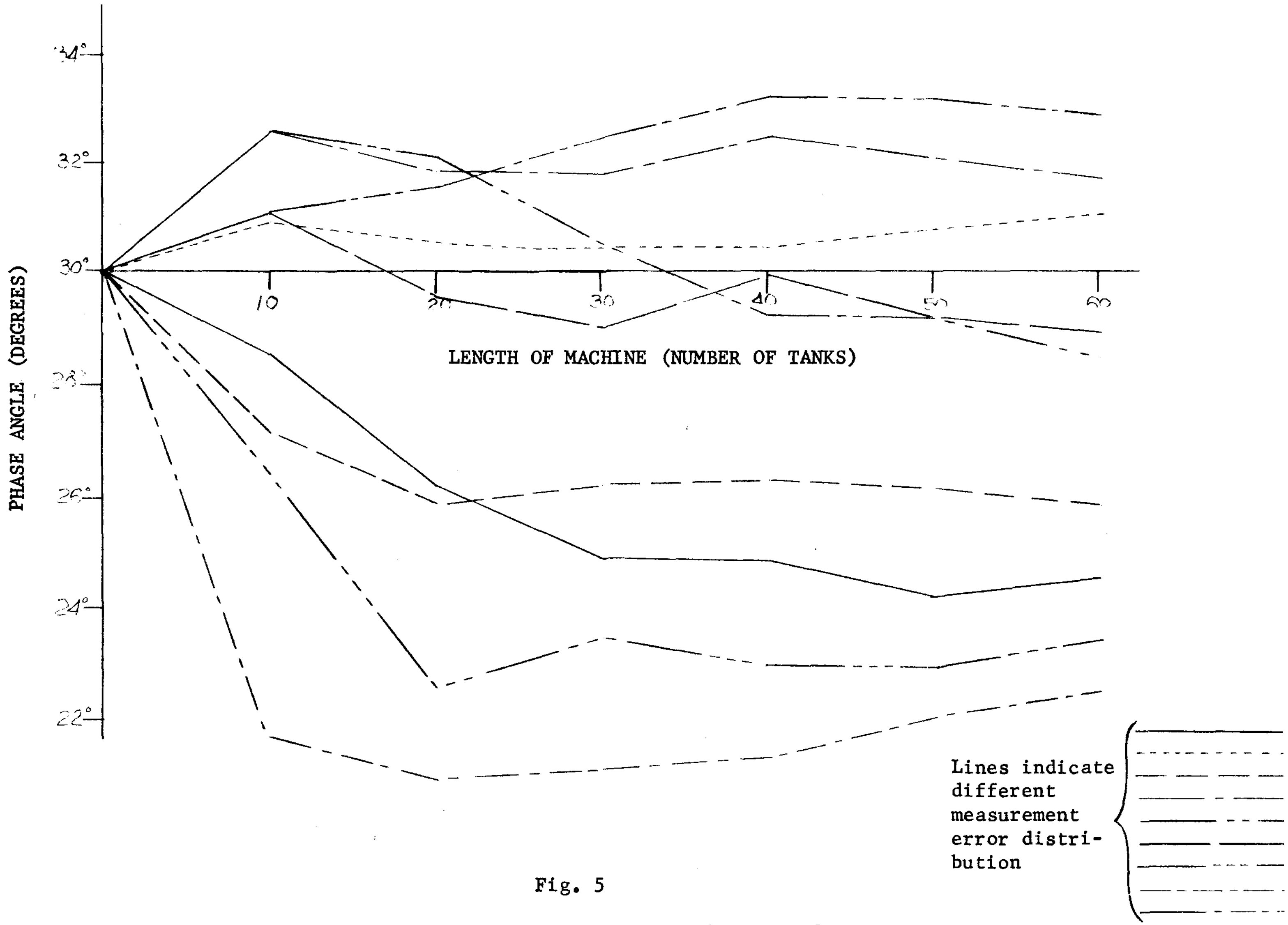

Fig. 5

Phase angle drift of the synchronous particle for different energy measurement error distribution. Standard deviation of energy measurement is σ = .05%.