LINAC FOCUSING

L. Smith
Lawrence Radiation Laboratory

Pursuing the investigations of transverse focusing for a stepped phase velocity linac the arrangement and parameters for focusing quadrupole magnets between linac sections were considered. This was done partly analytically and partly with a computer approach by H.S. Snyder. The stability diagrams given here are obtained from the computer results. These are similar to those used for a drift tube linac plotting horizontally a parameter which is a measure of rf defocusing and vertically a parameter which is a measure of the quadrupole focusing strength. Here, stability diagrams plotting

$$\frac{B'}{\beta\gamma} \quad \text{versus} \quad \theta^2 = \frac{\pi e E \lambda \sin\varphi}{mc^2\, \gamma^3\, \beta^3} \left(\frac{D}{\lambda}\right)^2$$

are given. The parameter θ^2 is the square of the argument in the hyperbolic functions in the transfer matrix. The division by $\beta\gamma$ provides normalization such that quite universal curves are obtained. Optimum quadrupole strength and arrangement will be found for minimum amplitude of oscillation. Considering the matrix expression for the change in r and $\dot{r}$ for a complete section consisting of an rf section and drift section with quadrupole magnets, this can be expressed by taking the "phase advance" $\mu = \pi/2$, where μ is a parameter entering into the matrix expression.* Stability limits are provided by the values for μ equal π and 0. In the present case the transfer matrix applies to a particle exposed to a constant rf defocusing force, which implies going through with constant phase. This strictly applies only to the synchronous

* Internal Report, LRL, LS-3

particle. The extension to include nonsynchronous particles is somewhat suspect but it was the most expedient way of approaching this problem.

A few cases with doublets in the drift section were considered. In principle this leads to two possible arrangements. First, (focus) (defocus) (defocus in rf section) (focus) (defocus) and second, (focus) (defocus) (defocus in rf section) (defocus) (focus). The last case looks attractive because the same transfer matrices apply in both planes. It was pointed out by H.S. Snyder that actually the first case is similar in both planes as far as β_{max} and μ are concerned. So the "wiggle" and the pattern of β is different in the two planes, but the relevant numbers are the same. Moreover, the second case leads to impractical results, as will be shown below, because the true repeat length becomes twice of that of one rf section plus one drift section.

The following three cases have been computed with a doublet in the drift section:

(focus)		(defocus)		(defocus in rf section)	(repeat)
B	A	B	C	D	C

whereby A = distance between quadrupole elements.

B = length of quadrupole elements

C = distance between quadrupole and rf section

and D = length of rf section

A	B	C	D	
0.05	0.10	0.05	3.0	(meters)
0.05	0.30	0.05	3.0	(meters)
0.10	0.20	0.05	3.0	(meters)

The computed stability diagrams are shown in Figs.1,2 and 3 respectively. The diagram given in Fig. 3 for the parameters stated seems to provide at present the most favorable arrangement, because of the gradients required in the quadrupole elements and because the drift space is still quite adequate in the early section of the accelerator as far as longitudinal motion is concerned.

Considering now the case mentioned before, namely (focus) (defocus) (defocusing in rf section) (defocus) (focus); this was done analytically and the resultant stability diagram is shown in Fig. 4. The parameters used were:

A = 0.05 m; B = 0.30 m; and C = D = 2.95 m.

The lower stability limit is similar to the cases mentioned above. What would be the π boundary as before becomes here a 2 π stability limit because of the repeat length of the system. Further a substantial stopband opens up in the stability region, defined by the two π boundaries as shown, which was absent in the aforementioned cases. This is rather unfavorable because it is not tolerable that the particles spend very much time in this region. It is concluded therefore that this arrangement would not be suitable for practical applications.

Two further cases were investigated whereby instead of a doublet only a single quadrupole magnet was used in the drift space.
The parameters used were:

	B = 0.10 m;	C = 0.05 m;	D = 3.0 m
and	B = 0.30 m;	C = 0.05 m;	D = 3.0 m.

The resultant stability regions are shown in Fig. 5. It was confirmed that in these cases the results are rather unfavorable, even though the magnet gradients would be quite low because of noncancellation of focusing (defocusing) as compared with a doublet.

Discussion

R.L. Gluckstern (Yale): What would be the result if you keep other parameters constant and vary A only?

L. Smith (LRL): The two elements of a doublet are "fighting" each other. If you consider these as δ functions, then when they are moved together nothing happens at all. A separation of these two δ functions is very important to give the focusing action.

R.L. Gluckstern (Yale): Is there any trouble going in or out of such a system? You cannot get to the symmetry point.

L. Smith (LRL): We have not looked into this to any extent. I should say that β_{max} occurs in the focusing element of the doublet; the beam will be largest in the focusing element, and not in the rf section. The variation is not too great, however, and there might be a 20% to 30% wiggle.

S. Ohnuma (Yale): Is there any special reason why you prefer a doublet quadrupole system as compared to a triplet arrangement?

L. Smith (LRL): I see no reason for a triplet. It would again require large field strength, I think, and adequate space.

S. Ohnuma (Yale): You do not expect smaller values for β_{max} in this case?

L. Smith (LRL): I doubt if it would be enough smaller to make it worthwhile.

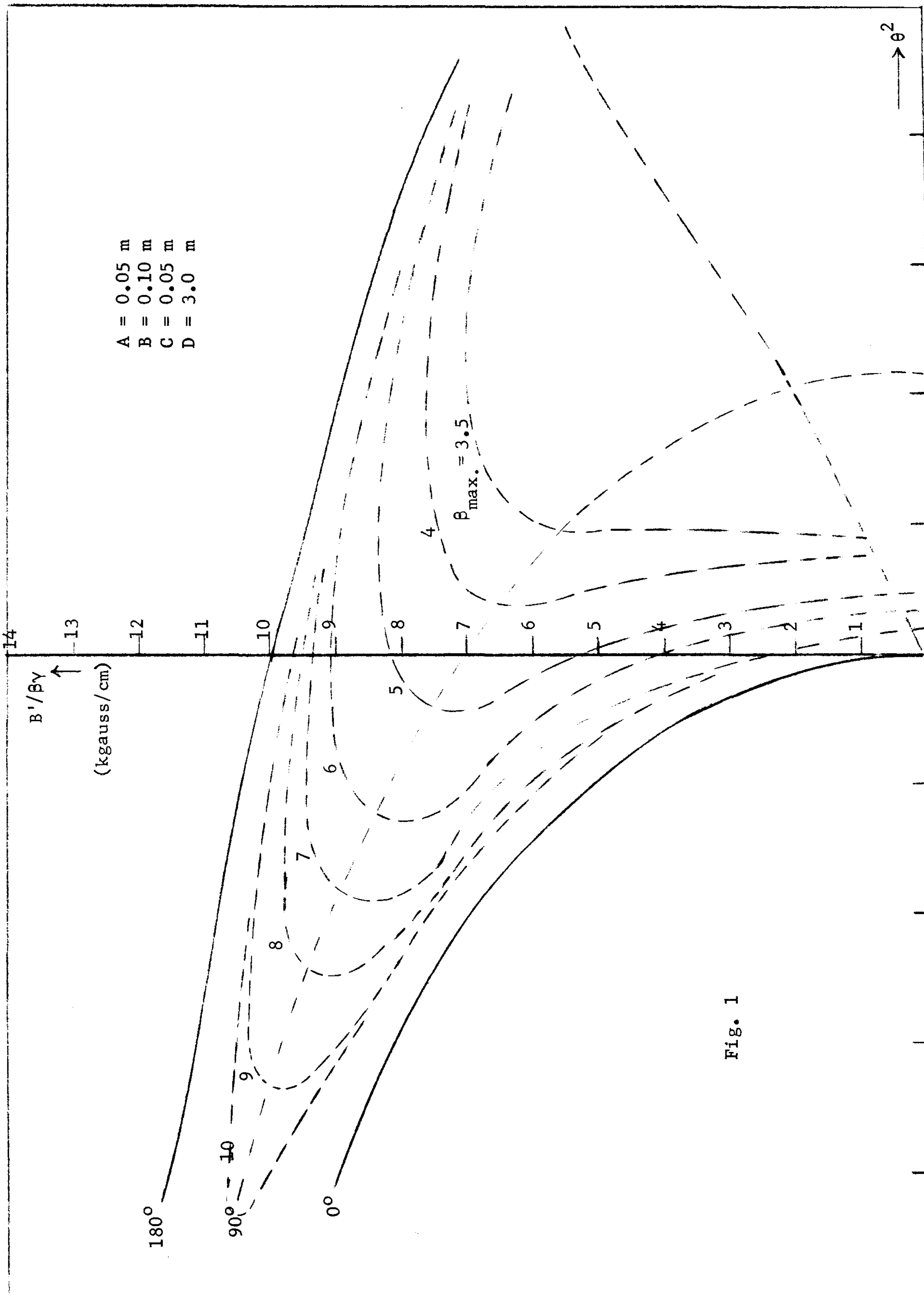

Fig. 1

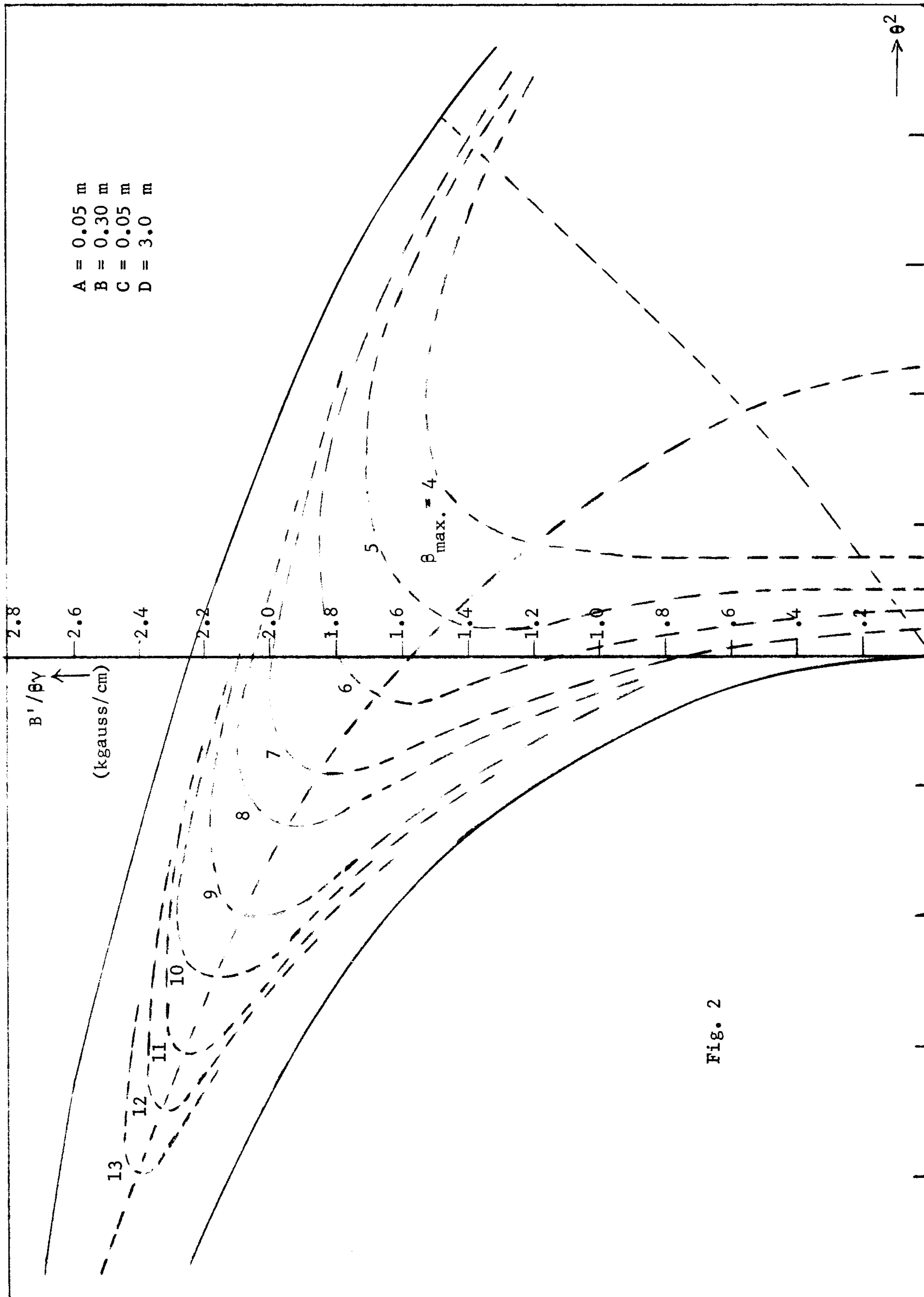
A = 0.05 m
B = 0.30 m
C = 0.05 m
D = 3.0 m
B'/βγ
(kgauss/cm)
θ²
β max. = 4
5
6
7
8
9
10
11
12
13
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
4
3
2
1
0
-1
-2
-3
-4

Fig. 2

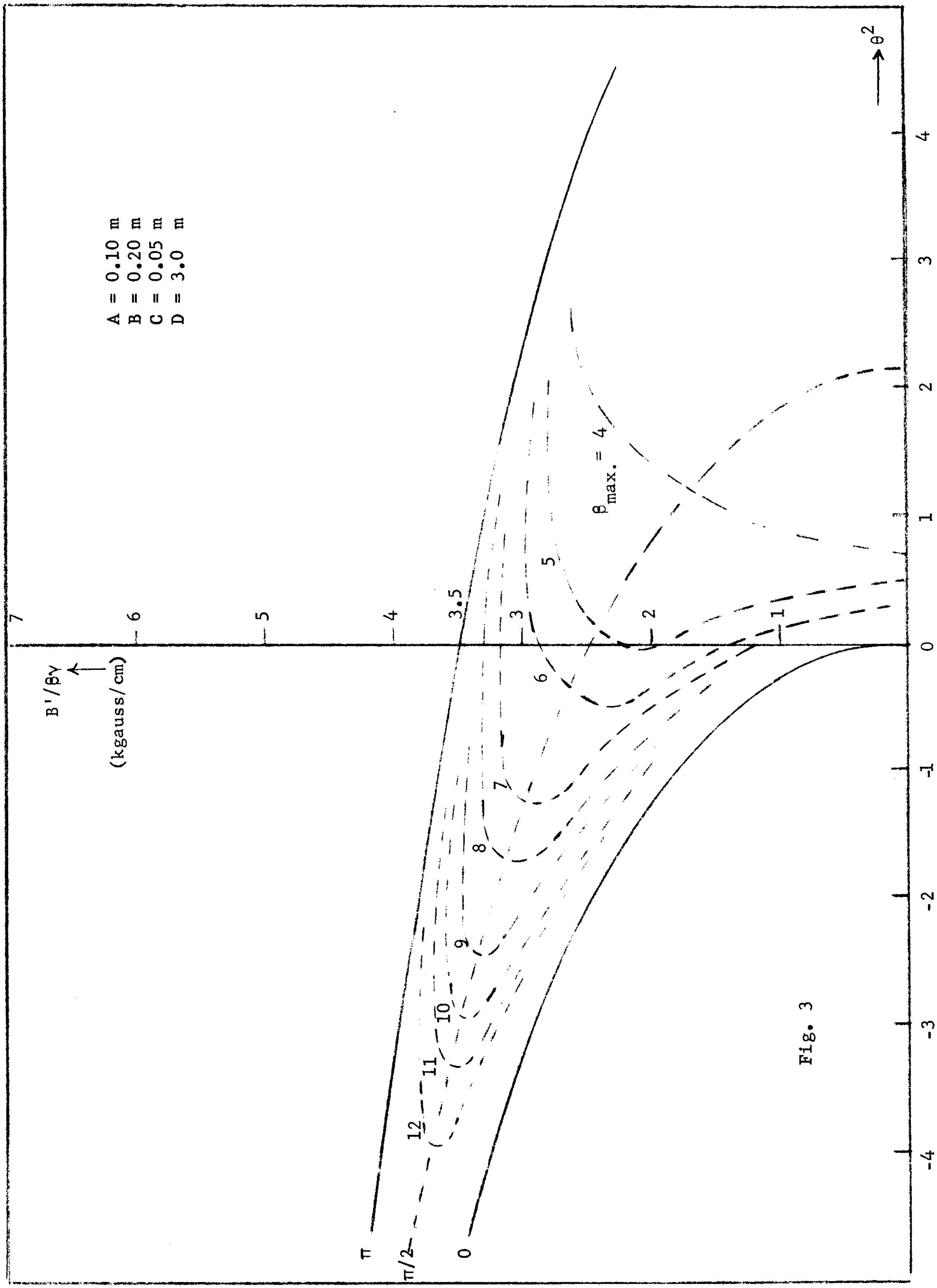

Fig. 3

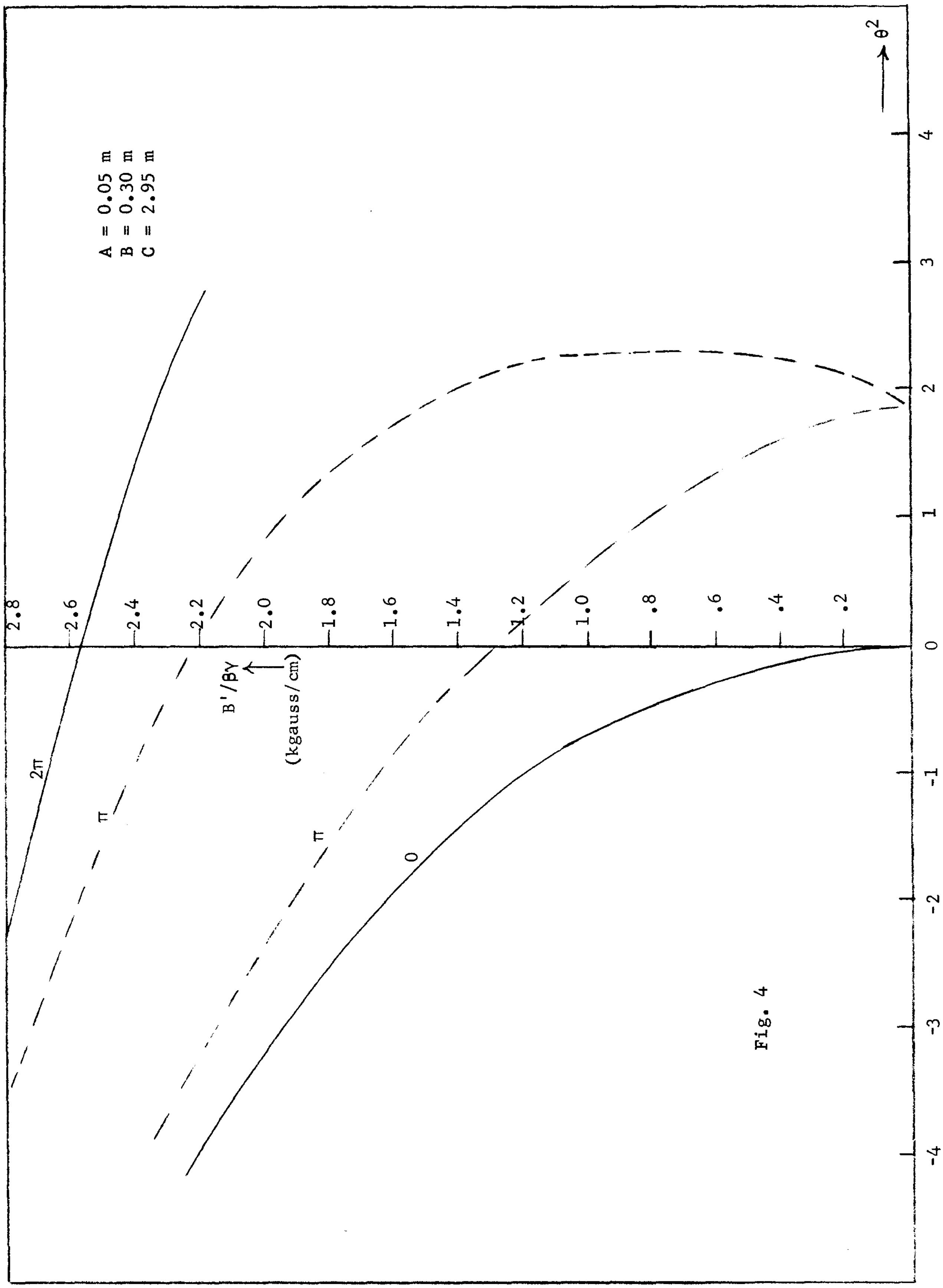
A = 0.05 m
B = 0.30 m
C = 2.95 m
B'/βγ
(kgauss/cm)
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
θ²
-4
-3
-2
-1
0
1
2
3
4
2π
π
π
0

Fig. 4

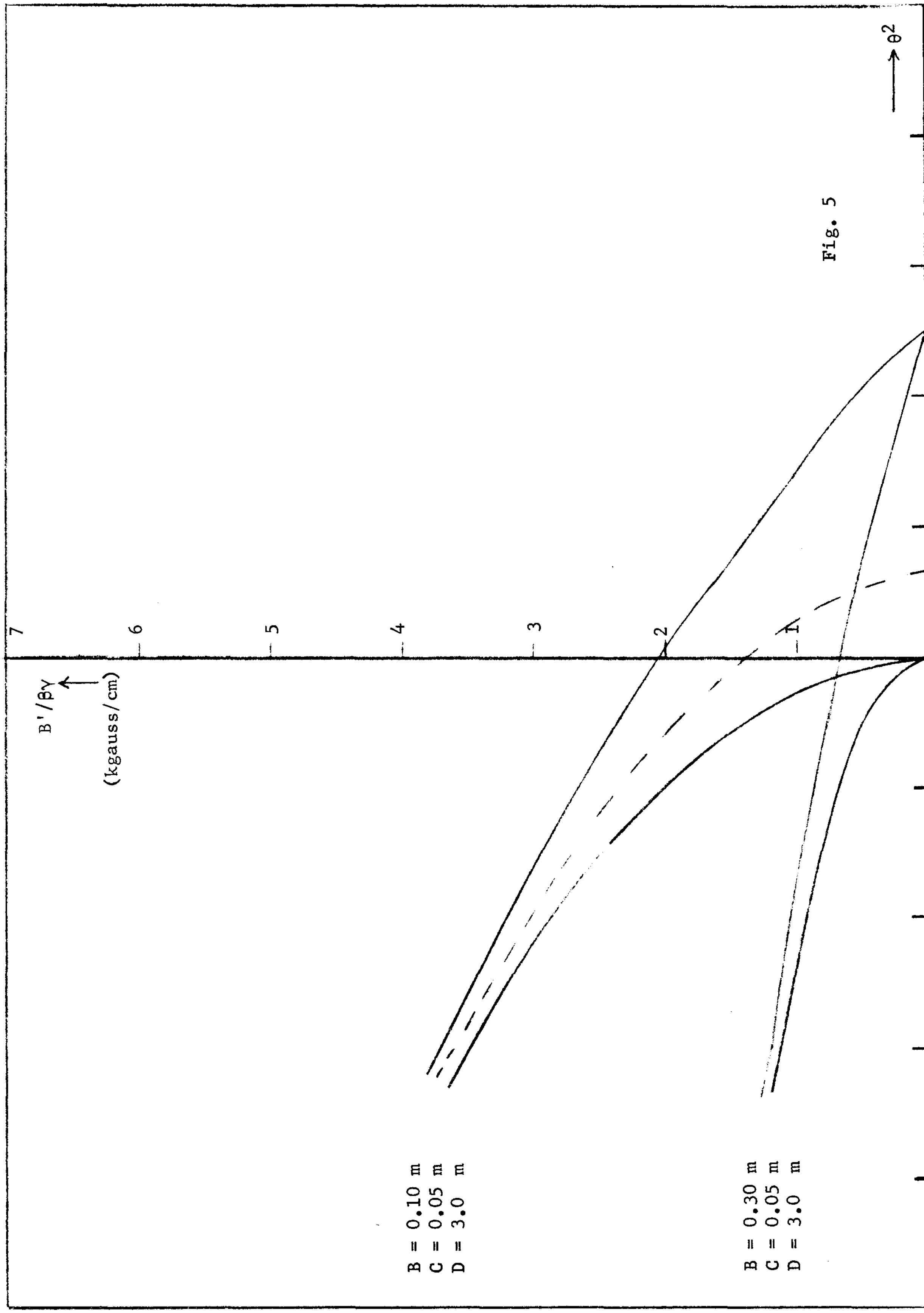
B'/βγ
(kgauss/cm)
θ²
7
6
5
4
3
2
1
0
-4
-3
-2
-1
1
2
3
4
B = 0.10 m
C = 0.05 m
D = 3.0 m
B = 0.30 m
C = 0.05 m
D = 3.0 m

Fig. 5