BEAM LOADING

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Rather than attempt to cover all aspects of work which has been done on this subject, a short description will be given of some work in progress at MURA. A general approach to beam loading involves several stages of calculation. The fields excited in the cavity by the power supply give rise via ordinary dynamics to current and charge distributions of the beam in the cavity. These current and charge distributions induce fields in the cavity which will modify the behavior of the beam. A complete treatment will involve completely closing this loop to find, for example, the equilibrium situation. It is evident that it must be possible to calculate the induced fields due to more or less arbitrary distributions of charge and current in the beam.

The present work is part of a series of problems being undertaken by Professor J. Van Bladel and his students in conjunction with MURA. The calculational basis was developed by E. J. Cristal and described in J. Appl. Phys. 32, 1715 (1961). The excitation of the cavity by the beam is being done by F. Kriegler.

The beam can be described as being an arbitrary distribution of charge along a line and this charge can then be made to move with velocity v through the cavity. Thus in essence, the cavity is being excited with various Fourier components of charge and current. Near the axis of the cavity in the neighborhood of the beam, the fields must approach those due to this line charge and current in free space. At the cavity walls, however, the fields must be close to those represented by the eigen functions in the cavity. Thus one represents the solution as a sum of cavity modes and free space solutions for the beam. Boundary conditions are satisfied in the calculation and fields are matched in the gap region of the cavity to obtain the complete solution.

Some early results are available which show the cavity voltage which is induced as a function of the velocity of current for the fundamental harmonic of the charge distribution. These results are shown in Fig. 1 for the case of the cavity portrayed in Fig. 2. One can see the resonant behavior of the cavity voltage as the velocity of the beam passes through the "synchronous" velocity. The next step in the process is to account for losses in the walls to obtain realistic estimates of the excitation of the cavity. In addition, some searching will be done for the higher modes in the cavity.

Discussion

H. B. Knowles (Yale): Have any measurements been made on the modes excited by the beam?

R. B. Neal (Stanford): Well, in general, the beam will excite the same kind of mode in the cavity that the cavity sets up to accelerate the beam. In the anomalous case you do get the excitation of other modes as well.

H. B. Knowles (Yale): Was it not tried at Stanford, to coast a beam into an unpowered cavity?

R. B. Neal (Stanford): Yes, the results are perfectly reciprocal. The beam will set up a wave which is just what could be obtained by reciprocity. If you calculate the energy loss in the beam it would set up a wave whose power is the same as the power required to accelerate the wave. May I add now a brief comment. I did like to report on a calculation that was done by P. B. Wilson, which applies to disk-loaded structures in standing wave operation. In the traveling wave case, the equation obtained by a number of people for beam loading is given by the expression:

$$V = \sqrt{2\tau}\ \frac{1 - e^{-\tau}}{\tau} \sqrt{P_o \ell r} \quad - i r \ell \left(1 - \frac{1 - e^{-\tau}}{\tau}\right)$$

where τ is the rf attenuation in the structure in nepers, i is the peak beam current through the structure, r is the shunt impedance per unit length, P_o is the input power, and ℓ is the length of a section. As can be seen the energy drops off linearly with current. There is an optimum attenuation parameter for each value of beam current.

The expression that P. B. Wilson has obtained for the standing wave disk-loaded structure looks quite similar.

$$V = \sqrt{P_o R_s} \left(\frac{2\sqrt{\beta}}{1 + \beta}\right) - \frac{iR_s}{2} \left(\frac{2}{1 + \beta}\right)$$

where β is the coupling coefficient, P_o is the input power as before, and R_s is $(r_{sh}\ell)/2$ so this is in effect the total shunt impedance of the standing wave structure. The energy drops off linearly with current, as before. The attenuation parameter in nepers in one case is analogous to the coupling coefficient in the other case. If you have no rf power into the accelerator, the second part on the right hand side term gives you energy loss by the beam in passing through the structure, due to the excitation. In the same way, this term would be the excitation term of the loss in energy of the beam as it passes through. One can now maximize energy with respect to β by taking $\frac{\partial V}{\partial \beta} = o$ resulting in the optimum value of β as a function of the current determined by

$\frac{\beta - 1}{2\sqrt{\beta}} = i/2 \sqrt{\frac{R_s}{P_o}}$. Going to higher currents this indicates that one wants to couple more heavily into the cavity. This is an expression that results in zero reflection from the cavity and therefore maximum utilization of the rf power. When you introduce a heavy current through this structure, the beam, in effect, acts as a shunt conductance across the cavity, and in order to stay matched to the cavity it is necessary to increase the coupling coefficient.

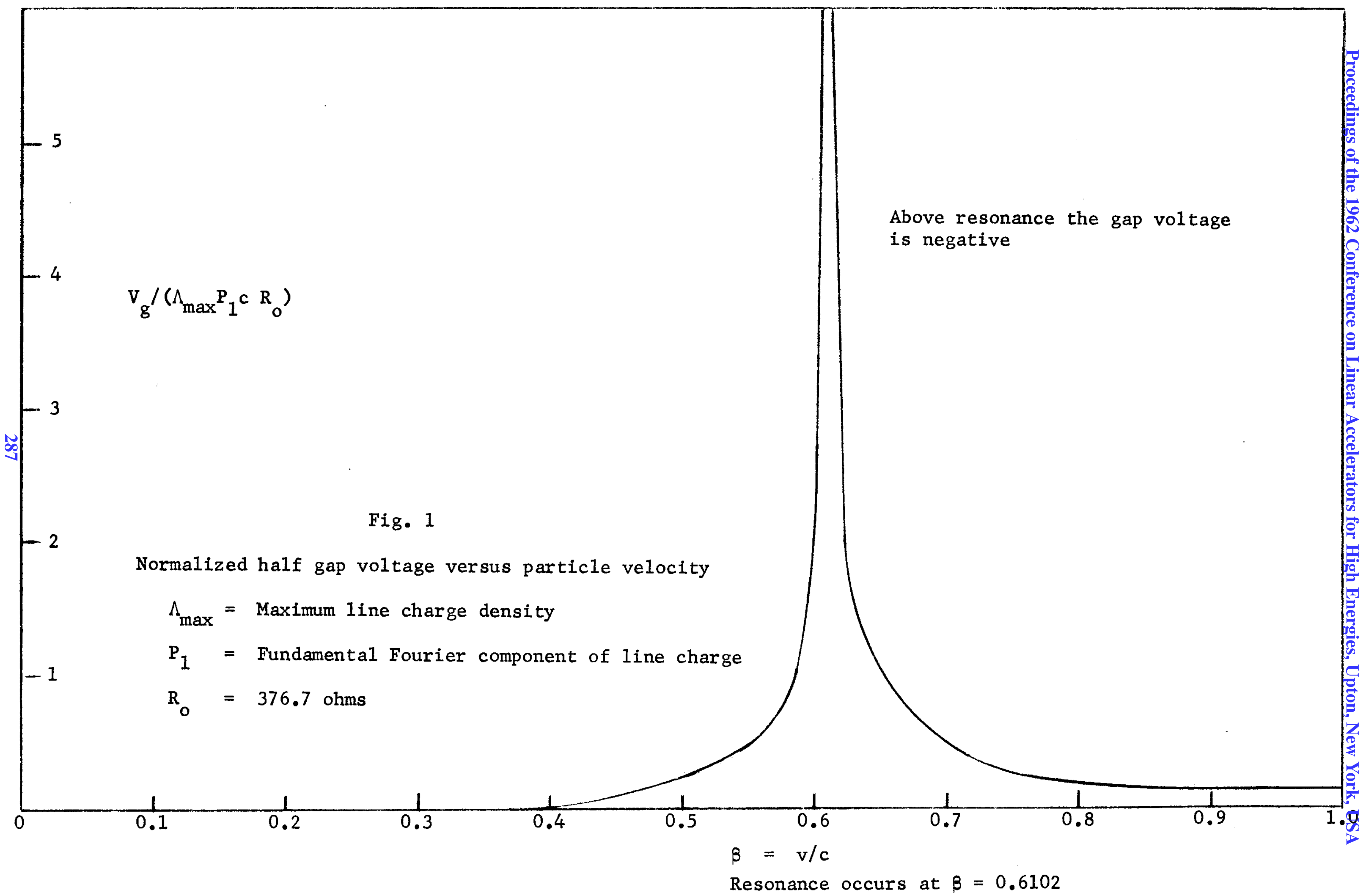


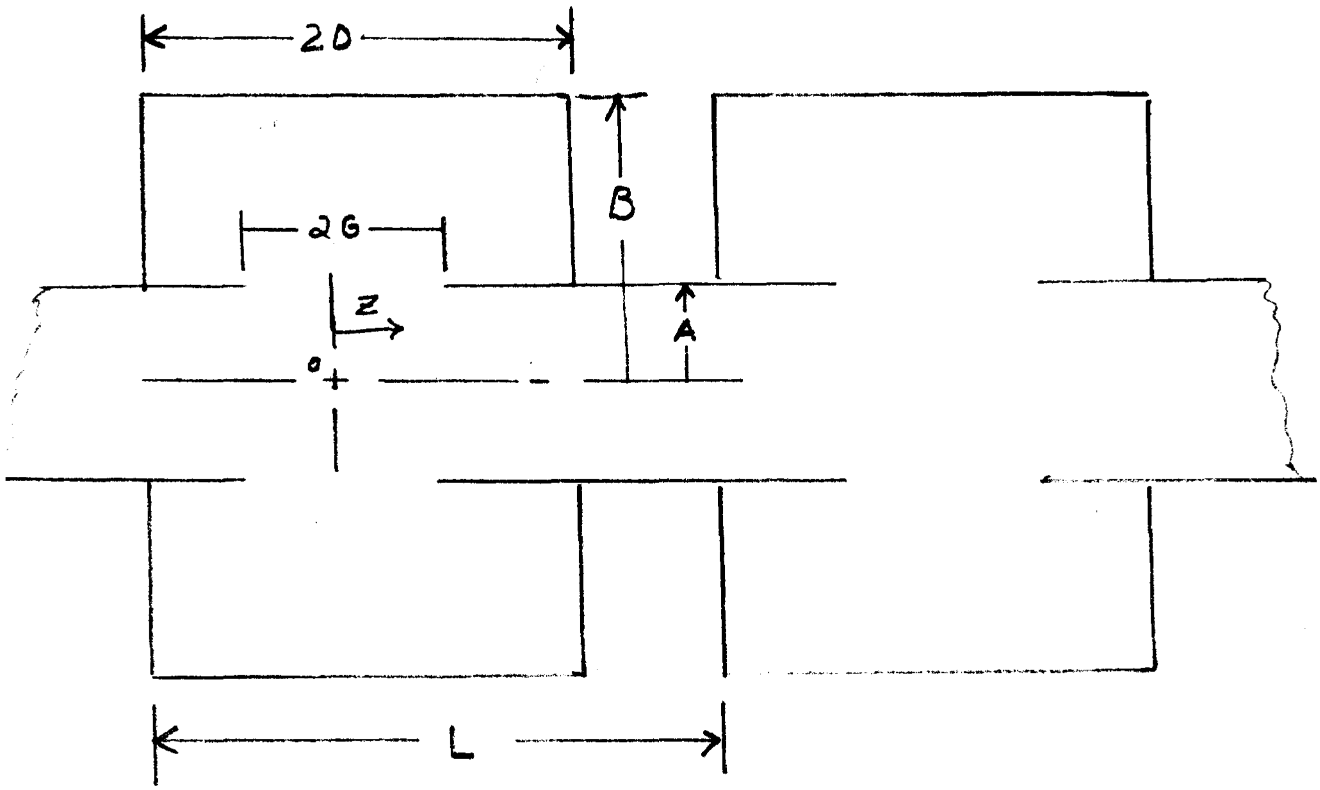
Fig. 1

Normalized half gap voltage versus particle velocity

Λ_{max} = Maximum line charge density

P_1 = Fundamental Fourier component of line charge

R_o = 376.7 ohms



$\alpha = \frac{2\pi A}{L} = 1$, $\mu = \frac{B}{A} = 3$, $\tau = \frac{2D}{L} = 0.75$, $\Psi = \frac{G}{D} = 0.5$

$$V_g = \int_0^G E_z(A,Z)\,dz$$

Fig. 2

Accelerator cavity