AN ACHROMATIC BEAM TRANSPORT SYSTEM*

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Injection into the proposed 10 Bev FFAG accelerator requires transport of the 200 Mev linac beam to the inside of the FFAG ring. This is because low energy orbits are at the inner circumference of this machine. The linear accelerator will be located 12 feet above the median plane of the main machine. The achromatic system needed to transport the linac output beam to the FFAG machine injection location will be considered here.

Fig. 1 shows the layout of the proposed system. Inside the magnet ring, the beam is brought down 12 feet into the FFAG median plane. Injection into an acceptable orbit is completed by a septum magnet in the main vacuum chamber.

It is essential that the beam transport system provides reasonable achromatic translation and acceptable focusing in both transverse directions, with practical tolerances on the parameters of the beam optical elements.

A Fortran program has been written for the 704 computer to evaluate possible beam transport systems. This program is described in MURA Internal Report 645. Ease of computer input and ease of interpretation of computer output have been stressed. The present program essentially does matrix multiplication, although it may readily be extended to other analytic transformations. The flexibility and high speed of the operation give a powerful means of studying transport systems, since it is possible to quickly evaluate many systems and select the best one. Typical computer data are given in

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*Report of work done by M. Shea and D.A. Swenson

Fig. 2, showing input values such as momentum spread, transverse phase space coordinates, etc. and results. The resultant phase space coordinates for a certain number of particles are plotted for both degrees of freedom in the form of asterisk graphs, also shown in Fig. 2.

Initially, a beam transport system given by Panofsky (Rev. Sci. Instr. 25, p.287, 1954) was investigated. This consists of two bending magnets and three quadrupole magnets in a symmetric arrangement. In one plane, say the X plane, the bending magnets provide translation and the quadrupole magnets, focusing in this plane, achromaticity. Wedge angles are used on the bending magnets to compensate for the fact that the quadrupole magnets are defocusing in the Y plane. With this system it seems possible to achieve:

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ transforming into } \begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and in the Y plane,}$$

$$\begin{pmatrix} y \\ y' \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ transforming into } \begin{pmatrix} y \\ y' \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} .$$

Translation through this system results in acceptable (x,x') phase space ellipses, but the resultant (y,y') phase space ellipses are severely elongated. The reason for this elongation is that in the above approximation the particles represented in the (y,y') phase space area have effectively undergone an infinite drift length. It has been found that this long effective drift length can be considerably shortened by the introduction of two more symmetrically located quadrupole magnets, defocusing in the X plane. The other requirements are still met by this system. Thus, the proposed system is, considering the X plane:

bend - focus - defocus - focus - defocus - focus - bend.

Phase space ellipses referring to the exit of each component of the proposed system, for an incident parallel beam, are shown in Fig. 3. Similar results for the (x,x') phase space for momentum deviations $\Delta p/p = \pm 0.005$ are given in Fig. 4 and Fig. 5. The resultant phase space ellipses are quite acceptable. Actually, for the 10 Bev FFAG accelerator it is intended to reduce $\Delta p/p = 0.5\%$ by a factor of 5 by means of a debuncher. This would reduce the effect shown in Fig. 5 by the same factor.

Considering the effective drift length for particles represented in the (y,y') phase space area, this has been calculated and the results are shown in Fig. 6. Here the effective drift length is plotted as a function of the ratio of the quadrupole magnet lens strengths of the elements indicated and for different length parameters. The results indicate the possibility of obtaining an effective drift length of zero value. The circled point in Fig. 6 refers to the parameters chosen for the beam transport system for the proposed FFAG accelerator.

Discussion

J.P. Blewett (BNL): It seems that a large aperture is needed in the x direction.

D.A. Swenson (MURA): A larger aperture is only necessary in the center quadrupole magnet; even there the radius is only 3 cm.

S. Ohnuma (Yale): Was this system designed for a particular phase space ellipse for the output of the linac?

D.A. Swenson (MURA): This was not necessary in a first approach, because of the rather short effective drift length.

J.P. Blewett (BNL): I cannot resist asking what would happen if a hole was bored in the back leg of the magnet to bring the beam inside on the median plane.

D.A. Swenson (MURA): It could be done, I suppose, but this would involve some complications.

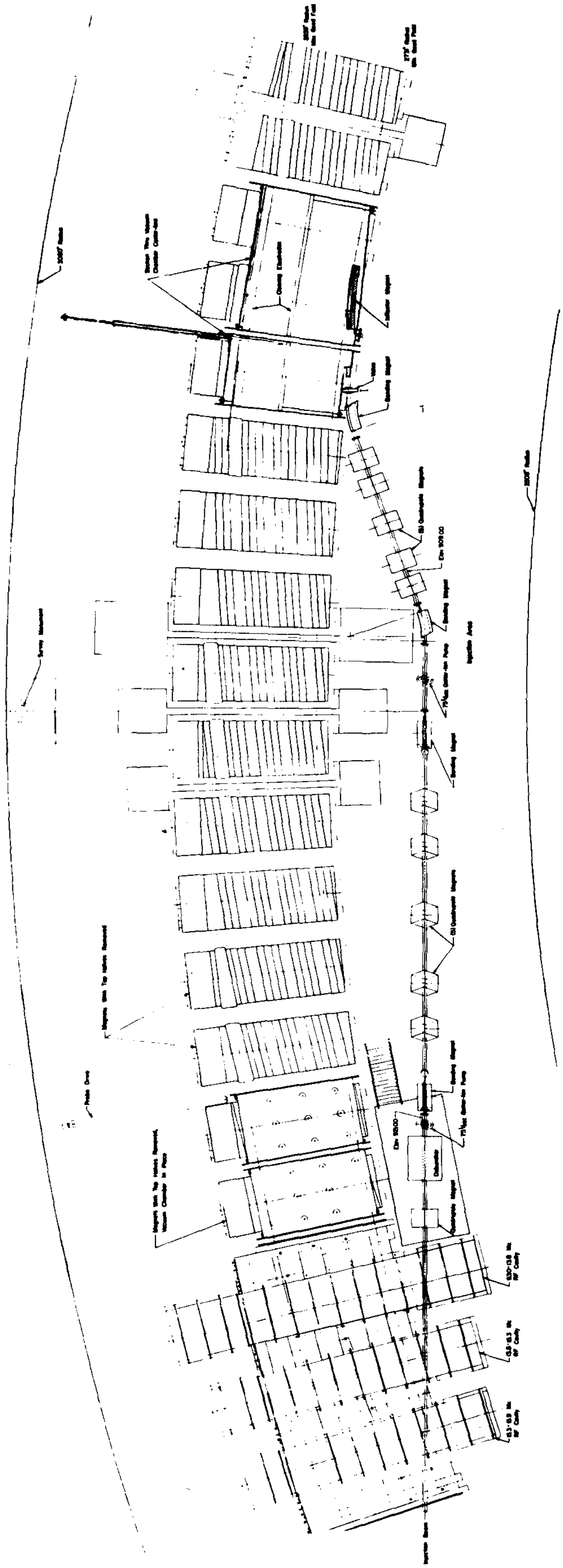

Fig. 1

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BEAM TRANSPORT PROGRAM          SWENSON                                   31 JUL 62

TRANSPORT          4002 -0.                 -0.                 -0    -0    -0
BEND          1       1  0.10000000E 03  0.20000000E 02          1    -0    -0
                     -0  0.20000000E 03 -0.                      1     1    -0
                     -0  0.30000000E 02  0.                     -0    -0    -0
QUADRUPOLE            2  0.20062500E 03  0.40000000E 03          1    -0    -0
                     -0  0.50000000E 02 -0.                      1    -0    -0
QUADRUPOLE            3  0.13375000E 03  0.40000000E 03          1    -0    -0
                     -0  0.50000000E 02 -0.                     -1    -0    -0
QUADRUPOLE            4  0.20062500E 03  0.20000000E 03          1    -0    -0
                     -0  0.50000000E 02 -0.                      1    -0    -0
QUADRUPOLE            5  0.20062500E 03  0.40000000E 03          1    -0    -0
                     -0  0.50000000E 02 -0.                     -1    -0    -0
QUADRUPOLE            6  0.13375000E 03  0.40000000E 03          1    -0    -0
                     -0  0.50000000E 02 -0.                      1    -0    -0
BEND                  7  0.20062500E 03  0.20000000E 02          1    -0    -0
                     -0  0.20000000E 03 -0.                      1    -1    -0
                     -0  0.                  0.30000000E 02     -0    -0    -0
DESTINATION           8  0.10000000E 03 -0.                      1    -0    -0
RUN                  23 -0.                 -0.                 -0    -0    -0
INPUT                73  0.10000000E 01  0.50000000E-03          3    -0    -0
                     -0  0.10000000E 01  0.50000000E-03         -0    -0    -0
OUTPUT                1  0.12500000E 01  0.50000000E-02          1     1    -0
MOMENTUM             -0  0.21496000E 07 -0.                     -0    -0    -0
DELTAP               -0  0.50000000E-02 -0.                     -0    -0    -0
CHANGE                1  0.27450000E 02 -0.                      5    -0    -0
CHANGE                2  0.46566100E 03 -0.                      2    -0    -0
CHANGE                3  0.37252880E 03 -0.                      2    -0    -0
CHANGE                4  0.42070370E 03 -0.                      2    -0    -0
CHANGE                5  0.37252880E 03 -0.                      2    -0    -0
CHANGE                6  0.46566100E 03 -0.                      2    -0    -0
CHANGE                7  0.27450000E 02 -0.                      6    -0    -0
BEGIN                -0 -0.                 -0.                 -0    -0    -0
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RESULTS AT EXIT FROM ELEMENT    1                                         31 JUL 62

X    1.177665  1.179572  1.172961  1.157682  1.134450   Y   0.818675  0.820930  0.816938  0.806729  0.790379
XP   0.002441  0.002481  0.002515  0.002544  0.002565   YP -0.002597 -0.002551 -0.002486 -0.002401 -0.002299

X    1.102843  1.063301  1.016126  0.961677  0.900368   Y   0.768015  0.739805  0.705965  0.666753  0.622465
XP   0.002581  0.002590  0.002592  0.002587  0.002576   YP -0.002178 -0.002042 -0.001889 -0.001723 -0.001543

X    0.832665  0.759085  0.680186  0.596569  0.508872   Y   0.573441  0.520052  0.462706  0.401837  0.337911
XP   0.002559  0.002534  0.002503  0.002466  0.002424   YP -0.001351 -0.001150 -0.000939 -0.000721 -0.000498

X    0.417760  0.323928  0.228090  0.130975  0.033322   Y   0.271413  0.202849  0.132742  0.061624 -0.009963
XP   0.002376  0.002323  0.002266  0.002204  0.002138   YP -0.000271 -0.000042  0.000187  0.000415  0.000640

X   -0.064126 -0.160627 -0.255446 -0.347862 -0.437172   Y  -0.081473 -0.152364 -0.222095 -0.290136 -0.355969
XP   0.002069  0.001998  0.001924  0.001849  0.001772   YP  0.000860  0.001073  0.001279  0.001474  0.001658

X   -0.522696 -0.603782 -0.679815 -0.750214 -0.814446   Y  -0.419093 -0.479027 -0.535316 -0.587530 -0.635273
XP   0.001695  0.001619  0.001543  0.001468  0.001395   YP  0.001830  0.001988  0.002130  0.002257  0.002366

X   -0.872019 -0.922497 -0.965496 -1.000687 -1.027804   Y  -0.678181 -0.715928 -0.748226 -0.774830 -0.795536
XP   0.001324  0.001257  0.001192  0.001132  0.001076   YP  0.002457  0.002529  0.002583  0.002616  0.002630

X   -1.046640 -1.057050 -1.058958 -1.052347 -1.037267   Y  -0.810189 -0.818675 -0.820930 -0.816938 -0.806729
XP   0.001025  0.000980  0.000939  0.000905  0.000877   YP  0.002624  0.002597  0.002551  0.002486  0.002401

X   -1.013835 -0.982228 -0.942686 -0.895512 -0.841062   Y  -0.790379 -0.768015 -0.739805 -0.705965 -0.666753
XP   0.000855  0.000839  0.000831  0.000828  0.000833   YP  0.002299  0.002178  0.002042  0.001889  0.001723

X   -0.779753 -0.712051 -0.638470 -0.559571 -0.475955   Y  -0.622465 -0.573441 -0.520052 -0.462706 -0.401837
XP   0.000844  0.000862  0.000887  0.000917  0.000954   YP  0.001543  0.001351  0.001150  0.000939  0.000721

X   -0.388257 -0.297145 -0.203313 -0.107475 -0.010360   Y  -0.337911 -0.271413 -0.202849 -0.132742 -0.061624
XP   0.000996  0.001044  0.001097  0.001155  0.001216   YP  0.000498  0.000271  0.000042 -0.000187 -0.000415

X    0.087293  0.184741  0.281242  0.376061  0.468477   Y   0.009963  0.081473  0.152364  0.222095  0.290136
XP   0.001282  0.001351  0.001422  0.001496  0.001571   YP -0.000640 -0.000860 -0.001073 -0.001279 -0.001474

X    0.557787  0.643310  0.724397  0.800429  0.870829   Y   0.355969  0.419093  0.479027  0.535316  0.587530
XP   0.001648  0.001725  0.001801  0.001878  0.001952   YP -0.001658 -0.001830 -0.001988 -0.002130 -0.002257

X    0.935060  0.992634  1.043112  1.086111  1.121302   Y   0.635273  0.678181  0.715928  0.748226  0.774830
XP   0.002025  0.002096  0.002164  0.002228  0.002288   YP -0.002366 -0.002457 -0.002529 -0.002583 -0.002616

X    1.148419  1.167254  0.080307  0.        0.        Y   0.795536  0.810189  0.        0.        0.
XP   0.002344  0.002395  0.001710  0.        0.        YP -0.002630 -0.002624 -0.        0.        0.
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RESULTS AT EXIT FROM ELEMENT 1 — 31 JUL 62

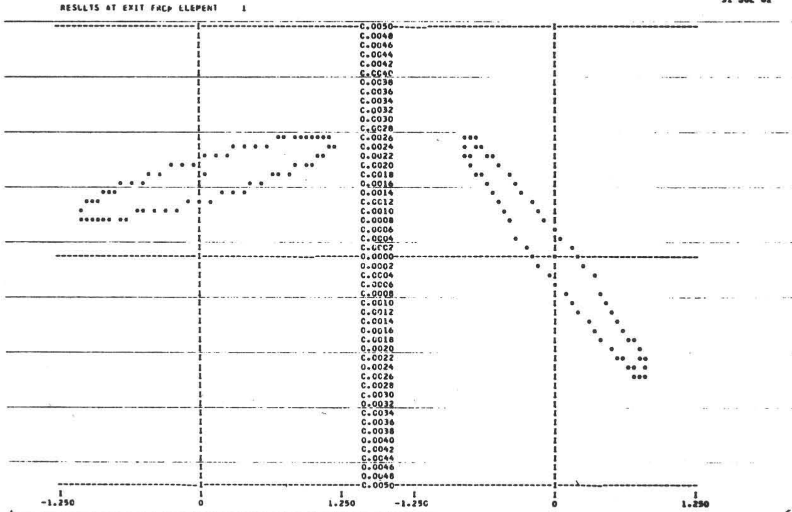
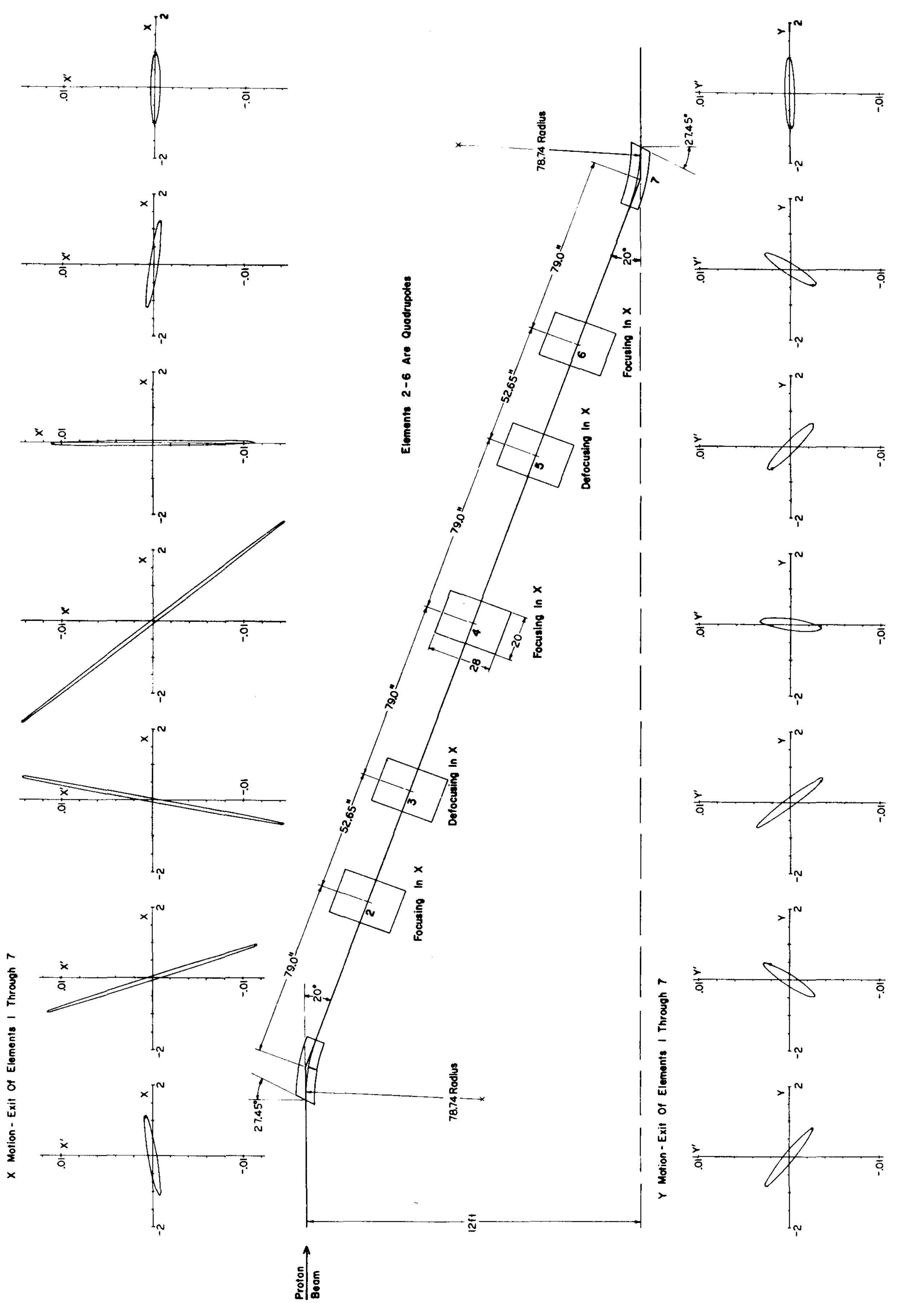
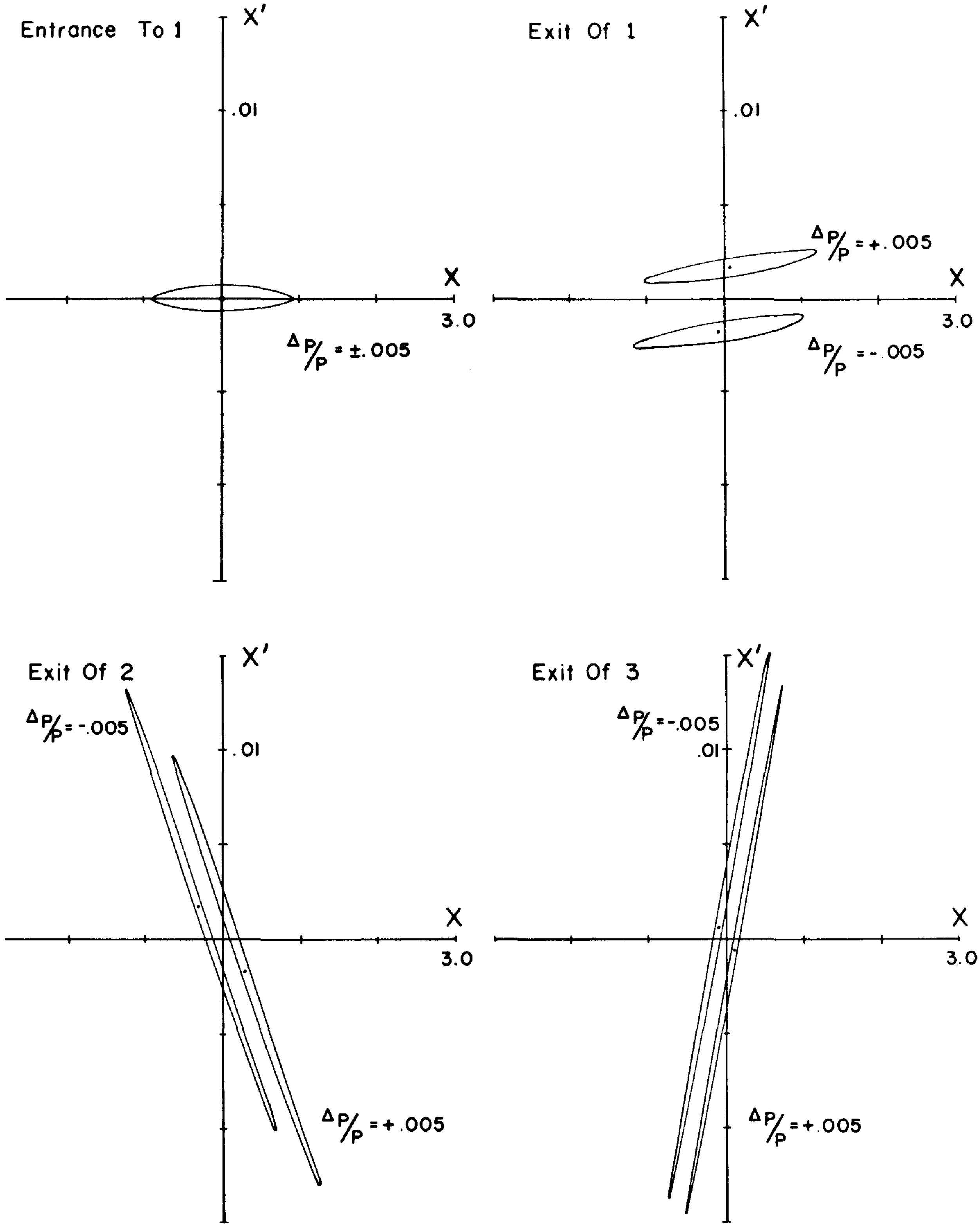


Fig. 2



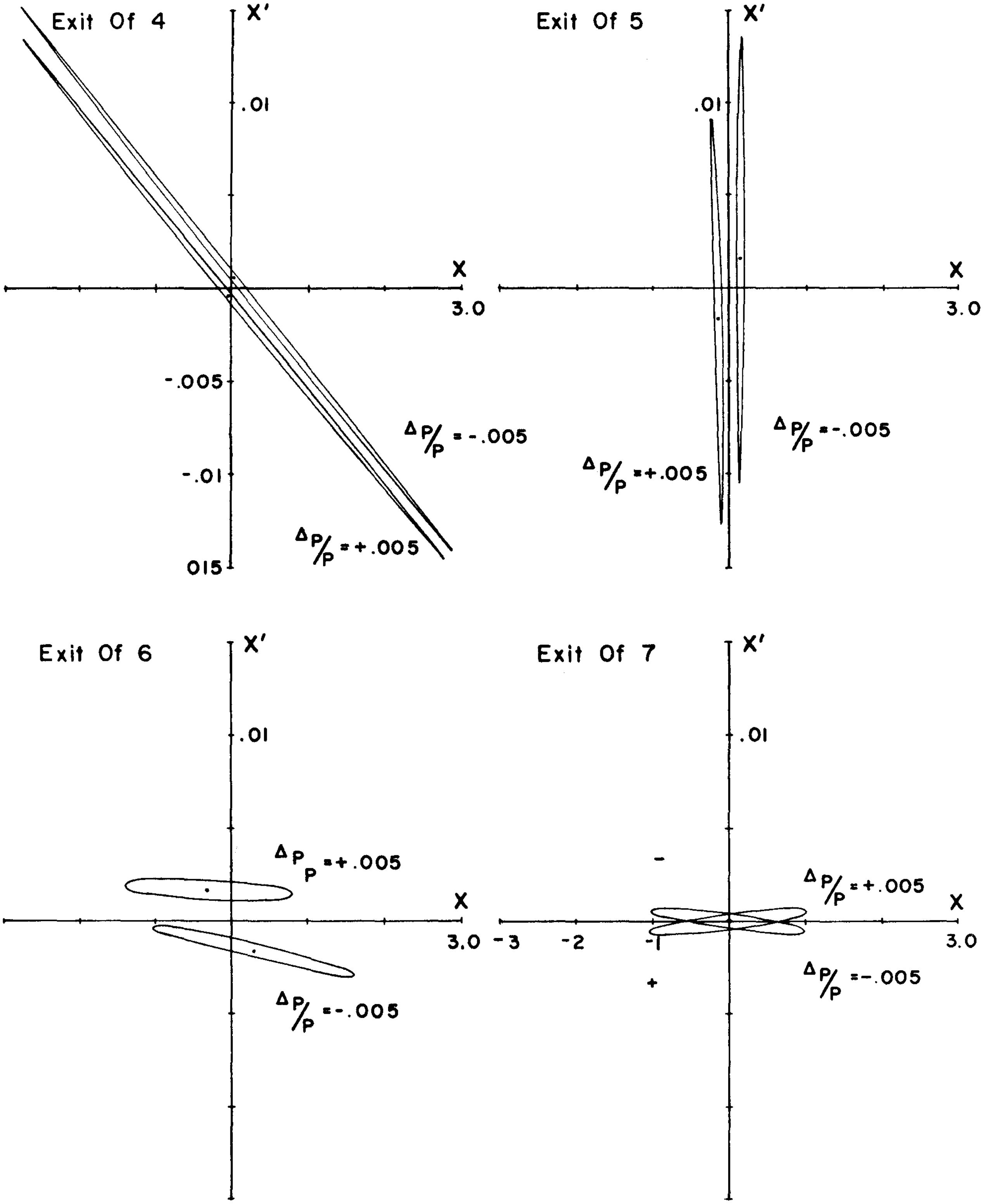
AN ACHROMATIC TRANSLATION FOR BEAM TRANSPORT SYSTEM

Fig. 3



Beam Transport Phase Plots

Fig. 4



Beam Transport Phase Plots (Cont.)

Fig. 5

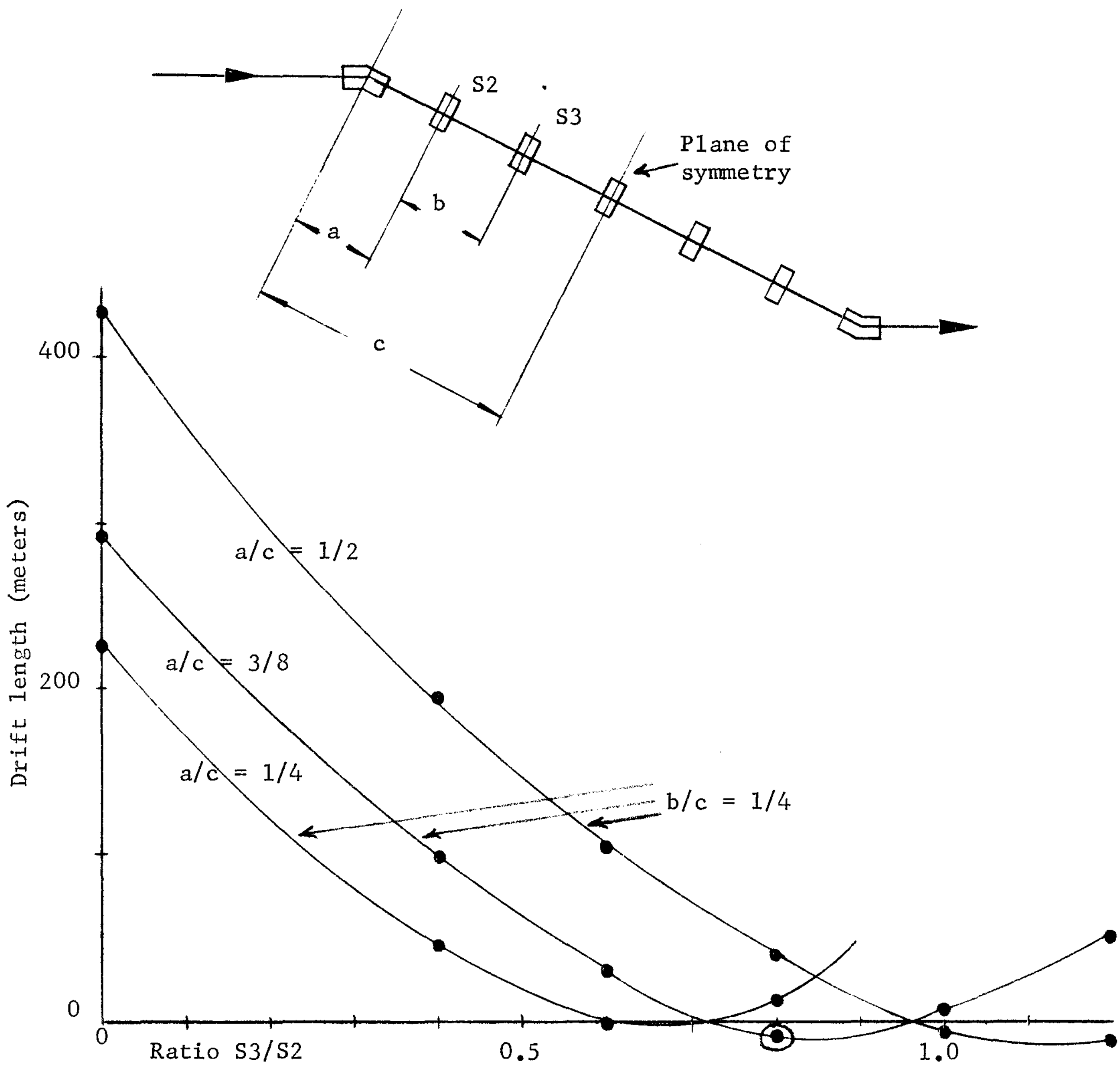

S2
S3
Plane of symmetry
b
a
c
400
200
0
Drift length (meters)
a/c = 1/2
a/c = 3/8
a/c = 1/4
b/c = 1/4
0
Ratio S3/S2
0.5
1.0

Fig. 6