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COUPLED PHASE AND RADIAL MOTION IN LINEAR ACCELERATORS*

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I would like to tell you something about the phase and radial dynamics program that we have been working with at MURA. We have prepared a computer program entitled PARMILA to trace the phase motion and the transverse motions of particles through a proton linear accelerator. The name of the program is derived from the phrase, "Phase And Radial Motion In Linear Accelerators". The program is prepared to simulate a linac of standard design, namely a linac composed of a set of resonant linac cavities (tanks), each containing a series of drift tubes, each of which contains a quadrupole focussing magnet.

The program proceeds by transforming the coordinates of a collection of particles through a set of transformations representing the linac. The transformations are chosen to represent the important effects of the linac accelerating and focussing structure on the particles.

Each particle has six coordinates, namely x, x', y, y', E and φ, where x, x', y and y' are the transverse displacements and angles of the particle trajectories in two transverse directions, E is the energy and φ is the phase of the particle with respect to the rf accelerating voltage.

If one ignores the coupling between the x, x' motion, the y, y' motion and the E, φ motion, one can study each of the three motions independently. This has been done on

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* Much of this material is also available as MURA TN-437.

many occasions, both analytically and digitally and is well described in the literature.[1-4]

It is extremely difficult to include the coupling between the transverse motion and the phase motion in the analytic approach. The present work is an attempt to study the effect of these coupling terms with the aid of digital computation.

In order to describe the nature of the coupling terms included in the program, we must outline the sequences of transformation performed by the program, and describe the functional dependence of each transformation.

The linac is treated as a series of cells, each cell beginning at the center of one drift tube and ending at the center of the next. The main loop of the program transforms the coordinates of the particles through one cell.

The first transformation is denoted in Fig. 1 as T1 and simulates the action of the quadrupole magnet on the transverse coordinates x, x', y and y'. The x, x' motion is independent of the y, y' and φ coordinates, but is a function of the particle energy E. This is one of the terms which couple the transverse motions to the phase motion.

The second transformation is denoted on Fig. 1 as T2 and simulates a free-space drift to the electrical center of the accelerating gap. There are no coupling terms present here.

Before proceeding to the transformation T3 at the center of the gap, we shall dispense with transformation

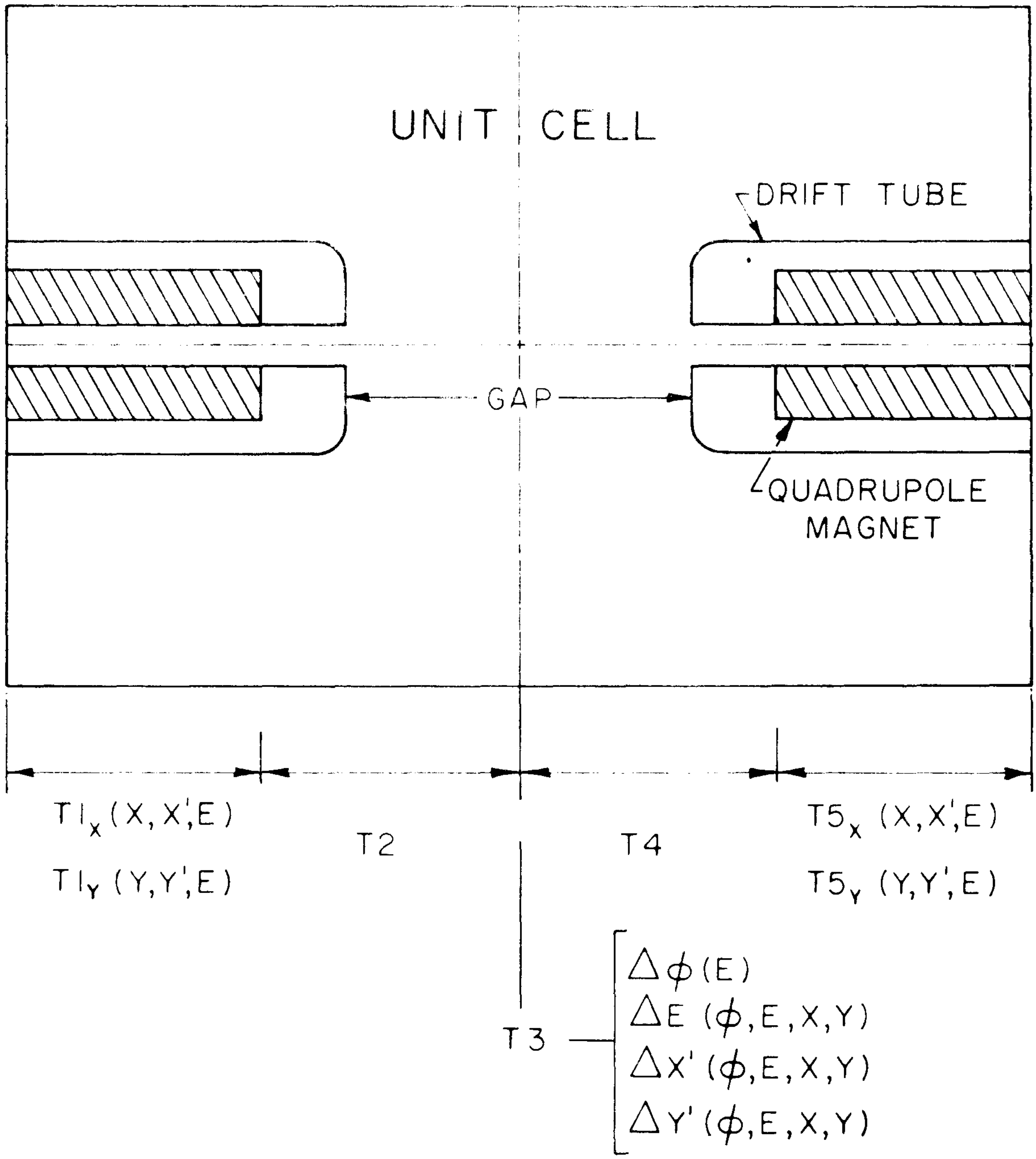
T4 and T5 by noting that they are similar, except in magnitude, to transformations T2 and T1, respectively.

The transformation T3 at the center of the gap simulates the change in energy of the particle, and the net defocussing experienced by the particle on crossing the gap.

The change in φ from the previous gap is a simple function of the length of the cell and the longitudinal component of the particle velocity. The change in φ per cell is calculated in two steps. It is calculated once to prepare the coordinates for output at the end of the unit cell, and once to prepare for the phase dependent calculations at the center of the gap. The difference between the actual velocity, and the longitudinal component is a possible coupling between the phase motion and the transverse motion. This effect is very small and is ignored.

The change in energy on crossing the gap is a function of φ and a longitudinal transit time function. The longitudinal transit time is a function of the particle energy E, and the effective length of the gap, which, in turn, is a function of the transverse coordinates x and y. Hence the change in energy is a function of φ, E, x and y. This represents a strong coupling between the phase motion and the transverse motion.

The defocussing action on the x' coordinate of a particle on crossing the gap is a function of φ and x and of a transverse transit time function, which is a function of E, x, and y.



FUNCTIONAL DEPENDENCE OF TRANSFORMATION

Figure 1

The radial displacement of the particle is checked at the entrance to the drift tube to see if the particle is within the available aperture, and at some point in each cell, the particle energy is checked to see if it has dropped more than 2 MeV below the synchronous energy. Both of these checks are used to signal the loss of particles from the beam. When a particle is lost, a record of where and how it was lost is stored in the memory.

Now there are a number of things that one can do with a program like this. One can search for the optimum arrangement for the quadrupole magnets, and the optimum formulations for their strength as a function of particle energy. We have in our program, a rapid way to specify the quadrupole strengths, such that θ^2 is any linear function of Δ, where θ^2 and Δ are those defined by Smith and Gluckstern.[3] A preliminary set of runs have been made for a (+ + - -) orientation of quadrupoles where θ^2 and Δ follow the five linear functions shown on the left-hand portion of Fig. 2. The x x' acceptance for each of the five formulations and their relative areas are shown on the right-hand portion of the same figure.

After making some choice for the orientation and strength of the quadrupole focussing magnets, one is free to study other properties of the particle dynamics; for example, the effect of the phase motion on the transverse acceptance, or the effect of the transverse motion on the phase acceptance.

I will now show an example of the effect of the phase motion on the transverse acceptance. For this purpose, I will neglect the y and y' coordinates and concentrate on the x, x', E and φ coordinates and the coupling between them. You might think at first sight, that there is no coupling between the x and y motions, but there is, by virtue of their mutual coupling to the E, φ motion. Also, the aperture through which all the particles have to go is a circular aperture in the drift tubes, and of ccurse, a large x value coupled to a large y value will get a particle lost sooner than in the case where one value is large while the other is small.

I now choose the initial coordinates of the particles at the entrance to the linac such that every particle has some given value of E and φ as indicated by a dot on the φ, E graph of Fig. 3, and such that the x and coordinates form a rectangular mesh of points filling the rectangle shown on the x x' graph of Fig. 3. It is a simple matter to run such a sample of particles through the program, and to determine which particles succeed in passing through the linac. In this way one can determine the transverse admittance for any pair of E, φ coordinates.

Some results of this process are shown in Fig. 3, where the admittance curves on the left correspond to the dots on the right bearing the same number. The distorted "fish" on the right gives the phase acceptance for axial particles (i.e., x = x' = 0).

As one sees, the area of the transverse admittance does not change too rapidly with φ. However, the admittance available to particles with all phases between the

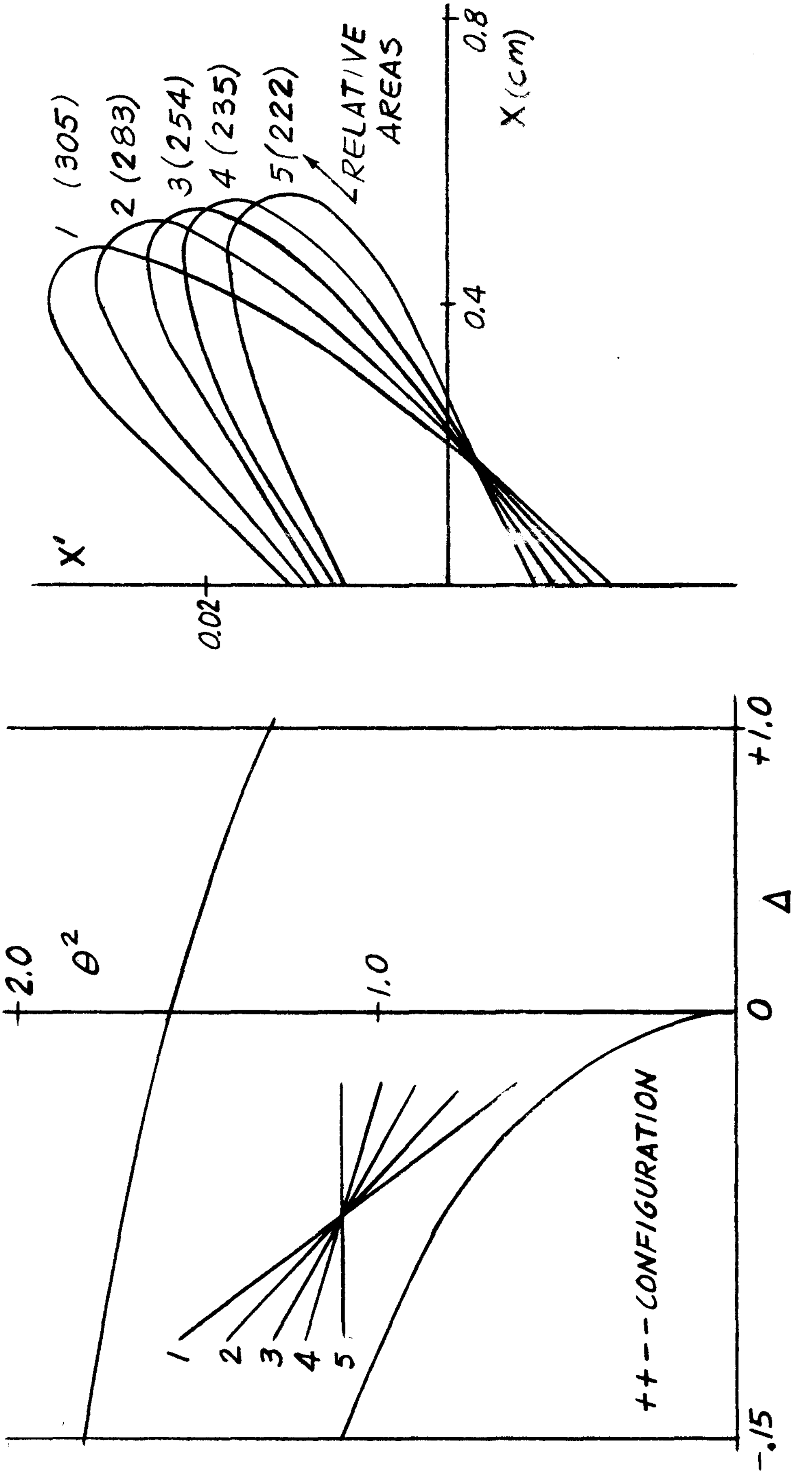

X'
0.02
0.4
0.8
X (cm)
1 (305)
2 (283)
3 (254)
4 (235)
5 (222)
RELATIVE AREAS
2.0
θ^2
1.0
1
2
3
4
5
++-- CONFIGURATION
-.15
0
Δ
+1.0

FIG. 2

first and the third case is an area common to all three curves, and is considerably smaller than the area of any of the larger curves.

In a similar fashion, we have studied the effect of transverse motion on the phase acceptance. In this case we choose the initial coordinates of the particles at the entrance to the linac such that every particle has some given value of x and x' as indicated by a dot on the x, x' graph of Fig. 4, and such that the E and φ coordinates form a rectangular mesh of points filling the rectangle shown on the E, φ graph of Fig. 4. The curves on the right correspond to the phase acceptance corresponding to the initial x, x' coordinates represented by a dot bearing the same number. The curve on the left is the admittance for particles with $\Delta\varphi = \Delta E = 0$.

In conclusion, I should say that the results presented here are simply examples of the effects which one should be able to study with a tool such as our PARMILA program. We hope to be able to use the results of such studies to guide us in the choice of the parameters for a 200 MeV proton linac.

BLEWETT: Where are most of the particles lost?

SWENSON: In the studies described in Fig. 3, the particles that were lost were lost radially. In the studies described in Fig. 4, particles were lost both radially and because they fell behind in energy. In Fig. 4, the particles accepted in situation 1, which were not accepted in situation 3, were lost radially.

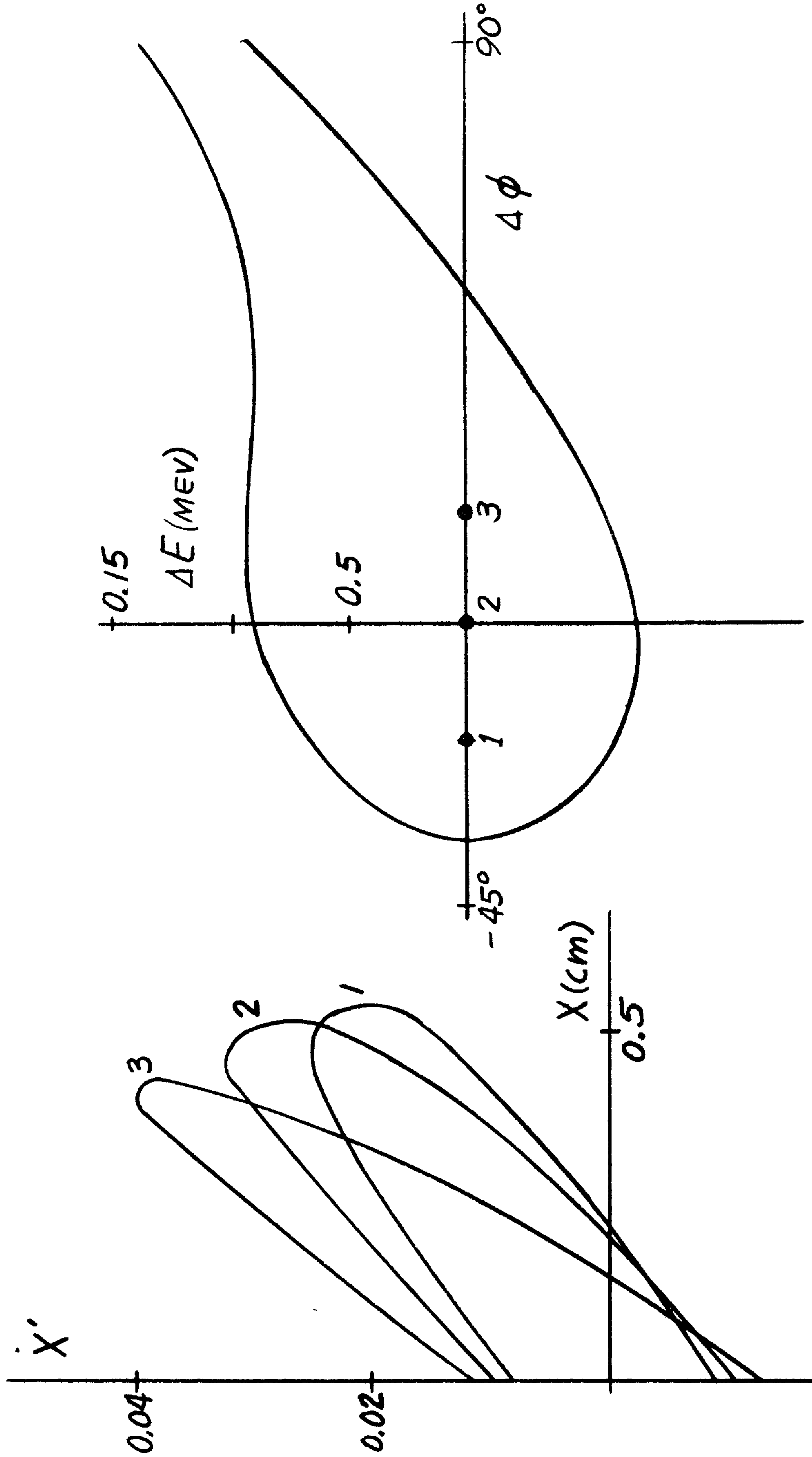


FIG. 3

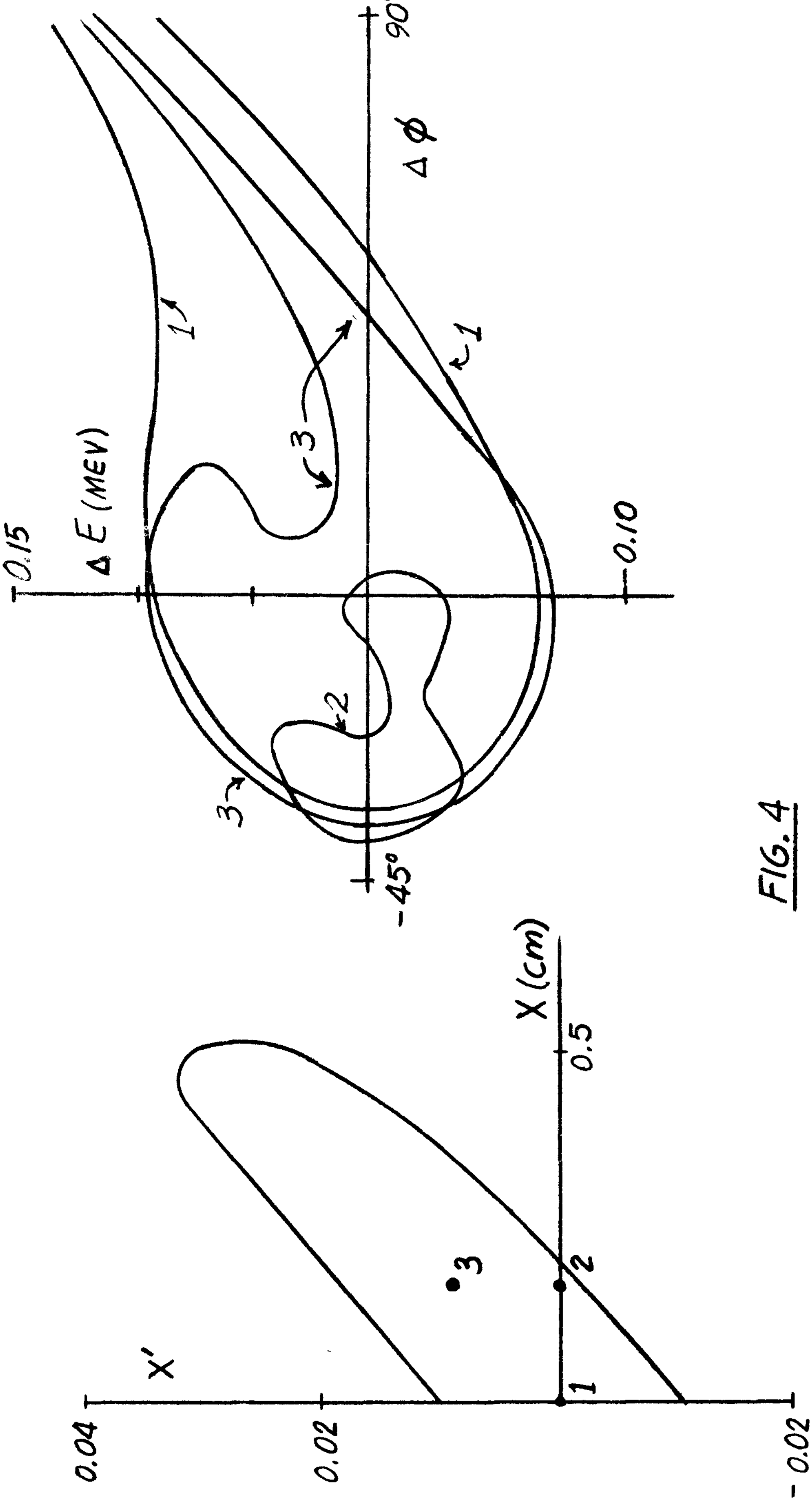


FIG. 4

WHEELER: Do you have any feeling as to how much the transverse acceptance is restricted by the phase spread?

SWENSON: In Fig. 3, you can see that the area common to all three curves on the left is about half of the area of any one curve. The graph on the right shows that this corresponds to a phase spread of about 40°. Such effects depend heavily on the other parameters of the linac and certainly on the strengths of the quadrupoles. I have been attempting to observe these effects as I vary the quadrupole strengths in a controlled way to get a feeling if there is some advantage in one setting over another. Although this is a perfect linac, not a misaligned linac, I want to look at the amplitudes of the transverse oscillations as a function of the quadrupole strengths.

QUESTION: What values of injection energy and frequency did you use?

SWENSON: For the particular diagrams I've drawn here, I've used 0.750 MeV and a 200 Mc/sec linac.

MILLS: Are there any places where particles start out near the axis and with small angles but with a large phase excursion, which are subsequently lost radially?

SWENSON: Yes.

TENG: This study is made for one transit time factor; is it possible to adjust the ends of the drift tube so that you can either decouple or make the coupling more advantageous?

SWENSON: I can specify g/L, for instance, to be a linear function of the number of the drift tube, so that it starts out as some small value and increases in the way that the linac cavity studies show to be optimum. The aperture in the drift tube is taken to be a linear function of linac length. The transit time functions are calculated on the basis of these dimensions and the x, y, E and φ coordinates of each particle. Every time a particle crosses the gap, a different transit time factor is calculated.

BLEWETT: It wasn't quite clear to me what you're using for the field pattern in the gap space.

SWENSON: The details of the field pattern are condensed into a transit time factor and a radial impulse factor which are derived from the amplitude of the wave component which is travelling in the direction of the particles with the synchronous velocity.

OHNUMA: I think what you're doing there is just expressing the entire field by one parameter so that you can calculate the energy gain.

BLEWETT: You are using the same sort of thing Luis Alvarez did, rather than the fields computed in the field calculation program.

SWENSON: No, I'm not. The field calculation program, "MESSYMESH" gives values of this field factor as well as the longitudinal transit time factor, and those have yet to be compared with the simple expression I use in PARMILA. This program is still at a very early stage.

TENG: How much can you change the field pattern so you can do something about the coupling of radial and phase motion?

I suspect you can do quite a lot by shaping the drift tube ends.

SWENSON: That would be reflected in different values for the thin lens parameters at the center of the gap. And one can certainly accommodate such effects in the program, if they can be realized in practice.

MILLS: This, of course, corresponds to taking purely the travelling wave component in the linac, and the question is, what are the effects of the other waves? We developed an expression which included the other waves, and I recall that their effects were not very large. At some later point I think we do want to put this into the calculation.

GLUCKSTERN: I have a very uneasy feeling about how accurate this will be if φ is very different from φ_s. If you're trying to find where those particles that get out of the bucket are lost radially, then I think you're in danger. I don't see anything short of real numerical integration in each cell.

SWENSON: That's probably correct.

References

(1) J. C. Slater, Rev. Mod. Phys. 20, 473 (1948).

(2) J. P. Blewett, Internal BNL Report JPB-18 (March 27, 1963).

(3) L. Smith and R. L. Gluckstern, Rev. Sci. Instr. 26, 220 (1955).

(4) D. Cohen, Internal ANL Report, ANLAD-57 (July 16, 1959).