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COMPUTER PROGRAMS FOR PARTICLE MOTION IN LINACS*

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Since the computer program, "PARMILA," which has just been reported by Don Swenson is quite similar to what we have at Yale, it would be more useful to talk here about specific features of our programs and what we want to find out from them rather than to give detailed explanations. Because of the expected high intensity of the primary proton beam it is essential in the design of a meson factory that we should not lose any particle beyond, say, 50 MeV. Up to 200 MeV, this is not too restrictive a requirement. However, when we change the rf frequency from 200 Mc/sec to 800 or 1,000 Mc/sec at around 200 MeV, the beam size (relative to the size of the stability region in the longitudinal phase space) is increased by a factor of 4 or 5. It is quite possible that we might subsequently lose some particles at higher energies.

It should be emphasized here that, in order to investigate how particles are lost, radial and longitudinal motions must be coupled together. Particles which are "unstable" in the longitudinal space might travel a long way before getting lost radially. Also, we cannot assume that all particle velocities are very

* The programs described here have been developed by J. N. Vitale, M. Lockerd, T. Ludlam and R. Bakeman.

close to the synchronous velocity since this is true only for (longitudinally) stable particles. This means the longitudinal transit time factor depends not only on the geometry of drift tubes (or irises) but also on the velocity of each particle.

We have decided to employ two different programs for simulating particle motions, one for the drift tube sections (low-energy region, up to 200 MeV) and the other for the iris sections (high-energy region). Fundamentally, of course, the equations of motion are the same but arrangements of focussing quadrupole magnets will be quite different.

In the low-energy region, unless we use very short tanks, quadrupole magnets must be placed in drift tubes. The length and the distance of magnets changes roughly in proportion to the synchronous velocity. Undoubtedly, some kind of matching magnets would be required between two adjacent tanks. Beyond 200 MeV, it is best to place magnets outside of each tank and the geometry of the focussing system will be the same along the entire length of this section.

Drift Tube Section

The program can be used for any magnet arrangement (distance, length and strength); it can also handle an intentionally tilted rf field (like the HILAC at Berkeley). Once the geometry of the drift tubes is selected (for example, Gluckstern's shaped drift tubes), we can calculate the magnitude of the fundamental component of the rf accelerating field, i.e., the longitudinal transit time factor for the synchronous particle. For particles with a different velocity, we use 9 parameters for interpolation formulas in each tank to modify the longitudinal transit time factor. These parameters can be easily calculated if the rf field is known on the axis. Strictly speaking, this modification should be done for all values of r (radial distance from the axis), the correction being different for different values of r. So far, however, the program uses only $I_o(k_1 r)$ (where $K_1 = (2\pi)/(\gamma_s \beta_s \lambda)$) as the "radial" transit time factor regardless of particle velocities. The approximation here is very good for particles inside the "fish" (the velocity is close to the synchronous velocity) and particles near the axis. However, it is not too clear how good this approximation is for particles which are very slow and eventually are lost radially. On the other hand, because of the rapid change of phase, such particles receive practically no net acceleration and the magnitude of the accelerating electric field might be relatively unimportant.

In Figs. 1--5, the velocity dependence of the longitudinal transit time factor is shown for three different spatial distributions of the rf field.

As the particle energy increases, the defocussing action of the accelerating field will decrease and we don't have to place magnets in every drift tube. When different magnet arrangements are used in two adjacent tanks, some kind of matching system (most likely a triplet) will be necessary between them in order to change the transverse emittance shape from one tank into another shape which is similar to the acceptance shape of the following tank. In connecting two tanks, our program assumes a 100% matching, although a design of such a system is by no means trivial.

Iris Section

Here the particle energy is sufficiently high so that focussing magnets can all be placed outside of tanks. One possible arrangement is shown in Fig. 6, where a doublet is used in order to have a rather big space in the middle for other equipment. If a triplet is used, it will be difficult to have such a space without increasing the tank-to-tank distance. Also, a shorter separation of quadrupole elements generally requires a stronger magnetic field. For the configuration shown in Fig. 6, the optimum values of β_{max} and g are given in Fig. 7 as a function of the rf defocussing parameter, Θ. Here β_{max} is the parameter defined in Courant and Snyder[1]; g is related to the magnet strength

$$H(\mathrm{kG/cm}) = 0.31295\ (g/0.15)^2\ (\gamma_s\ \beta_s)$$

and Θ is defined by

$$\Theta = \frac{L}{\lambda}\left(\frac{\pi e \epsilon_o \lambda(-\sin\varphi_s)}{m_o c^2 (\gamma_s \beta_s)^3}\right)^{1/2}$$

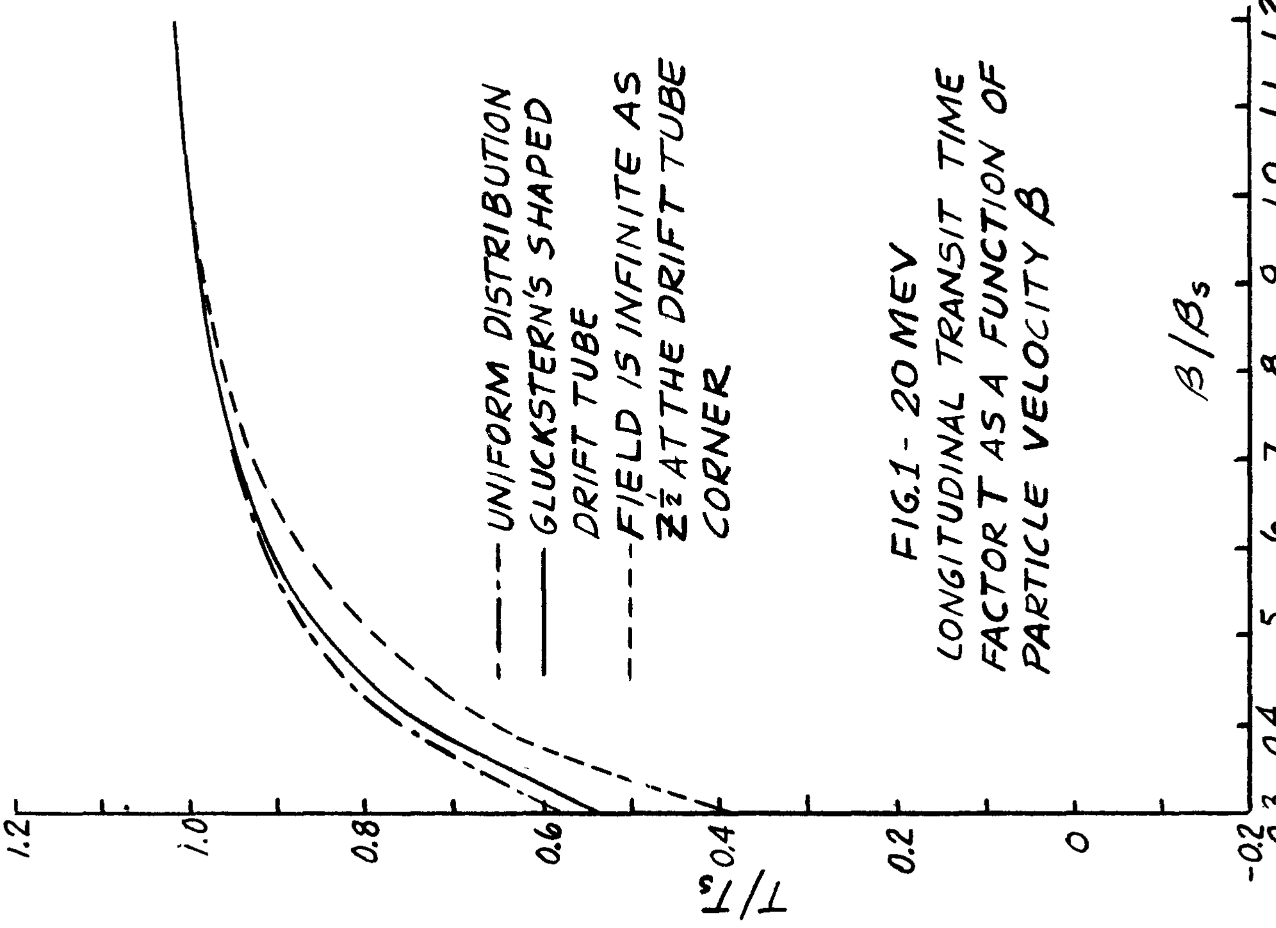

FIG.1 - 20 MEV
LONGITUDINAL TRANSIT TIME FACTOR T AS A FUNCTION OF PARTICLE VELOCITY β

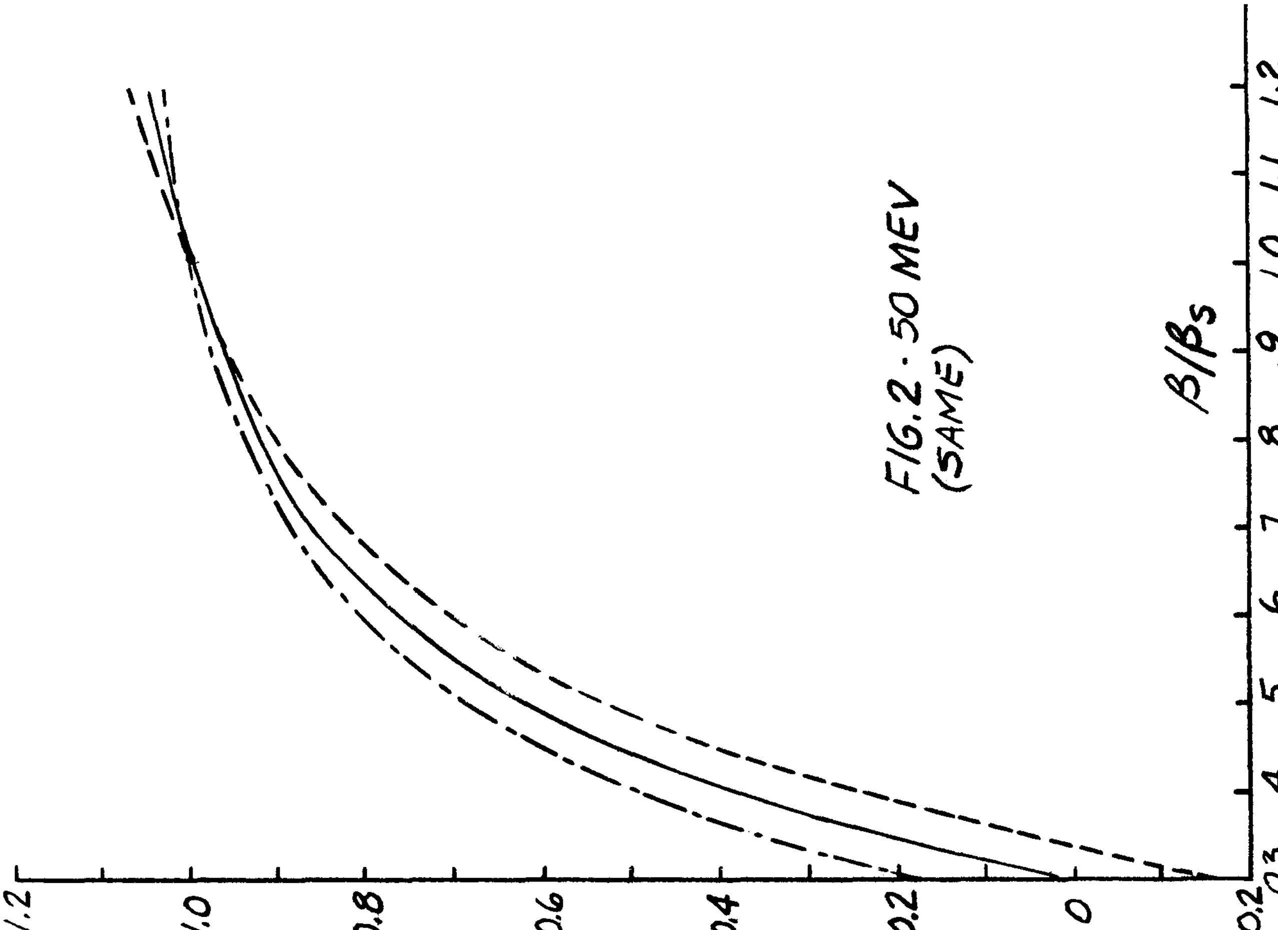

FIG.2 - 50 MEV
(SAME)

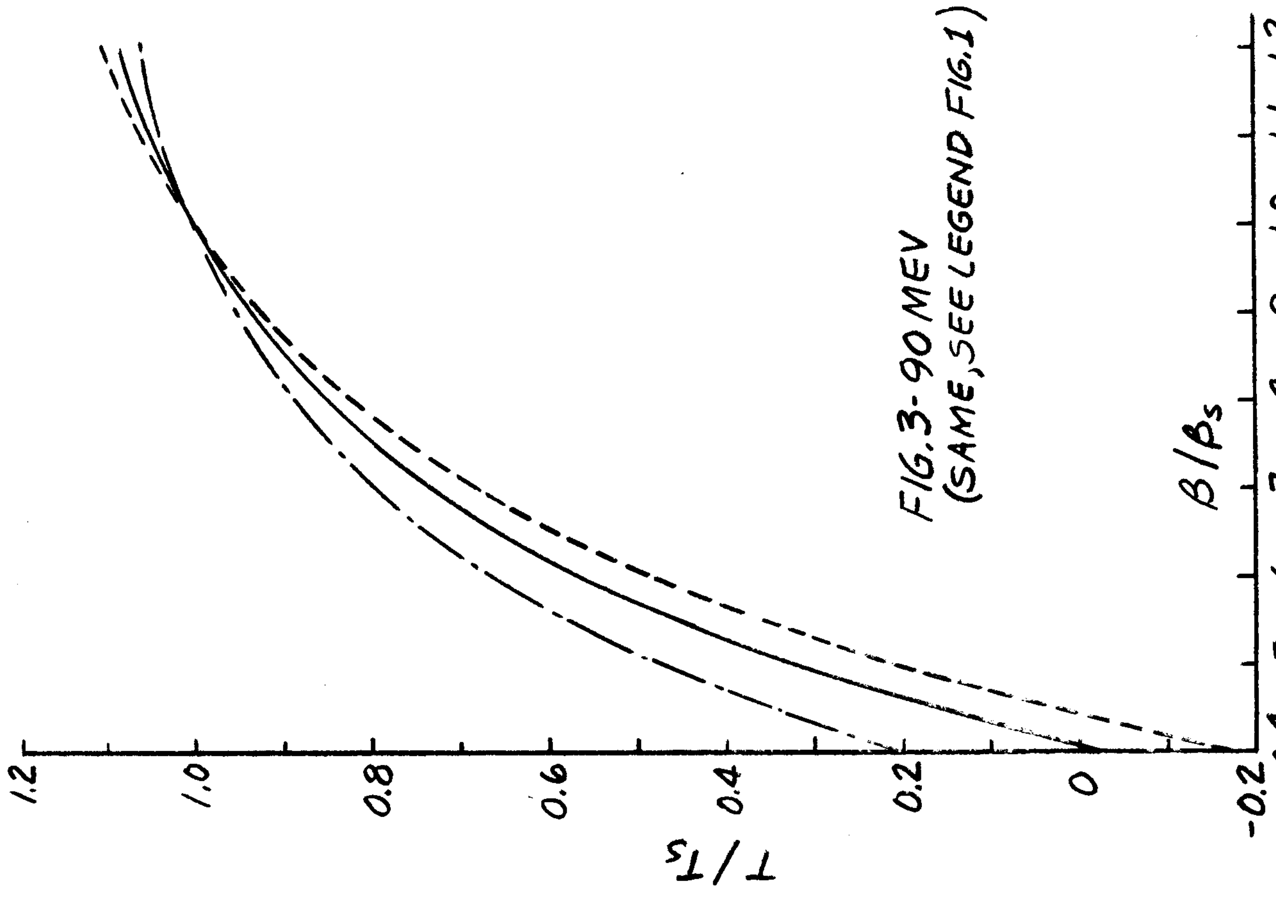

FIG.3- 90 MEV
(SAME, SEE LEGEND FIG.1)

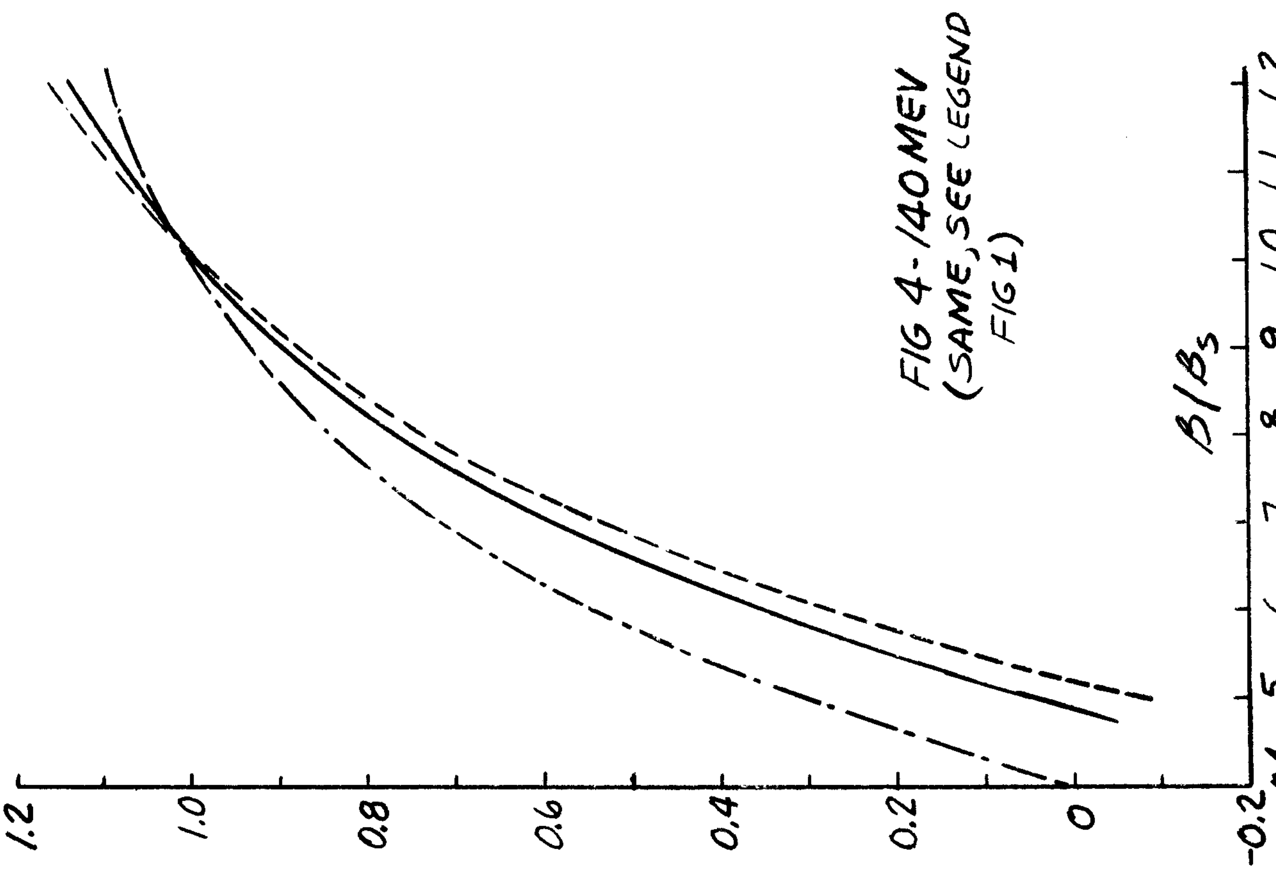

FIG 4- 140 MEV
(SAME, SEE LEGEND FIG 1)

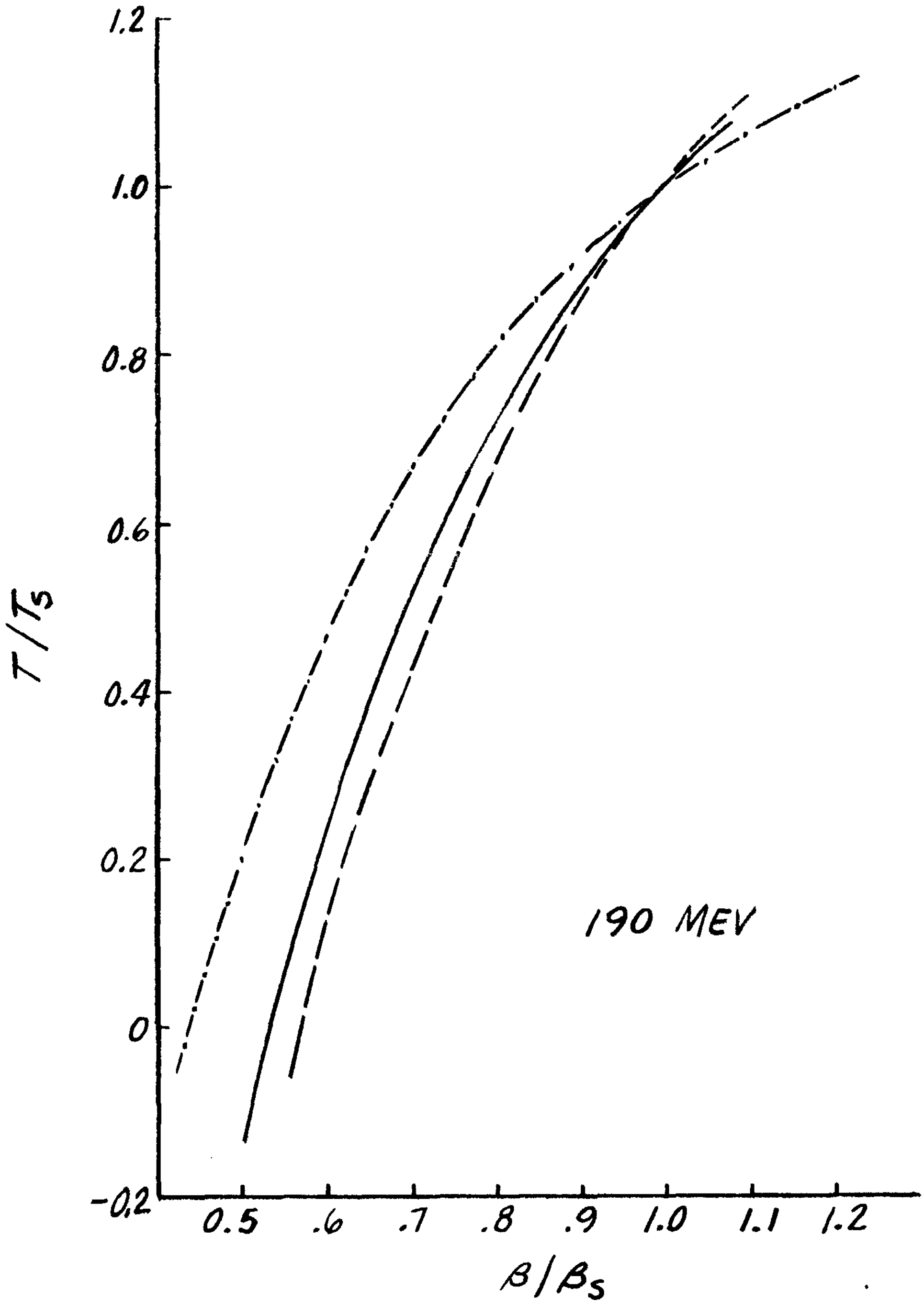

FIG. 5

(SAME, SEE LEGEND FIG. 1)

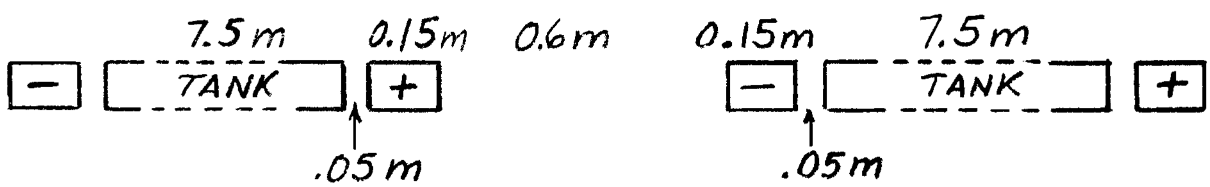

FIG. 6
QUADRUPOLE DOUBLET ARRANGEMENT
IN THE IRIS-LOADED WAVEGUIDE

where L = tank length, and φ_s = synchronous phase and

$$\frac{d\gamma_s}{dz} = \frac{e\varepsilon_o}{m_o c^2} \cos \varphi_s$$

In terms of gradient, this system requires from 0.95 kG/cm to 2.2 kG/cm for 190 to 1,000 MeV. If the transverse phase space area is πA meter-radians, the maximum transverse deviation is roughly $(A \cdot \beta_{max})^{1/2}$ meters. For example, assuming 10 π cm-mrad at 750 keV, we expect a maximum excursion $\simeq$ 1.3 cm in the iris section. However, it should be emphasized here that this value is based on a perfect matching of the beam shape (in the transverse phase space) to the acceptance of the iris section. To what extent this could be achieved depends on how well one would be able to measure the beam shape from the drift tube section.

We have just started using the programs but we hope the following points will be clarified soon without any difficulty:

(1) The best arrangement of focussing magnets in the drift-tube and iris sections, and the acceptance and emittance of each section.

(2) How particles are lost, i.e., whether uniformly along the machine or mostly in a relatively small region.

(3) Is the longitudinal size of the outcoming beam from the drift-tube section small enough to permit five times expansion (200 Mc/sec to 1,000 Mc/sec)?

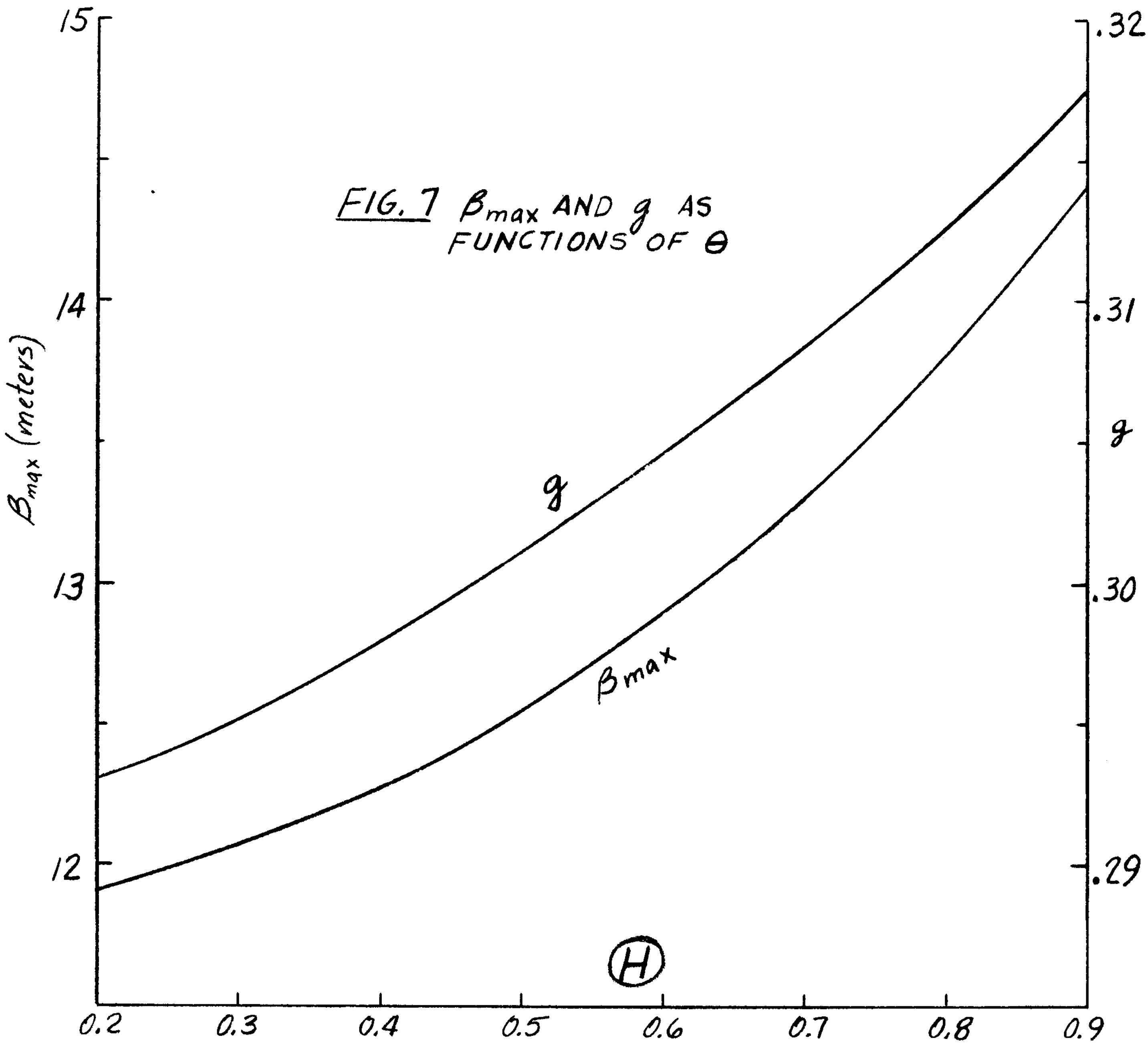
FIG. 7 β_{max} AND g AS FUNCTIONS OF Θ
15
14
13
12
β_{max} (meters)
.32
.31
.30
.29
g
g
β_{max}
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Θ

(4) Effects of the initial longitudinal position on the size and the shape of the transverse acceptance. How important is it to concentrate the beam (initially) near the synchronous point?

LEISS: Have you considered putting quadrupole triplets between sections? Studies made at SLAC indicate the alignment tolerance of the machine with quadrupole triplets and quadrupole doublets between sections is radically different.

OHNUMA: You refer to the calculation by Dick Helm. We haven't done anything about triplets. We don't want to take too much space between sections because for the 750 MeV machine there are about 60 tanks, and even if you take only 1 meter, you have 60 meters of drift sapce. The engineers have asked for a big space in the center part of the drift space, for vacuum pumps, current transformers, etc. We have attempted to set the distance between two magnets at something like 60 cm out of 1 m. I asked several people about magnet optical parameters of the triplet case, compared to the doublet case, but as far as I know nobody has done any extensive studies. I suspect that the improvement is not big enough to go to the triplet combination.

LAMB: As I understand the SLAC report it is a question of alignment tolerances.

HUBBARD: There seem to be only one or two out of the many different kinds of misalignment which are helped by triplets. The others are not affected. One advantage is that the principal planes are in the center so that if there is cocking, it doesn't make much difference.

BERINGER: Isn't this just a thin lens approximation? In other words, if you run an astigmatic combination with the principle plane in the middle, you have effectively, a thin lens path in both planes and naturally it is going to be rather insensitive to that type of misalignment.

COURANT: Doesn't this depend on what kind of errors and correlations in the misalignment you assume between different elements of doublets and triplets?

FEATHERSTONE: I assume that in your system, doublet or triplet, the components are all aligned with respect to each other and you just align this one system with respect to the machine.

BERINGER: It seems to me that there are two parts to the radial transit time factor which should be separable in a simple way. One part has to do with the periodicity of the structure, and I disregard this. The other part is the truly radial part which relates to the geometry of the gap. This part has never been treated in a satisfactory manner in the literature, although the possibility certainly exists today (with either the MURA calculation or Gluckstern's calculation) of making a correct geometrical treatment of this part, and including it in the calculation of the beam dynamics.

OHNUMA: I think it is largely a matter of how much computer time you are willing to use. One can prepare a tape which is a complete description of the fields and feed it into a computer. On the other hand, you could proceed step by step, representing fields by one parameter, two parameters, and so on. In our program, right now, particles on the axis ($x = y = 0$, $x' = y' = 0$) are treated exactly, regardless of their velocity. This means we take not only the fundamental component but all components of the Fourier series. For particles off the axis, the treatment is exact only if $\beta = \beta_s$ because only $I_o(K_1\ r)$ or $I_1(K_1\ r)$ comes into the transit time factor. This is not true if the particle velocity is quite different from the synchronous velocity. We have to take all terms with $I_o(K_m\ r)$ or $I_1(K_m\ r)$. We could do the ideal thing, of course, but it would be very time consuming.

BERINGER: I'm suggesting that there is a geometrical separation which shouldn't be too difficult. Is this an incorrect assumption?

BLEWETT: I don't think so. What about the possibility of just representing the fields in the neighborhood of the axis where we can do something with analytic expressions?

GLUCKSTERN: We have talked about doing what Blewett suggests. You can certainly get the radial and longitudinal fields off the axis from the z-dependence of the field on the axis. In principle, all you need is a one-dimensional field pattern. Even that might be represented by a few terms in the Fourier expansion so you may still get away with, say, eight coefficients from which you can calculate everything. It would be messy but possible. Certainly it would be less troublesome, I think, than trying to put in a complete field pattern.

MILLS: If you do the calculation as Ohnuma described, you use one parameter for the energy gain of the on-axis particle at the synchronous energy. If you ask for the radial momentum transfer of the synchronous particle then you find that there is another coefficient which describes this, the "coupling coefficient" which is tabulated in the MURA calculation.

GLUCKSTERN: But this must work only in the vicinity of the synchronous energy?

MILLS: Probably so.

OHNUMA: Particles are lost mainly when their velocity is much smaller than the synchronous velocity. I'm afraid those two parameters alone wouldn't be enough for such particles.

BLEWETT: There is another troublesome little point of this sort when you are working with the high energy part of the machine. You find that quite a lot of particles, at least as far as phase motion is concerned, can be trapped in $2\beta\lambda$ mode.

COURANT: I have done some computation on this $2\beta\lambda$ mode for the particular geometry of the Brookhaven linac. In this computation a very simple-minded version of what we have heard here today was employed. We compute the phase motion with a transit time factor on the axis and then compute the transfer matrices for horizontal and vertical motions with quadrupoles. I found that we had particles trapped in the $2\beta\lambda$ mode very nicely as far as phase motion is concerned, but for the transverse motion the beam is highly unstable.

VAN STEENBERGEN: In connection with phase spills from the bucket, what would be the maximum permissible length between tank sections, especially at low energies? I think you would lose particles if you have too large a distance between sections.

OHNUMA: The maximum permissible length depends not only on the energy but also on the initial bunched beam. If it has a long tail to begin with, and you don't want to lose any particles then you shouldn't have, in principle, any drift space between the tanks. This is, of course, not practical. If the drifting space is, say, less than 2 m, practically all particles would stay in the bucket but this is not good enough for our purpose. We are primarily concerned with the loss of particles at higher energy where damage is serious. Therefore, we want to make the bunch as small as possible. You must remember that it will be increased by a factor of 4 to 6 when the rf frequency is changed. For example, if we take 1 meter of space between the first and the second tank (~10 MeV), the final phase spread is, for a particular bunch we are using, ±.095 radians at 190 MeV, whereas if there is no space there, the phase spread is ±0.75 radians. After the frequency change from 200 Mc/sec to 800 Mc/sec, it will be ±.38 and ±30 radians, respectively. We feel this is important enough to take into consideration.

References

(1) E. D. Courant and H. S. Snyder, Annals of Physics 3, 1 (1958).