October 21, 1963

THE EFFECTS OF PHASE AND FIELD ERRORS ON THE LONGITUDINAL MOTION IN A LONG PROTON LINAC

G. W. Wheeler and T. W. Ludlam
Yale University

In present drift tube linacs, errors in fabrication, alignment and field level can cause the beam to grow rather than damp in both longitudinal and transverse phase space. Since some errors will inevitably be present, at best the damping will not be as good as in a perfect machine. In accelerators where phase stability is important, as it is for proton linacs below 1 GeV, such uncontrolled growth can lead to the loss of particles from the bunch. In a high energy proton linac, this problem becomes much more severe if there is a change in frequency along the structure. It is fairly well established that 200 Mc/sec is the highest convenient frequency for low energy machines. However, for energies above about 200 MeV, the increase in shunt impedance with frequency dictates a change to a higher frequency since higher frequency structures appear practical to fabricate for proton velocities near and above $\beta = 0.5$.

This transition in frequency causes the phase spread of the bunch as measured at the new frequency to increase by the ratio of the new frequency to 200 Mc/sec. Thus, a bunch with a spread of 15° at 200 Mc/sec will have a spread of 60° at 800 Mc/sec or 90° at 1200 Mc/sec. Most of the phase oscillation damping occurs below 200 MeV. In the ideal machine, the damping only amounts to a factor of about 2 between 200 and 750 MeV. Thus with a stable phase angle between 25° and 30°, one is in trouble at 1200 Mc/sec immediately even without considering the effects of errors.

Several types of errors should be distinguished. Fabricational errors in a section of waveguide will cause the iris spacing to be different from $\beta\lambda/2$ (in the π-mode) and will cause local field variations, both of which will stimulate phase oscillations. However, if these errors are random, they will tend to cancel within one cavity because the cavities will probably be shorter than 1/4 phase oscillation wavelengths (the phase oscillation wavelength is about 100 meters with the low gradients which we have been considering). Radial misalignment of the cavities will contribute directly to the transverse oscillation amplitude. The adjustment of distance between cavities and of the phase difference between the rf signals in adjacent cavities are indistinguishable and can be a major source for stimulating phase oscillations. Incorrect adjustment of the average field level in a cavity will have the same effect.

In designing a high energy linac, particularly of the meson factory type, it is extremely important to assure ourselves that particles are not lost from the beam during acceleration after the particle energy has reached a few

MeV. For an average current of 1 mA, a loss of 0.1% corresponds to 1μA of high energy protons striking the accelerating structure. This is the same current level which is making present day synchrocyclotrons extremely radioactive. In a linac this loss would occur over a length of about 2000 ft. and so would not be so serious, but another factor of 10 would be intolerable.

In order to estimate the degree of precision in phase and field amplitude control required of the rf system and to help in determining the choice of frequency for the high energy portion of the accelerator, we have examined the behavior of the beam in longitudinal phase space when subject to errors in average field amplitude and intercavity phase adjustment. This work does not include all possible types of error nor does it consider the transverse motion at all. However, because of its simplicity, the program uses relatively little computer time. At a later time, a more complete study should be carried out with the programs discussed by Swenson and Ohnuma.

These problems were first considered analytically by Lloyd Smith(1) and then in a manner similar to ours by the Rutherford Laboratory.(2) Our program is tailored to fit the parameters of the very high intensity linac meson factory which we have been considering.(3)

The motion in phase and energy for axial particles in the iris-loaded section of a high energy linac from 190 MeV to a final energy of 1 GeV has been simulated by a program in IBM 709/7090/7094 Fortran. In addition to the stated purpose, this longitudinal motion code will shed some light on the extent to which linear (small amplitude) theory is applicable. The particle motion through the machine is described by the differential equations:

$$\frac{d\gamma}{dz} = \frac{eE_o}{m_oc^2} \cos\varphi,$$

$$\frac{d\varphi}{dz} = -\frac{2\pi}{\lambda}\left(\frac{1}{\beta_s} - \frac{1}{\beta}\right)$$

in which eE_o is the energy gain per unit length and λ is the free space wavelength. These equations are solved numerically at each iris throughout the machine.

Figure 1 shows the region of phase stability ("fish diagram") at 190 MeV. The elliptical bunch of eight particles shown is based upon the output bunch from the 200 Mc/sec drift tube section, and represents a conservative estimate of the boundary inside which all the particles of the actual bunch lie after making the transition to an 800 Mc/sec section. At 1200 Mc/sec a bunch of this size would be difficult to achieve. Except for special cases, this standard bunch was used for all 800 Mc/sec runs. The phase spread of this input bunch is $\pm 30^o$ and the energy spread is $\pm$ 0.79 MeV. The synchronous phase is -0.451 rad (-25.8^o), and the bunch is centered at -0.31 rad (-17.76^o). The rate of energy gain increases from 1.2 to 1.6 MeV/m through the iris section.

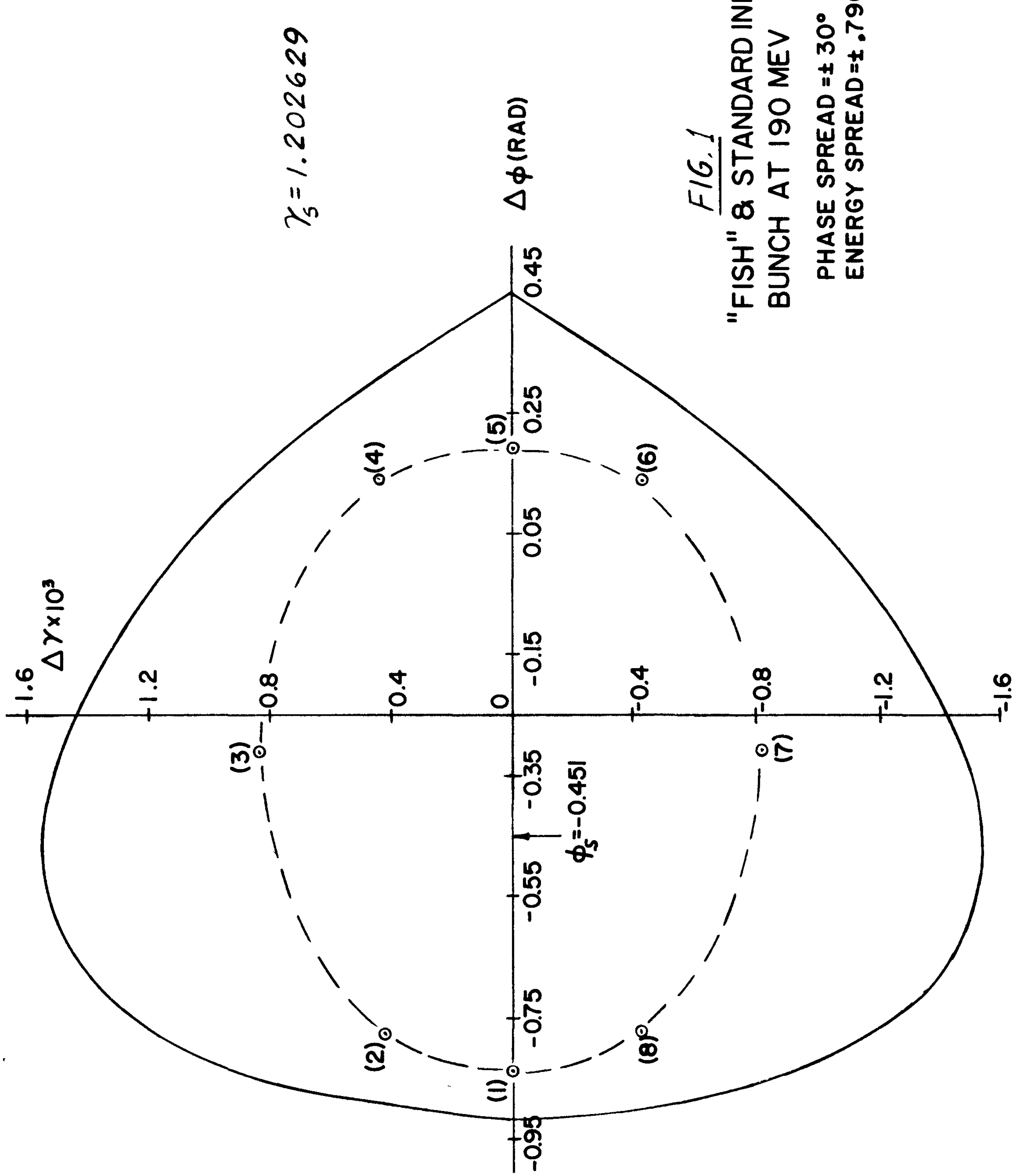


FIG. 1
"FISH" & STANDARD INPUT BUNCH AT 190 MEV
PHASE SPREAD = ± 30°
ENERGY SPREAD = ± .7902 MEV

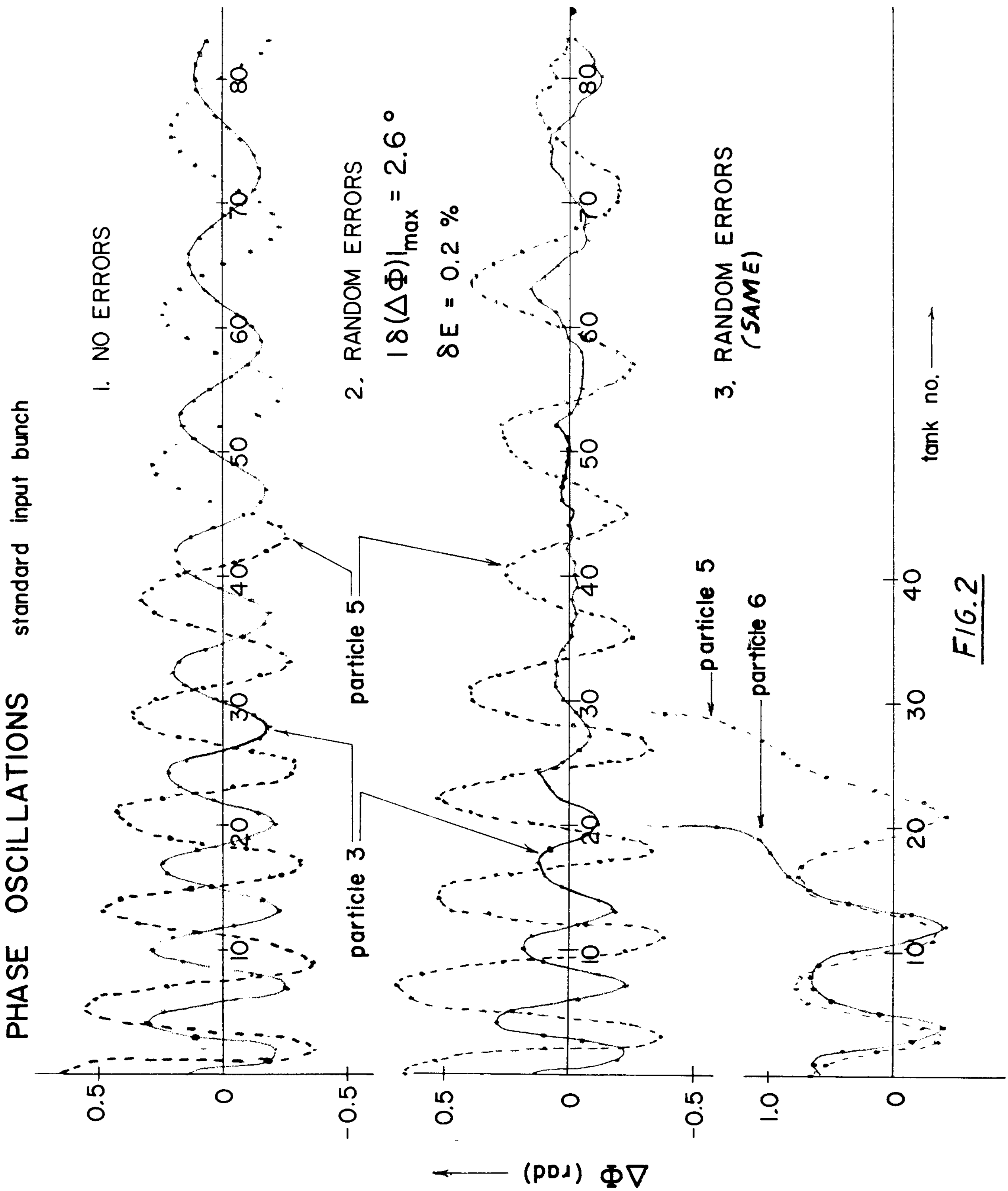

PHASE OSCILLATIONS
standard input bunch
1. NO ERRORS
2. RANDOM ERRORS
$|\delta(\Delta\Phi)|_{max} = 2.6°$
$\delta E = 0.2\ \%$
3. RANDOM ERRORS (SAME)
particle 3
particle 5
particle 5
particle 6
$\Delta\Phi$ (rad)
tank no.
0.5
0
-0.5
1.0
10
20
30
40
50
60
70
80

FIG. 2

Figure 2-1 shows the phase excursion through the machine (no errors applied) for two typical particles -- #3 and #5 of the standard input bunch. According to linear theory, the phase oscillation wavelength is given by:

$$\lambda_\Phi = \frac{(2\pi\lambda m_o c^2 \gamma_s{}^3 \beta_s{}^3)^{1/2}}{eE_o(-\sin\varphi_s)}$$

in which λ (800 Mc) $= 0.375$ m

$m_o c^2 = 938.211$ MeV

$-\sin\varphi_s = 0.43587.$

At 750 MeV (tank #61): $\gamma_s = 1.800$

$\beta_s = 0.835$

$eE_o = 1.573$ MV/m.

Then, in the linear approximation, $\lambda_\varphi(750) = 104$ m.

In Fig. 2-1, the average value of λ_φ between 700 and 800 MeV is 107.2 m for particle #3, and 98.5 m for particle #5.

In the adiabatic approximation, the damping of the phase oscillation amplitude is given by:

$$\frac{\Delta\varphi_1}{\Delta\varphi_2} \cong \frac{(\gamma_{s2}\beta_{s2})^{3/4}\ (eE_{o2})^{1/4}}{(\gamma_{s1}\beta_{s1})^{3/4}\ (eE_{o1})^{1/4}}$$

and that of the corresponding oscillation in energy is

$$\frac{\Delta\gamma_1}{\Delta\gamma_2} = \left(\frac{\Delta\varphi_1}{\Delta\varphi_2}\right)^{-1}$$

For a typical particle (#3, Fig. 2-1) these ratios (taken at the 2nd and 8th peaks) are

$$\frac{\Delta\varphi_8}{\Delta\varphi_2} = 0.535$$

$$\frac{\Delta\gamma_2}{\Delta\gamma_8} = 0.546.$$

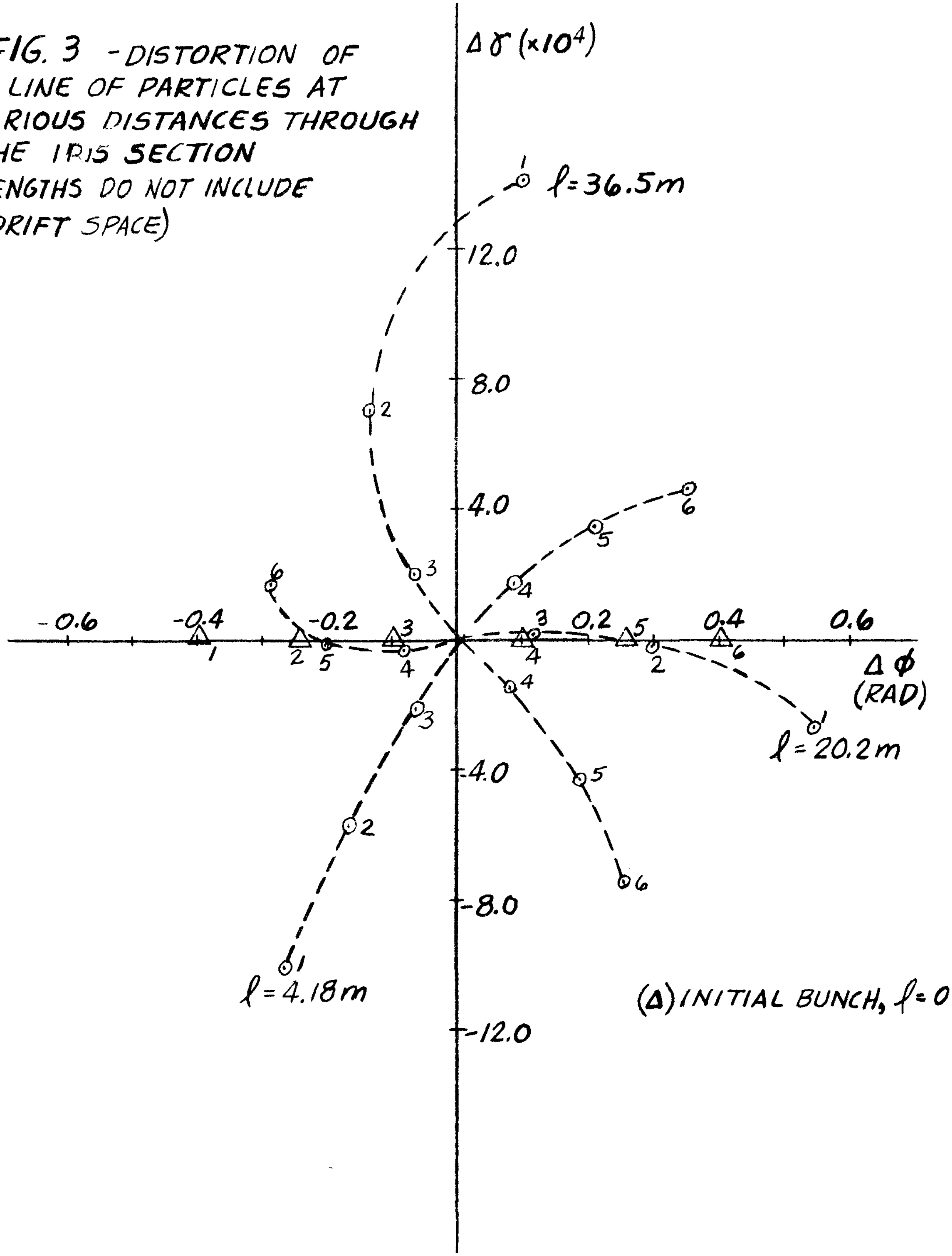
Adiabatic theory predicts that these ratios be equal, and that the value should be that given by:

$$\frac{(\gamma_{s2}\beta_{s2})^{3/4}\ (eE_{o2})^{1/4}}{(\gamma_{s8}\beta_{s8})^{3/4}\ (eE_{o8})^{1/4}} = 0.584.$$

Studies of the relative motion of individual particles through the machine shows an early breakdown of small-amplitude theory; the particle motion becomes markedly non-linear in the first few tanks. Figure 3 shows an initially straight line of particles as it moves through the first five tanks of the iris section. The distortion of the line clearly indicates differing rates of particle rotation and changes of amplitude in different regions of the stability diagram.

In order to study the effect of random machine errors, the code may be made to apply random disturbances to φ

FIG. 3 - DISTORTION OF A LINE OF PARTICLES AT VARIOUS DISTANCES THROUGH THE IRIS SECTION (LENGTHS DO NOT INCLUDE DRIFT SPACE)



(called $\delta\ (\Delta\varphi)$) at the end of each cavity where, $\Delta\varphi$ is the ideally correct phase shift between two cavities, and random errors (δE) to the average field gradient (E) in each cavity. The maximum value of these errors is an input quantity. The program generates random numbers between -1 and +1 and normalizes the magnitude to this maximum value. No correction is made for the small change in the synchronous phase caused by these errors. Eighty runs have been made to 1 GeV using the standard bunch of 8 particles with

$$\left|\delta(\Delta\varphi)\right|_{max} = 2.6^{o}, \quad \left|\delta E\right|_{max} = 0.002(E_{o})$$

where E_o is the field gradient (MV/m).

These numbers were chosen on the basis of what is presently felt to be achievable in practice. Many more runs are needed for accurate statistics, but these are sufficient to indicate whether or not errors of this magnitude are at all tolerable.

The physical significance of applying the phase errors in this way may be seen in Fig. 4a. Here, the phase information for the n^{th} cavity is taken from the $(n-1)^{th}$ cavity. The "correct" phase shift ($\Delta\varphi$) is introduced by a suitable phase shifting device. Since such a device is not perfect, it will have an error, $\delta(\Delta\varphi)$, which is passed to the n^{th} cavity. This error thus shifts all cavities from the n^{th} onward by that amount. Any error between the n^{th} and $(n+1)^{th}$ cavity is not related to the error between the $(n-1)^{th}$ and n^{th}. An alternative method is shown in

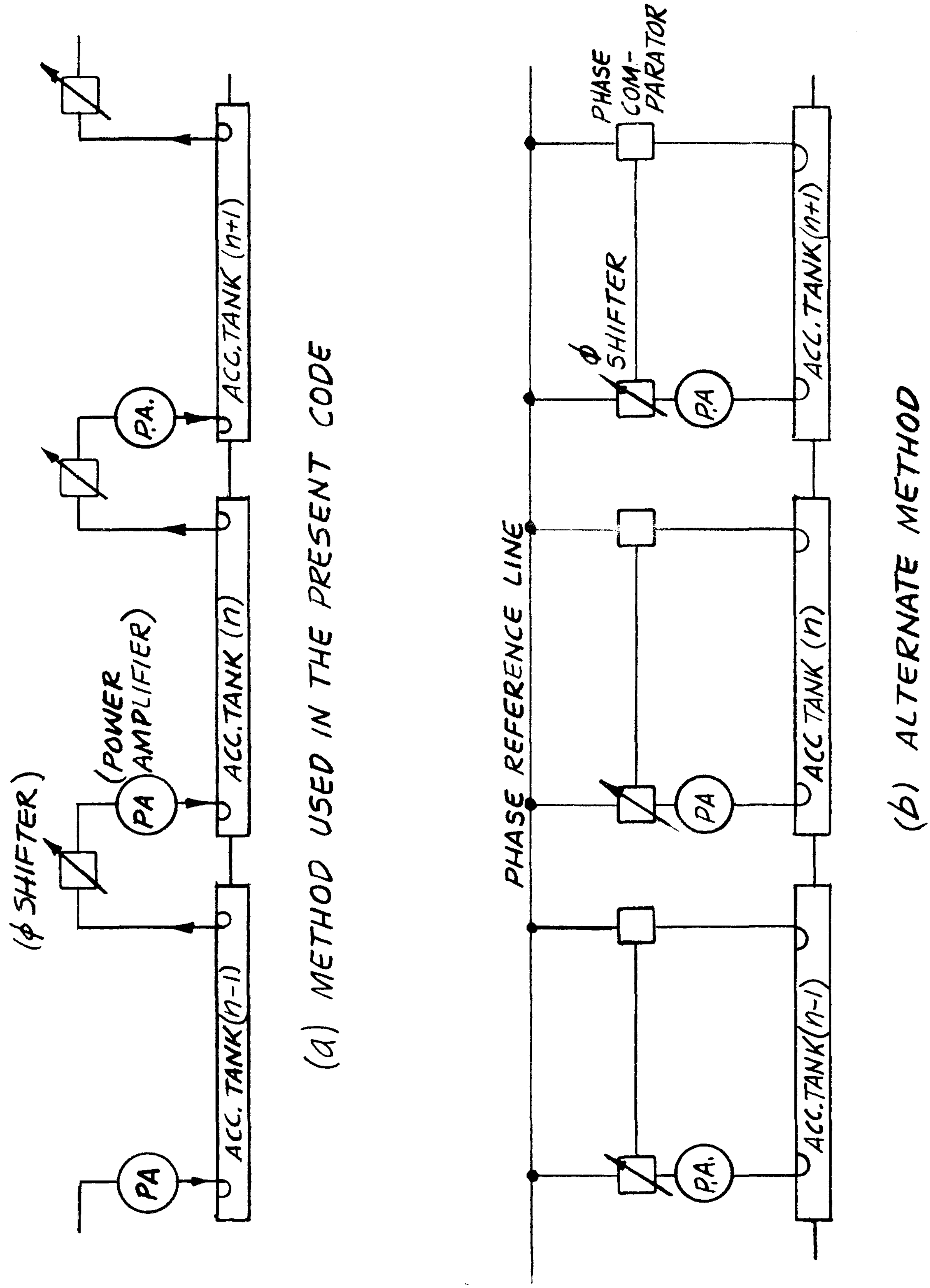


FIGURE 4
METHODS OF INTRODUCING PHASE ERROR

Fig. 4b. Here, an error in the n^{th} phase comparator and shifter will introduce an error of $\delta(\Delta\varphi)$ between n-1 and n while at the same time an error of $-\delta(\Delta\varphi)$ between n and n + 1. We plan to check the effect of this type of system also.

Of the 80 cases run, particles were "lost" as shown in Table I. The criterion for a lost particle is that it leave the phase stable region and cease to oscillate in γ and φ. Such a particle will continue to be accelerated for a short period and will, for a time, remain stable in the transverse motion, and so will not immediately be lost. Some of the "lost" particles may even be recovered by a fortuitous error in some later tank.

TABLE I

Particle	of Times Lost
2	1
4	1
5	4
6	3

(The particles of the standard input bunch are numbered as shown in Fig. 1.) It appears that the center of the bunch ought to be shifted closer to the synchronous phase.

In 20 runs made with $|\delta(\Delta\varphi)|_{max} = 3.5^{o}$, particle #1 was lost once, particle #4 three times, particle #5 twice and particle #6 twice. In ten cases with $|\delta(\Delta\varphi)|_{max} = 1.5^{o}$, no particles were lost.

Thus, it seems that the choice of $|\delta(\Delta\varphi)|_{max} = 2.6^o$ represents the largest amplitude of random error that a bunch of this size can tolerate. (Disturbances on the field given by $|\delta E|_{max} = 0.2$ have almost no effect on the particle motion.)

Figure 2-2 shows the phase motion through the machine for particles #3 and #5 (the same two particles shown in 2-1) with this type of error applied. Figure 2-3 shows the phase motion for two particles which were lost (the values of $|\delta(\Delta\varphi)|_{max}$ and $|\delta E|_{max}$ are the same here as in 2-2).

In addition to random errors, certain types of systematic errors have been investigated. These are disturbances applied purposely to drive a given particle away from the synchronous phase. One of these types is an error, introduced at the end of each tank, whose value varies sinusoidally through the machine with the same frequency as that of the phase oscillation, and with a constant amplitude, called AMP. Thus, the particle sees at the end of each tank a destructive error whose magnitude is roughly proportional to the value of $(\varphi - \varphi_s)$ at that point. The resulting set of errors is far more unfavorable to the particle motion than is likely to occur in reality. A particle with $(\varphi - \varphi_s)$ initially equal to 0.66 rad. (#5 in Fig. 1) can withstand errors of this type with values of AMP up to about 0.01 rad ($\sim 0.6^o$).

A more gentle scheme for systematically throwing a particle out of the phase-stable region is to introduce a destructive disturbance in phase with given magnitude (again called AMP) each time the phase oscillation reaches a maximum or minimum. The particle is then given a destructive "kick" in phase about 14 times during its passage through the machine. A particle initially at $(\varphi-\varphi_s) = 0.66$ rad will tolerate errors of this sort up to AMP = 0.04 rad ($\sim 2.4^{\circ}$).

Some attention has been given to the question of whether the beam can be retained if the field in one tank goes to zero. If the bunch encounters a tank in which E = 0, its γ will drop away from γ_s as it passes through that tank. Each particle will shift downward on the $\Delta\gamma$-$\Delta\varphi$ plot by an amount equal to the design energy gain in the tank. Those particles which are still within the fish after experiencing this drop will continue to be accelerated. Those which are not will be lost. One might hope to help the situation by increasing the field in the tanks adjacent to the down tank, so that the bunch arrives at this tank centered above γ_s and then, after experiencing a drop which presumably carries it below the lower boundary of the fish, is brought back to the phase-stable region by the increased gradient on the high-energy side of the down tank.

Figure 5 shows the energy gain and the fish size (maximum possible deviation from the synchronous energy) in each tank. Up to about tank #53 the energy gain per tank is greater than the extent of the fish in energy, so that no particle which is within the fish at the beginning of the down tank will remain there after passing through it. This implies that even if the bunch is to be retained by increasing the gradient in nearby tanks it will have to be outside of the fish at one time or another. This scheme is possible

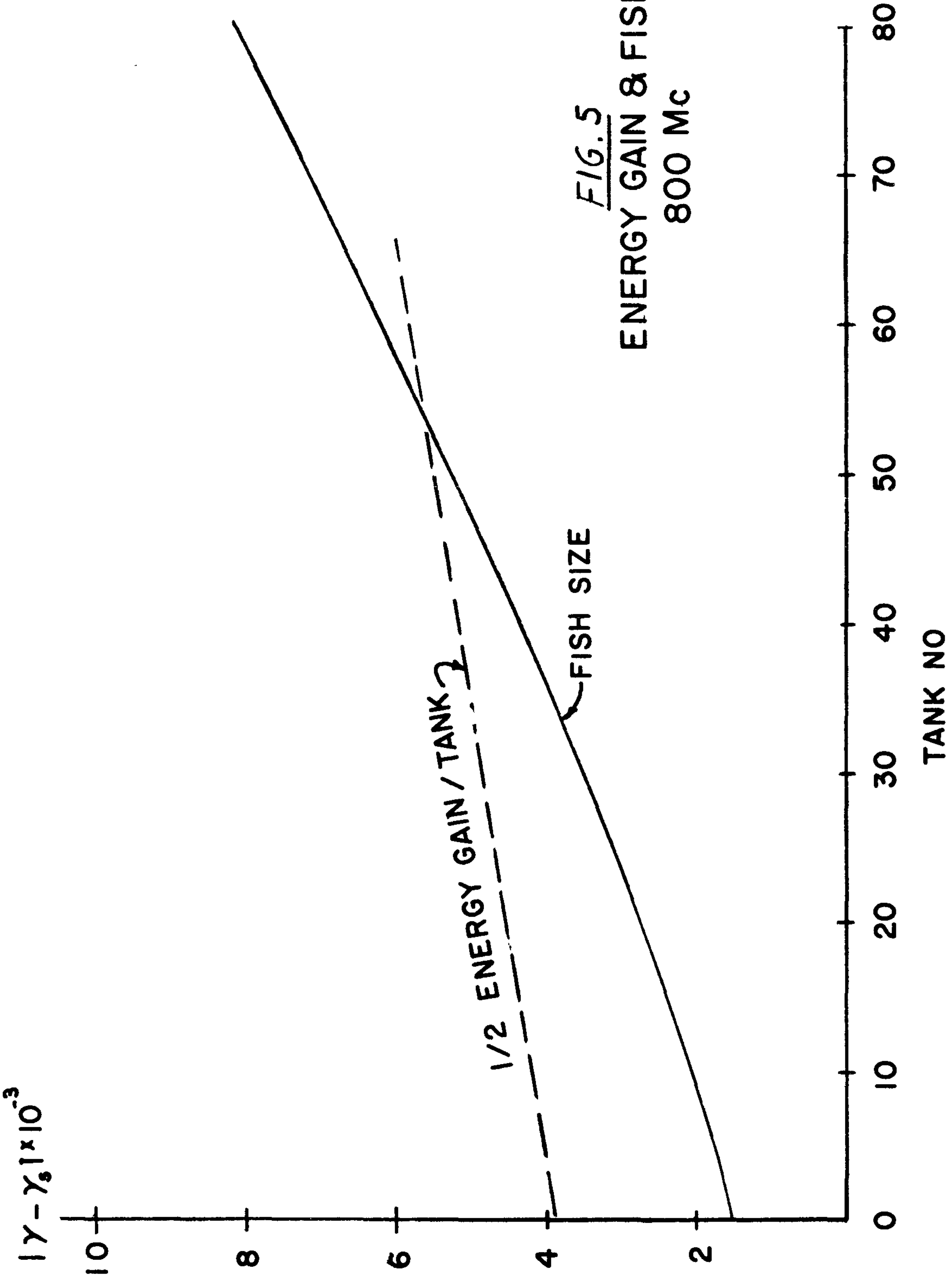


FIG. 5
ENERGY GAIN & FISH SIZE
800 Mc

(and has been carried out with tank #65 down) only if the gradient is increased sharply on either side of the down tank, a procedure which is prohibited by power limitations. With tank #65 turned off, the beam was retained by increasing the gradient by 50% in tanks #64 and #66. Increasing the gradient by smaller amounts over a series of tanks proved unsuccessful.

To sum up, it may be concluded from these investigations that:

(a) Though linear theory does not apply throughout the machine, it is a valid guide in predicting such quantities as phase oscillation wavelength.

(b) An initial bunch of particles with ± 30° phase spread is usable and will tolerate realistic machine errors for an 800 Mc/sec iris-loaded section.

(c) It is apparently impossible to retain the beam through a down tank by adjusting the gradients of other tanks, except perhaps for tanks near the high energy end.

GLUCKSTERN: What's the difference between the second and third diagrams of Fig. 2 as far as particle #5 goes?

LUDLAM: It is a different case for the same maximum errors but with a different set of random errors.

VAN STEENBERGEN: You could have a certain maximum error which would be distributed randomly over all the tanks. You could also have a random distribution of the energy error within the maximum ΔE which is distributed in some organized fashion over the tanks. I think this is a more difficult case.

WHEELER: There are several possible ways of controlling the phase. These will give different results. We haven't explored all the ways of adjusting the machine yet.

HUGHES: How does this result compare with the sort of rms treatment that Lloyd Smith gave?

WHEELER: Actually, these results are generally consistent with Smith's estimate. His estimate, of course, was for shorter tanks and substantially higher gradients, but he came to the conclusion that phase errors could be of the order of $\pm 1^{\circ}$.

BLEWETT: What are the relative importances of the 0.2% field errors and the errors in phase?

LUDLAM: It turns out to be very unimportant. If you run a case with no phase error and a 0.2% field error, and a corresponding case with no errors at all, you can scarcely see the difference. So really whatever happens here is due to the error in phase. The reason that ΔE is chosen so small is that it is felt that in constructing the system, ΔE can be kept within this limit.

OHNUMA: I want to mention that ΔE is the error in absolute average level so that we haven't investigated anything about the tank-flattening.

MILLS: In changing ΔE, do you change it in such a way that the synchronous phase remains the same, or when there is a given error in the excitation of the cavity does that also introduce an error in synchronous phase?

LUDLAM: The effect of the error is to change, slightly, the synchronous phase.

MILLS: Did I understand correctly that you're assuming each tank is perfectly flat?

WHEELER: Yes.

LEISS: One thing, of course, that will cause a correlation between the errors of cavities is the beam loading, since the beam loading wave is tied to the beam and not to the power source. Has this been considered?

WHEELER: This has not been included here. We are certainly aware that this particular problem exists.

GLUCKSTERN: During the early stages of consideration of the problem of these phase and amplitude errors, there was some talk about measuring energy and phase at some stages along the machine and servoing in some corrections. I imagine that because of things like the beam loading, there is eventually going to have to be some feedback. If the machine somehow can recognize when the phase amplitude is building up or the energy has deviated too far from synchronism, and if corrections can be put in rapidly enough so that the particles can be kept oscillating in narrow amplitudes then the tolerances on these errors could be significantly relaxed. What's the current thinking on this?

WHEELER: This would be a very convenient way of doing it. The problem at the moment is how to measure the beam phase. A direct measurement of the beam phase could generate a signal that would compensate for beam loading.

COURANT: If one could have a deflecting cavity properly phased with respect to the accelerating cavity then the amount of deflection introduced would be a function of the beam phase. In principle, something like this could be used to measure the phase.

GLUCKSTERN: Do I understand then that at the present time, one is thinking of doing this by deadreckoning, using some preset value of phases and amplitudes and then if direct methods of measuring the beam phase do prove feasible, these will be incorporated later?

WHEELER: Yes.

LEISS: On the Orsay machine, I know that they use simple cavities for pick-ups between sections. The signals are used for phase control of the machine, and it is not difficult at all to get a big enough signal for phase control.

LAMB: This measures the center of gravity of the bunch.

LEISS: That's all you really care about unless you're trying to detect the phase oscillations building up.

BERINGER: If you're talking about losing particles longitudinally, which we are here, the measuring of the center of gravity alone will tell you nothing.

VAN STEENBERGEN: Dr. Blewett calculated the acceptance for the Brookhaven linac and finds that it is slightly different from the theoretical acceptance. It was found at Brookhaven that an injection energy which is slightly different (higher) from the theoretical one is better. This is only for one tank. If one has now several tanks in series, can one still proceed to write a program completely realistically in view of this changed optimum injection energy?

WHEELER: The effect you're referring to is, of course, for injection at $\beta \simeq .04$ in the Brookhaven machine. I think that the distortion in the stable region which you observed in the Brookhaven machine is probably washed out by 200 MeV. I don't think you'd see much of this.

References

(1) L. Smith, LRL Internal Report LS-3, November, 1960.

(2) R. B. R-Shersby Harvie, "Minutes of the Conference on Linear Accelerators for High Energies" BNL, August, 1962.

(3) Design Study Staff, Yale University, Internal Report Y-6, October, 1962.