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BEAM LOADING EFFECTS IN PROTON LINACS

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Introduction

A bunched beam of charged particles passing through a cavity interacts with the fields in the cavity. It couples, in addition, to other resonant modes, exciting fields which may react on the beam. The present discussion is centered on two effects, the interaction with the externally applied accelerating field and subsequent reaction on the beam, and the generation of transverse modes capable of deflecting the beam.

Interation of the Beam with the Accelerating Mode

Effect of Beam on Cavity Impedance -- Equivalent Circuit:

It can be shown that the equivalent circuit of the cavity, loaded by a periodic beam pulse is as shown in Fig. 1.

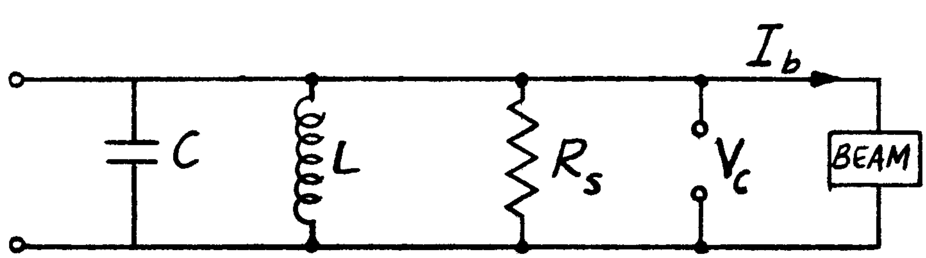

Figure 1

The quantities R_s, L and C refer to the unloaded resonant cavity, related to the resonant frequency, ω, and Q by

$$\omega^2 = \frac{1}{LC} \; ; \qquad Q = \frac{R_s}{\omega L} = \omega C R_s \tag{1}$$

and the current I_b is an equivalent impressed beam current given by

$$I_b = f_b \, I_o \, e^{-i\varphi_b} \tag{2}$$

where I_o is the average beam current (during the pulse), $f_b I_o$ is the component of the beam current pulse resonant with the cavity and φ_b is the phase of the center of the beam pulse (assumed symmetric for convenience) relative to some fixed standard in the power source.

The elements L and C play a major role in the transient response of the cavity to the incident power. The usual method of compensating for beam loading in the transient response of short pulses consists of injecting the beam as the cavity voltage rises. The complications of these transient phenomena will not be investigated here. Instead we shall confine our attention to the steady-state response of the cavity to the beam pulse. In these considerations the elements L and C may be omitted.

Two models have been considered for the power source. The first consists of a generator in series with some sort of internal or matching impedance.

The second consists of a generator fed through an isolator. In either case one finds a relation of the form:

$$V_c = V_o - Z_{eq.} \quad f_b I_o e^{-i\varphi_b} \tag{3}$$

where V_o (taken to be real) is the steady state cavity voltage in the absence of beam loading and $Z_{eq.}$ is an equivalent impedance consisting of the parallel combination of the shunt resistance R_s and the internal impedance in the first model, or the transformed characteristic guide impedance in the second model.

Beam Dynamics with Beam Loading: The reaction of the beam to the modified cavity field depends on the magnitude $|V_c|$ and phase φ_c of the cavity accelerating voltage. The beam reacts in fact only to the combination $|V_c|$ cos $(\varphi_b - \varphi_c)$ which describes the energy gain of each bunch in the cavity. From Eq. (3) one finds

$$|V_c| \cos(\varphi_b - \varphi_c) = \mathrm{Re}(V_c e^{i\varphi_b}) = V_o \cos \varphi_b - f_b I_o R_{eq.} \tag{4}$$

and $R_{eq.}$ is the real part of $Z_{eq.}$ If the design voltage gain and synchronous phase are V_{os} and φ_s, the deviation in voltage gain from synchronous is

$$\Delta V = V_o \cos \varphi_b - f_b I_o R_{eq.} - V_{os} \cos \varphi_s \tag{5}$$

The beam therefore acts as if there were a new synchronous phase given by

$$\cos \varphi_{bs} = \frac{f_b I_o R_{eq.} + V_{os} \cos \varphi_s}{V_o} \tag{6}$$

and performs oscillations about this phase, the limits on phase stability being those associated with φ_{bs}. If one wants to retain a value φ_s as the loaded synchronous phase it is then necessary to modify V_o as the current builds up according to the relation

$$V_o = V_{os} + \frac{f_b I_o R_{eq.}}{\cos \varphi_s} \tag{7}$$

In a matched situation, $R_{eq.} = R_s/2$. In this case one has

$$\frac{\Delta V_o}{V_{os}} = \frac{f_b I_o R_{eq.}}{V_{os} \cos \varphi_s} = \frac{f_b}{2\cos^2 \varphi_s} \cdot \frac{I_o V_{os} \cos \varphi_s}{V_{os}{}^2/R_s} \tag{8}$$

In this form it is obvious how the fractional increase in source voltage is related to the ratio of beam power to cavity power. (For a tightly bunched beam $f_b = 1.00$.)

According to the scheme just presented, one will have to modify each power source as the beam increases in a way which depends on $f_b I_o$ and $R_{eq.}$, quantities which will vary along the accelerator. Clearly, one should learn a great deal about the properties of the power source and the way in which its voltage and phase are controlled. Numerical studies of the beam-loaded dynamics will then be possible. Further study of this problem is obviously called for.

Interaction of the Beam with the Deflecting Mode

The phenomenon of beam blowup in travelling wave electron accelerators now appear to be well understood.[1,2] The bunched beam couples to a transverse band and generates a wave with approximately the same phase velocity as the accelerating mode. This mode causes a deflection of the beam. As additional beam enters the guide the fields build up further until they are capable of deflecting the beam into the irises. According to this understanding, the pulse length before beam cut-off is inversely proportional to the beam current, as is confirmed in observations.

Considerations of this type appear at first to be not directly applicable to standing wave accelerators, where there is only a discrete spectrum of deflecting mode phase velocities. In addition, it appears more desirable to construct the stimulated fields from the complete spectrum of resonant modes. Nevertheless, the phenomenon of beam blowup also occurs for standing wave accelerators. Our purpose is to explore beam blowup in a standing wave accelerator, and to discover which parameters are relevant, and in what way any damaging modes can be suppressed.

Qualitatively, what happens when a beam pulse enters a cavity is that it couples to many modes. When the pulse leaves the cavity, oscillations have been built up and continue, in the no-loss approximation. A subsequent pulse will cause additional oscillation. If the separation of pulses is just right, the fields from successive pulses will be in phase and will lead to a build-up of fields which may then interact with the beam pulses. If it happens that the beam couples appreciably to the modes in a tranverse band, the result may be that the build-up of transverse fields occurs to the point where the beam is deflected into an iris.

Quantitatively one may estimate the effect as follows. If the beam consists of pulses separated in time by $2\pi/\omega_o$ with an average current I_o, the charge in each pulse is $2\pi I_o/\omega_o$. The motion of this single pulse of charge through the cavity leads to a build-up of field proportional to

$$E_t(z,t) \sim \frac{i\pi I_o}{\omega_o \epsilon} e^{i\omega_j(t-t_o)} E_t^j(z) \int_0^L E_t^j(z') e^{-\frac{i\omega_j z'}{v}} dz' \qquad (9)$$

where ω_j is the frequency of the troublesome mode, t_o is the time at which the pulse enters the cavity, and $E_t^j(z)$ are the normalized transverse cavity fields.

Successive pulses clearly lead to fields which are out of phase by ω, $2\pi/\omega_o$ or by

$$2\pi \left[\frac{\omega_j}{\omega_o} - n \right] \qquad (10)$$

where n is the closest integer to ω_j/ω_o. The maximum field amplitude will therefore occur after the order of

$$M = \frac{\omega_o}{\omega_j - \omega_o n} \tag{11}$$

pulses and will lead to a field amplitude M times as large as that given in (9).

The transverse deflection due to this field will be given by integrating $E_t(z,t)$ at values of t given by $t = z/v + t_o'$. This will lead to a transverse momentum of the order

$$\frac{\Delta p}{p} = \frac{1}{p} \int \frac{dz}{v}\, e\, E_t(z, \frac{z}{v} + t_o') \simeq \frac{M I_o e}{p v} \frac{\left| \int_o^L E_t^j(z') e^{-\frac{i\omega_j z'}{v}} dz' \right|^2}{\omega_o \epsilon \int \left| E_t^j(z') \right|^2 dv} \tag{12}$$

where the denominator has been included to allow an arbitrary normalization of E_t^j. The combination involving E_t^j is a transverse equivalent to the quantity $R_s/Q = rL/Q$, where $r = R_s/L$ is the shunt resistance per unit length.

Equation (12) can be put in a form for comparison with previous results for backward wave oscillator theory[2,3]. The typical number of pulses contributing to blowup can be written in terms of the group velocity as

$$M = \frac{\omega}{\Delta\omega} = \frac{\omega}{\Delta\beta v_g} \sim \frac{\omega L}{v_g} \tag{13}$$

since the difference in propagation constants $\Delta\beta$ for adjacent modes is π/L. Writing $pv \simeq eV_o$ corresponding to the beam energy one has

$$\frac{\Delta p}{p} \sim \frac{\omega L}{v_g} \frac{I_o}{V_o} \frac{rL}{Q} \qquad (14)$$

The "starting current" I_o is set by limiting the angular deflection to of order λ/L. This leads to an expression similar in form to that given in both references, providing reassurance that the phenomenon is understood qualitatively in both standing wave and travelling wave terms.

One can now return to Eq. (12) for consideration of the seriousness of the effect. A crude estimate of the magnitude of the angular deflection for a single proton linac section being considered for the Yale meson factory is of the order

$$\Delta\theta \simeq \frac{\Delta p}{p} \sim \frac{10^{-5} \text{ to } 10^{-6}}{(\omega_j/\omega_o) - n}$$

Since angles of the order of 10^{-3} radians are present in the transverse focussing system, only a resonance of the transverse mode frequency with the beam bunch frequency within 0.01 of an integer will lead to difficulty. In view of the fact that the physical parameters of each section will be different due to changing velocity, it is highly unlikely that any resonance will be serious for more than one or two sections.

To summarize, we have formulated the phenomenon of beam blowup in a standing wave proton linac in such a way that the expressions resemble those previously obtained for beam blowup in travelling wave electron linacs. In view of the existence of (1) a transverse focussing system and (2) changing parameters within a section and from section to section, the effect is not so serious in proton linacs. In addition, if an accidental resonance should occur, methods exist for suppressing or moving the transverse band to alter the resonance condition.

Studies are continuing to determine the precise form of the beam-field coupling, the effect of magnetic fields and the influence of other neighboring modes which reflect the dispersive character of the structure. It is hoped that the estimates can be made quantatively more reliable.

Discussion: (Not reported. Ed.)

References

(1) Transient and Beam-loading Phenomena in Linear Electron Accelerators. J. E. Leiss and R. A. Schrack, Internal Report, National Bureau of Standards, Washington, D. C. (October, 1962).

(2) P. B. Wilson, High Energy Physics Laboratory (Stanford) HEPL-297.

(3) See for example Kompfner and Williams Proc. IRS 41, 1602 (1953).