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EFFECT OF ERRORS IN REPETITIVE STRUCTURES

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As is well known, a practical limit on the length of repetitive electromagnetic structures is determined by the sensitivity of the field distribution to local dimensional errors. The theory of mode flattening in drift tube accelerators has been previously developed on the basis of a model with dielectric filler in a cylindrical cavity. This leads to an expression for the variation of axial field with distance of the form

$$\frac{d^2E_z}{dz^2} + \left[\frac{\omega_o^2 - \omega^2(z)}{c^2}\right] E_z = 0 \qquad (1)$$

where $\omega(z)$ is the local resonant frequency of each cell and ω_o is the overall resonant frequency. In terms of the cell to cell behavior, one can write for small errors

$$\frac{\delta^2 E_n}{E_n} \simeq \frac{8\pi^2 \ell^2}{\lambda^2} \left(\frac{\Delta\omega_n}{\omega_o}\right) \qquad (2)$$

where ℓ is the length of each cell, $\Delta\omega_n/2\pi$ is the frequency error in the cell, and δ^2 is the second difference with respect to cell number, n.

The situation in iris-loaded cavities is quite different, since the cells are coupled to one another much more loosely. In order to investigate this more fully, we have assumed the validity of a model whereby the structure is replaced by a sequence of LC circuits coupled inductively, as shown in Fig. 1

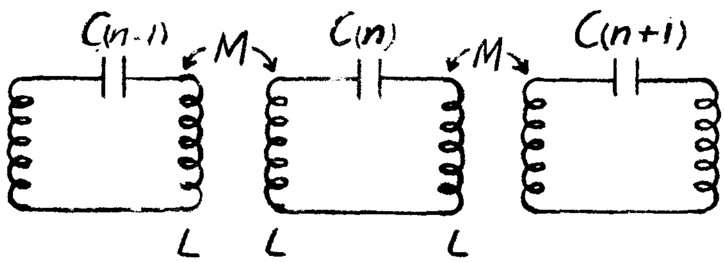

FIG 1

The coupling is kept constant from cell to cell, and the frequency errors are attributed to small variations in the capacitances. For small errors it can be shown that the circuit voltages obey the equation

$$\frac{\delta^2 V_n}{V_n} \simeq \frac{2}{\lambda} \left[1 - \lambda - \frac{\omega_n^2}{\omega^2} \right] \tag{3}$$

where $\omega/2\pi$ is the driving frequency and $\lambda = M/L$ is the coupling constant.

For the zero mode, $\delta^2 V_n = 0$ and

$$\omega_o = \omega_n \, (1 - \lambda)^{1/2} \tag{4}$$

For the π-mode $V_{n-1} = V_{n+1} = -V_n$ and $\delta^2 V_n = -4V_n$. Thus

$$\omega_\pi = \frac{n}{(1 - \lambda)^{1/2}} \quad , \ \pi\text{-mode} \tag{5}$$

The bandwidth is given by

$$BW = \frac{\omega_\pi - \omega_o}{\frac{1}{2}(\omega_\pi + \omega_o)} \cong \lambda \qquad (6)$$

for small coupling.

If all circuits are now resonant at slightly different frequencies, one can write for the 0-mode

$$\frac{\delta^2 V_n}{V_n} \cong \frac{4}{\lambda}\left(\frac{\Delta\omega_n}{\omega}\right) \cong \frac{4}{BW}\left(\frac{\Delta\omega_n}{\omega}\right) \qquad (7a)$$

and for the π-mode, with $V'_n = (-1)^n V_n$

$$\frac{\delta^2 V'_n}{V'_n} \cong \frac{4}{\lambda}\left(\frac{\Delta\omega_n}{\omega}\right) \cong \frac{4}{BW}\left(\frac{\Delta\omega_n}{\omega}\right) \qquad (7b)$$

It can be shown from Eq. (1) for the drift tube structure, that the bandwidth is approximately given by

$$BW \cong \frac{\lambda^2}{8\ell^2} \qquad (8)$$

in which case (1) can be written as

$$\frac{\delta^2 E_n}{E_n} \cong \frac{\pi^2}{BW}\left(\frac{\Delta\omega_n}{\omega}\right) \qquad (9)$$

In view of the fact that (9) has been derived from a continuous structure, and (7a) and (7b) are valid only for small coupling, the agreement is very good. All the expressions confirm the fact that a small bandwidth will lead to large errors. For N cells, one finds for the order of magnitude of the variation in voltage

$$\frac{\Delta V}{V} \sim \frac{1}{BW} \left(\frac{\Delta\omega}{\omega}\right) N^2 \qquad (10)$$

for operation at the 0- or π-mode. This equation may be written in the alternate form suggested by R. Beringer:

$$\frac{\Delta V}{V} \sim \frac{\Delta\omega}{MS} \quad , \qquad (11)$$

where MS is the mode spacing which, for the 0- or π-mode, is proportional to BW/N^2. It can be verified that Eq. (11) indeed represents the situation for all modes. Since the mode separation is much greater within the band than at its ends, one should investigate the possibility of using a mode other than the 0- or π-mode to see if improvement in tolerances is overbalanced by decrease in power efficiency.

NAGLE: If you look at Carne's plot of ω vs. β, for example, the dispersion diagram pointed up at the end of the band. The slope is not zero at the top of the band.

GLUCKSTERN: That's only true for the resonant slot.

NAGLE: In your analysis k was considered to be constant.

GLUCKSTERN: I appreciate the fact that the dispersion curve is usually not symmetric. What you claim is that one has to bring in many modes around the π-mode; the $1/n^2$ behavior would only be valid in the vicinity of the end of the band and might not apply elsewhere.

NAGLE: If you take the derivative of the cosine at the top of the band and expand the finite difference, the first term will go as $1/n^2$; that's a reflection of the fact that the derivative is zero. But if you took a curve such as Carne showed, then the first derivative would go as $1/n$.

GLUCKSTERN: I think the curve that Carne had applies only to the case of the resonant coupling, because otherwise the curve will have zero slope at the top. The real question is just where the next mode is. Is it in the region where the expansion works? The model presented gives symmetric modes only if k is not a function of ω.

NAGLE: It comes from the fact that ω is a function of mode number.

BERINGER: I should like to emphasize something that is perfectly clear from both the preceding papers, and which I suspect to be a very important design feature if it is borne out. There is a basic difference between designing a standing wave structure near the ends of the passband and somewhere in between, which can lead to enormous differences in price, complexity, length of section that one can tolerate, and so on, because in one case the dispersion curve is a quadratic function and in the other case, essentially a linear function.

LEISS: Why is there such a great interest in having "flatness" in the cavity?

GLUCKSTERN: If one designs the cavity according to a certain flat field with a certain φ_s, the value of φ_s governs the width of the stable phase region. If you have a field variation which becomes as large as 10%, (assuming $\cos \varphi_s = 0.90$) then, dynamically, the particle has to ride right at the peak, because otherwise it won't gain enough energy to keep in step. The width of the stability region then becomes zero and the particles are lost from the bucket. So you have to maintain some accuracy on the field levels throughout in order that

φ_s remains between 20^o and 30^o.

NAGLE: When you're trying to transmit power from the drive point down the tank you find, in standing wave operation, that there is a phase shift required from cell to cell, and the more power you have to transmit (for example, with beam loading), the larger the phase shifts have to be. If you look at phase shift from the drive points to the end you find that it goes as the square of the number of cells, inversely as the coupling strength, and inversely with Q; so, if you couple heavily, the phase shift will be less. So the point I want to make is that as the beam comes on, that phase shift will also change, and so you will find the particles all move to a different part of the fish as it moves down the tank and comes to the next tank, where it encounters a completely different set of conditions and this will drive phase oscillations.