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INJECTION INTO FFAG ACCELERATORS

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This report treats the filling of the accelerator acceptance with beam by means of multiturn injection. The stable phase-space area for radial motion in a fixed field alternating gradient accelerator is typically at least two orders of magnitude larger than the beam emittance of a linac injector. It is therefore useful to efficiently inject many turns so long as the space-charge limit is not a factor. The maximum number of turns which can fit is set by the ratio of acceptance to emittance, in accordance with Liouville's theorem. The total captured beam current depends also on phase-space density of the beam.

In MURA's 12.5 GeV FFAG proposal accelerator, the design intensity of 2×10^{14} protons per second requires a circulating current at injection of 0.34 A. This current is less than one-seventh of the space charge limit (including the effect of image forces). Present day linac technology does not yield sufficient current for single-turn injection. A 20 mA beam would require 17 turns while a 50 mA beam would require but 7 turns. It should be possible, as will become evident later, to inject sufficient turns to approach the space-charge limit.

While bypassing certain other schemes, which have been demonstrated to work, we shall restrict our discussion to the perturbed equilibrium orbit, or field bump, scheme.[1] This scheme has been effectively demonstrated on the 50 MeV electron accelerator[2] at MURA and would seem to offer the best hope for efficient injection into a high energy proton machine.

Consider a circular injection orbit (ignoring any scalloping due to alternating field gradients) with injection at an inside radius, as shown in Fig. 1. The injected beam oscillates about the equilibrium orbit and some n turns later, depending on the tune or number of betatron oscillations per revolution, will normally return to strike the inflector septum. This will happen on the fourth revolution if the tune is quarter-integral, say 9.75. Emittance plots of the beam on successive turns in relation to the accelerator acceptance are shown in Fig. 2. For the beam to clear, one must move the orbit during these four turns to larger radii in the vicinity of the inflector by an amount equal to injection beam plus septum width. This is accomplished by deflecting the orbit to pass near the inflector at the start of injection and letting it slowly recede with time back toward its unperturbed position. If this movement is slow enough, the beam already injected follows the orbit away from the inflector without growth of oscillation amr1itude.[1] In Fig. 2, this orbit movement is equivalent to movement of the septum to larger negative x values. Continuous injection during the period of orbit movement results in a continuous spread of oscillation amplitudes in the circulating beam. The acceptance area becomes covered to an extend depending on the efficiency of the process.

The orbit perturbation in Fig. 1 produced by two field bumps is held to a localized region to minimize turn shifts. The injection scheme does work when a single field bump is used.[1,2] In this case, however, the perturbed orbit consists of a betatron oscillation about the unperturbed orbit all around the machine, except at the bump location. Because of the nonlinear restoring force for large betatron amplitudes, a large perturbation amplitude can produce significant tune changes. This can result in particles returning to the septum in a smaller number of turns and can actually drive particle oscillations onto a resonance. Some of these effects have been observed in the 50 MeV accelerator which uses a single bump scheme. Whenever the tune is not well defined, one may need to move the orbit a distance in some n turns equal to the spread in betatron amplitudes of the injection turn beam plus septum width. In such circumstance the optimum beam width at the inflector can be determined as a function of the emittance. The amplitude spread is then proportional to the square root of the emittance for small betatron amplitude (perturbed orbit near the septum) and decreases for increasing amplitude.

One may ask what is the limit on efficiency of filling accelerator acceptance area under ideal conditions. These include nearly constant tune, uniform bump fields and appropriate turnoff rate for these fields which give a localized orbit perturbation. We now consider two cases: (1) programming the beam optics so that the beam width at inflector

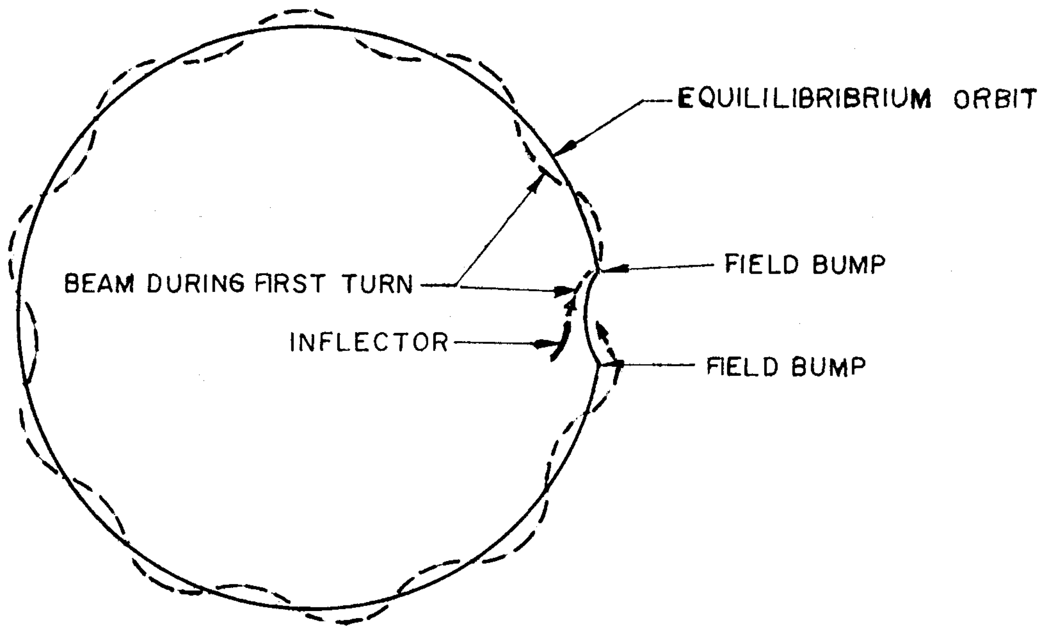

FIG. 1

Radial oscillations of beam about perturbed equilibrium orbit at injection.

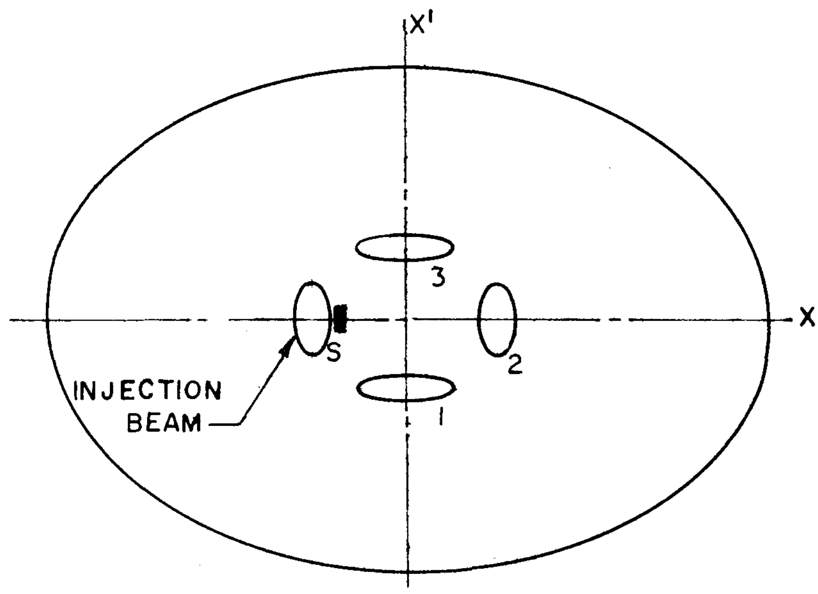

FIG. 2

Radial acceptance of accelerator and emittance of beam at injection and on successive turns.

exit is continually varied during bump field turnoff to have always the optimum width, and (2) injection of a beam of constant size.

Programmed Beam Optics

For simplicity we assume the phase-space area for the accelerator and for the beam to be of elliptical shape. The acceptance plot of Fig. 2 will become circular as shown in Fig. 3 upon multiplication of the angle x' by β, the matrix element parameter of Courant and Snyder.[3] The average value of β is the orbit radius divided by the tune. The beam emittance area, $\epsilon = \pi ab$, becomes the area, $\pi ab\beta$.

For quarter-integral tune, the orbit must move (2a + S)/4 in one turn in order for the beam to clear the septum. For clearance at the end of one turn, we have

$$b\beta = A - (a + S) + \frac{2a + S}{4} = A - \frac{a}{2} - \frac{3S}{4} \quad . \qquad (1)$$

Substitution for b in terms of ϵ gives

$$a = A_S \left[1 - \sqrt{1 - \frac{2\epsilon\beta}{\pi A_S^2}} \right] \qquad (2)$$

where $A_S = A - (3S/4)$. One sees that for septum clearance, the beam width, 2a, is permitted to decrease with increasing amplitude, A.

For amplitudes such that $2\epsilon\beta/\pi A_S^2 << 1$ (most of the range in practice) one has approximately

$$a << \frac{\epsilon\beta}{\pi A} \qquad (3)$$

The beam width can thus decrease until A reaches $A_M/(2)^{1/2}$

when the beam touches the stability boundary $A = A_M$. Beyond this point, the width must increase until A approaches A_M if no beam is to be lost. This calls for $b\beta = A_M^2 - A^2$ and

$$a = \frac{\epsilon\beta}{\pi A_M} \frac{1}{(1 - (A/A_M)^2)^{1/2}} \qquad (4)$$

The acceptance becomes filled as shown in Fig. 4, each quadrant having a similar pattern of ellipses. The number of such ellipses is the number of turns. For a zero-thickness septum, this number is approximately

$$N \cong \left(\frac{A_M}{a_1}\right)^2 \frac{\pi}{2} - \log \frac{(2)^{1/2} A_M}{a_1} \qquad (5)$$

where $a_1 = (2\epsilon\beta/\pi)^{1/2}$. When a thin septum thickness, S (with $(S/a_1) << 1$), is assumed, an approximate expression for the number is

$$N \cong \left(\frac{A_M}{a_1}\right)^2 \left[\frac{\pi}{2} - \frac{0.8\ SA_M}{a_1^2}\right] \qquad (6)$$

If the emittance ellipses were distorted so as to cover the acceptance completely, one would have a number $N_{max} = \pi A_M^2/\epsilon\beta = 2A_M^2/a_1^2$. The fraction of this number contained in expression (5) is slightly less than $\pi/4$. One notes that the number of turns is inversely proportional to the emittance.

In the MURA proposal accelerator, the radial stability limit at injection is $A_M = 20.9$ cm. Assume a linac emittance of 5×10^{-3} cm-rad. for a 50 mA beam at 200 MeV (extrapolated from the CERN figure of 10×10^{-3} cm-rad. for

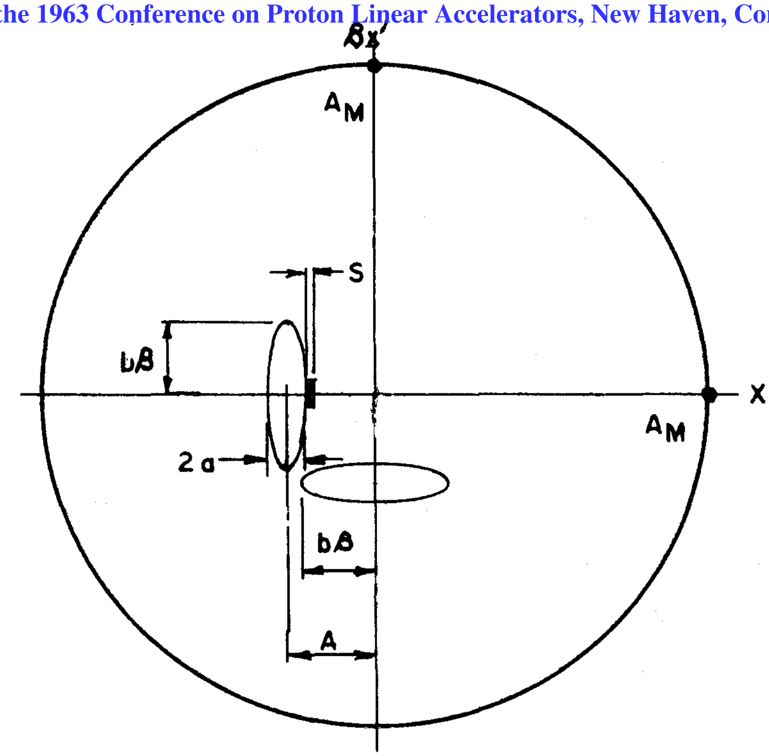

Fig. 3. Accelerator acceptance and beam emittance at injection and on first turn.

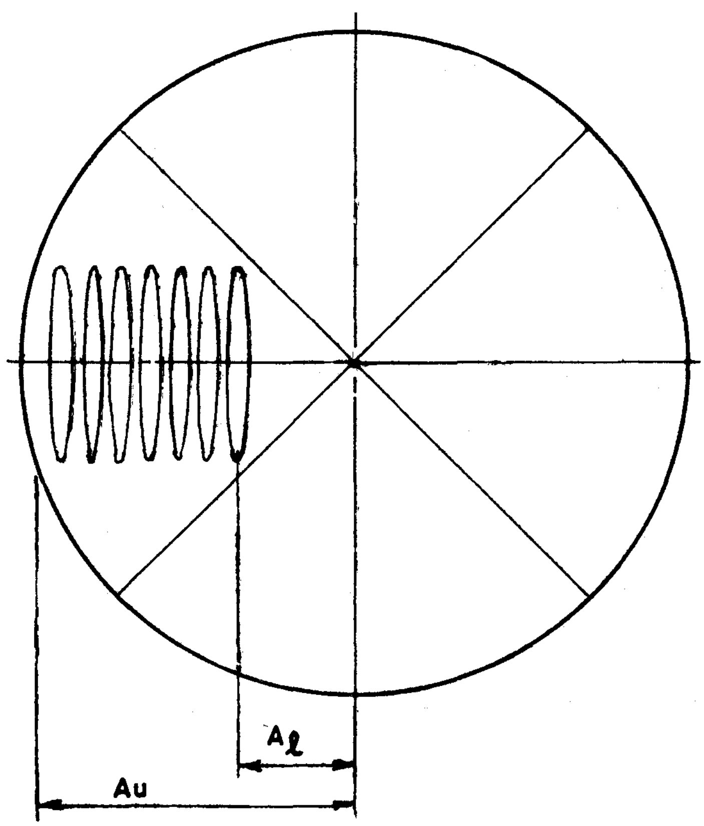

Fig. 5. Filling of acceptance for fixed beam size. All quadrants are filled similarly.

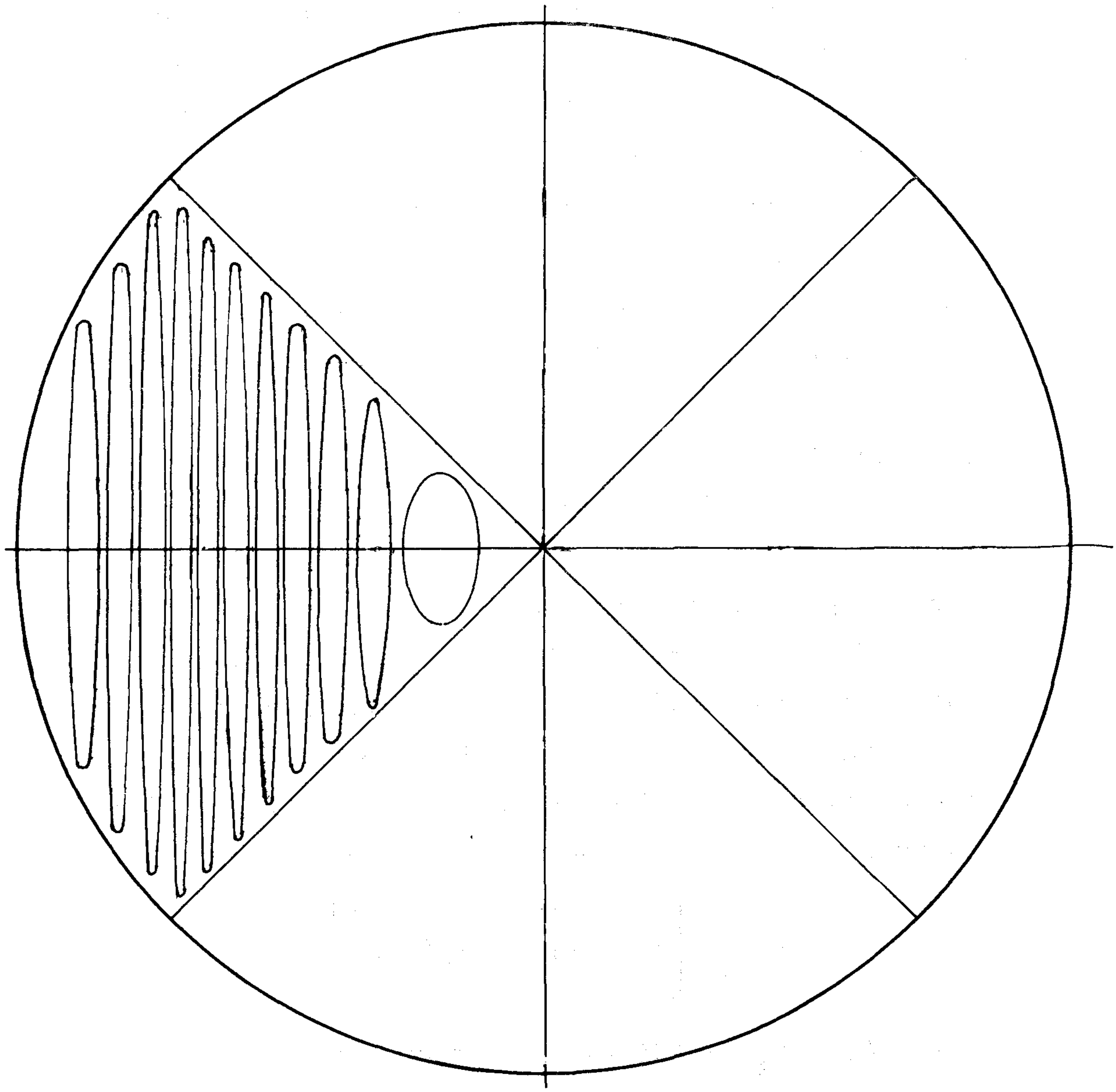

Fig. 4. Filling of acceptance for programed beam optics. All quadrants are filled similarly.

50 mA at 50 MeV). Under ideal conditions for continuously programmed beam optics, one now obtains approximately 230 turns injected for zero septum thickness and 165 turns for a 1 mm thick septum. One would not hope to approach this ideal. The lower number here, however, corresponds to about three times the space-charge limit of the accelerator for a 50 mA beam and some 20 times the design figure.

The required rate of orbit movement decreases with time until A reaches 0.7 A_M and would be given by a simple and natural kind of bump field turnoff. For $A > 0.7\ A_M$, the optimum bump field turnoff is less easily achieved.

Constant Beam Size

It is simpler to inject a beam of constant configuration, which is the usual practice. How well can one fill the acceptance in this case?

If one requires that a minimum of beam be lost on the inflector, there will exist a minimum amplitude A at which to begin injection and an optimum beam width for most efficient injection. The acceptance area will be filled as shown in Fig. 5. The beam width is given by

$$a = A_S \left[1 - \sqrt{1 - (a_1/A_S)^2} \right] \approx \frac{1}{2} \frac{a_1^2}{A_S} \qquad (7)$$

where $A_S = A_\ell - (3S/4)$. The maximum amplitude for injection without loss is

$$A_u = \sqrt{A_M^2 - (b)^2} \, . \qquad (8)$$

The number of turns injected is

$$N = \frac{A_u - A_\ell}{(2a + S)/4} = \frac{4\left[\sqrt{A_M^2 - A_S^2} - A\right] A_S}{a_1^2 + S\,A_S} \qquad (9)$$

where $A_S = A_\ell - (3S/4)$. By differentiation, one obtains an optimum A_S to maximize N as

$$A_S = -\frac{a_1^2}{S}\left[1 - \sqrt{1 + \frac{A_M S}{a_1^2}}\right] \qquad (10)$$

Use of the same emittance and accelerator parameters as in the programmed case gives approximately 125 turns for a zero-thickness septum and 100 turns for a septum one millimeter thick. The optimum beam width is 0.33 cm and the initial amplitude of injection, A_ℓ = 8.7 cm. Ideal performance therfore corresponds to about twice the space-charge limit of the accelerator. Space-charge forces would influence the injection process, of course, before one would reach the space-charge limit.

If one's goal were really to fill the accelerator to its space-charge limit, one might choose to multiturn inject into vertical phase space as well as into radial space, particularly if one's practical filling efficiency were very low.

Dependence on Linac Beam Properties

As already noted, the number of turns which can be injected is inversely proportional to the beam emittance. Experience has shown the emittance to be approximately proportional to the beam current. For a zero-thickness septum then, one would in principle expect to inject the same amount of charge by multiturn injection with low as with high intensity homogeneous beams. Arie Van Steenbergen has pointed out that beams from the ion source are often not homogeneous at high intensity. With appropriate collimation of an inhomogeneous beam into the linac, one might, therefore, expect to obtain a beam of higher average density in phase space. Even with homogeneous beams, better performance could be expected at high than at low intensity when the septum width is taken into account, because a smaller fraction of the acceptance area would then be left unfilled. These arguments, of course, assume plenty of vertical phase space to accept the higher intensity beams of larger vertical emittance.

All of our discussion of acceptance filling efficiency has been based on little beam loss during injection. Filling efficiencies might be improved somewhat by choice of narrower injection beam widths. This would result in appreciable beam loss on the septum and elsewhere creating worse radiation and heating problems.

BLEWETT: It would be very nice if you could transfer emittance from the yy' area to the xx' area or vice versa. Is there some reason why this is impossible?

COURANT: You could transfer it by just a rotation of the coordinates, but you cannot do it if initially the emittances in the phase space are in a ratio of 10 to 1, that is, you cannot change that ratio to 5 to 2, or something like that.

CURTIS: Is it true that if you try a transfer from x to y, and supposing you wanted to narrow down the y motion, you couldn't make it any smaller in y than you started with in the x?

NAGLE: If the x motion and the y motion are uncoupled, then the volume in the x space and in the y space is separately conserved. If they're coupled it is not true and there is a new pair of coordinates.

COURANT: Yes, that's right. If they're coupled there is a new pair of conjugate coordinates, which you can call z and η. There's a theorem to the effect that in that case, phase space in the new z coordinate system is equal to that of the old x system, and that in the η coordinate system is equal to that of the old y system.

VAN STEENBERGEN: You have the feeling that you should really be able to transfer the problem in the x plane to the y plane.

COURANT: You need some dissipative mechanism, because the condition that all the phase volume is conserved is really only one condition on the transformation. When you have a Hamiltonian system, you have a certain number of Poisson bracket relationships which set a lot more restrictions besides the over-all conservation of phase space, and these can be expressed in terms of more detailed sub-phase space conservation.

CURTIS: I think the result of that would be that the only way you can get transfer is by coupling, but even then, if you want to damp an energy spread, you can't damp it down to an arbitrarily small value.

COURANT: With this variable matching of the injector beam, from turn to turn, is it also implied that the spacing between one turn and the next (in other words, the rate at which the bump shrinks) is also unchanged?

CURTIS: Yes. I may not have said that explicitly, but the field bump should fall off more slowly as time goes on.

References

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(2) M. F. Shea, C. D. Curtis, D. A. Swenson, and D. E. Young, Bull. Am. Phys. Soc. II, 7, 276 (1962).

(3) E. D. Courant and H. S. Snyder, Annals of Physics 3, 1 (1958).