Proceedings of the 1964 Linear Accelerator Conference, Madison, Wisconsin, USA

PARTICLE MOTIONS AND THE FOCUSING SYSTEM IN PROTON LINACS*

S. Ohnuma
Yale University

Introduction

Last year, at the conference on proton linacs, computer programs developed at Yale for particle motions and some of their preliminary results were presented.[1] Since then, there have been a number of modifications in our design parameters, particularly in the iris-loaded section. The results here are mostly based on the design parameters of the proposed 500 MeV injector for the AGS in Brookhaven which was presented by G. W. Wheeler. The computer programs are essentially the same as the one discussed in Ref. (1).

Buncher

No change has been made to the buncher which was presented last year.[2] The essential requirement here is that the beam must be tightly bunched near the synchronous point ($\Delta\varphi = \Delta\gamma = 0$). The distribution of particles from a double buncher is shown in Fig. 1. If the ion source can produce 100 mA, about 79 mA will be confined to within $\Delta\varphi = \pm 14^{\circ}$ and $\Delta W = \pm 20$ keV. On the other hand, there are still particles near the boundary ($\sim .5$ mA) which come out of the drift tube section with relatively large values of ($\Delta\varphi$, $\Delta\gamma$). For a linac of higher intensity, a bunching scheme that gives very small fraction of beams near the boundary would become important.

Drift Tube Section

Phase and transverse oscillations of particles 1 and 2 in Fig. 1 are given in Figs. 2-3 together with those of the synchronous particle. The focusing system used here is shown in Fig. 4. Two different doublet arrangements, (A) and (B), have been tried in the last two tanks to see the change in the emittance shape. In Tanks No. 1 and No. 2, the strengths of quadrupole magnets are chosen such that they correspond to optimum values for the synchronous particle. For non-synchronous particles, the optimum condition is different and the amplitude of the transverse oscillation may increase instead of damping down. (see Fig. 3.) Since the

---

* The work reported here has been done by R. Bakeman, T. W. Ludlam, J. N. Vitale and S. Ohnuma.

Particle Distribution from
Double buncher

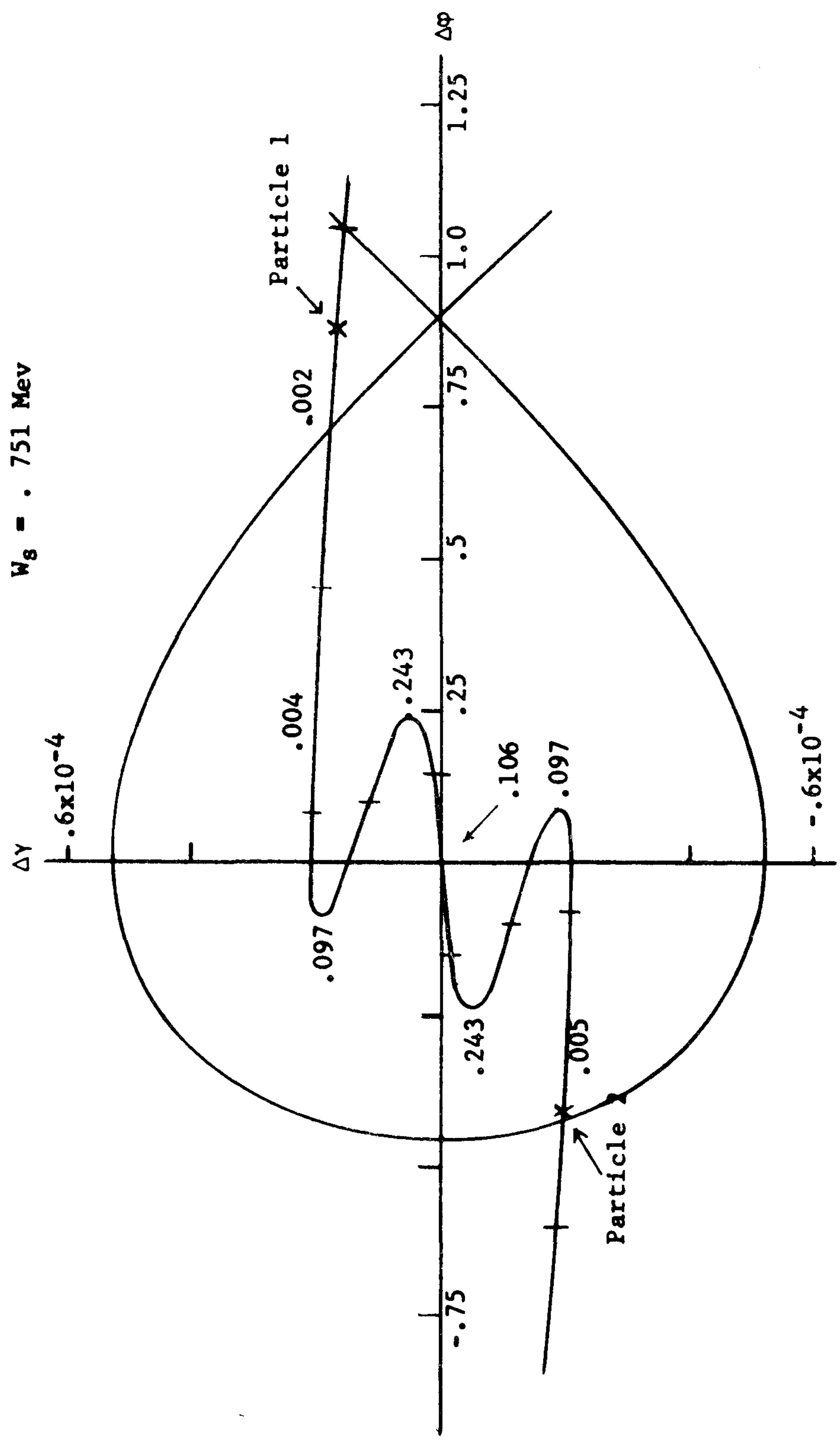

FIG. I

focusing system is continuous from Tank No. 1 to Tank No. 2 with a very short drift space, no matching magnet is required. This, however, is not true between Tank No. 3 and No. 4 where the system suddenly changes. At least one set of triplet is necessary in order to achieve a near perfect matching in x- as well as y-direction. After that, the system is continuous (i.e., the change is "adiabatic") and the matching system can be eliminated entirely.

The acceptance in x - x' space of the drift tube section for the synchronous particle is shown in Fig. 5. When the smallest bore radius is .75 cm, the area is 17π cm-mrad. For most of the non-synchronous particles, the acceptance area remains fairly close to the 17π cm-mrad but the shapes are quite different. Thus, if the input beam is matched to the shape given in Fig. 5, the effective acceptance value for particles in ($\Delta\varphi = \pm 14^{\circ}$, $\Delta\gamma = \pm 20$ keV) region is reduced to $\sim 16\pi$ cm-mrad. For particles 1 and 2, the shaded area in Fig. 5 is not acceptable and the area is less than 15π cm-mrad. It is, of course, possible to get a larger acceptance by simply taking a larger bore radius. This, however, will give a larger output bunch in ($\Delta\varphi$) - ($\Delta\gamma$) space and may cause particle loss when the frequency is increased in the high energy section. If (NNSS) configuration is used instead of (NSNS) type, the acceptance for the synchronous particle is 14π cm-mrad. The change of the acceptance shape for non-synchronous particle from that of the synchronous particle is more serious than (NSNS) configuration.

The maximum radial excursion of the beam in Tanks No. 3 - No. 7 will be substantially smaller than shown in Fig. 3 if the focusing system is (NSNS) throughout instead of doublets with gradually increasing repeat lengths taken here. The particular advantage of the doublet system will be discussed later.

It would be useful for the purpose of shielding designs to see where particles are lost radially. The location depends on the initial position of particles in x - x', y - y' and ($\Delta\varphi$) - ($\Delta\gamma$) spaces. For example, most of the particle 1 in the shaded area in Fig. 5 are lost at drift tubes No. 7, No. 9, and No. 13 of the first tank and at drift tubes No. 2 and No. 6 of the second tank, whereas the particle 2 are lost at drift tubes No. 5 and No. 7 of the first tank and No. 2 of the second tank. By assuming a particle distribution, one can calculate the distribution of lost particles along the drift tube section. However, extensive study of this problem has not been made so far. The results we have accumulated clearly indicate that the loss will be entirely in the first two tanks and most likely below ~ 20 MeV.

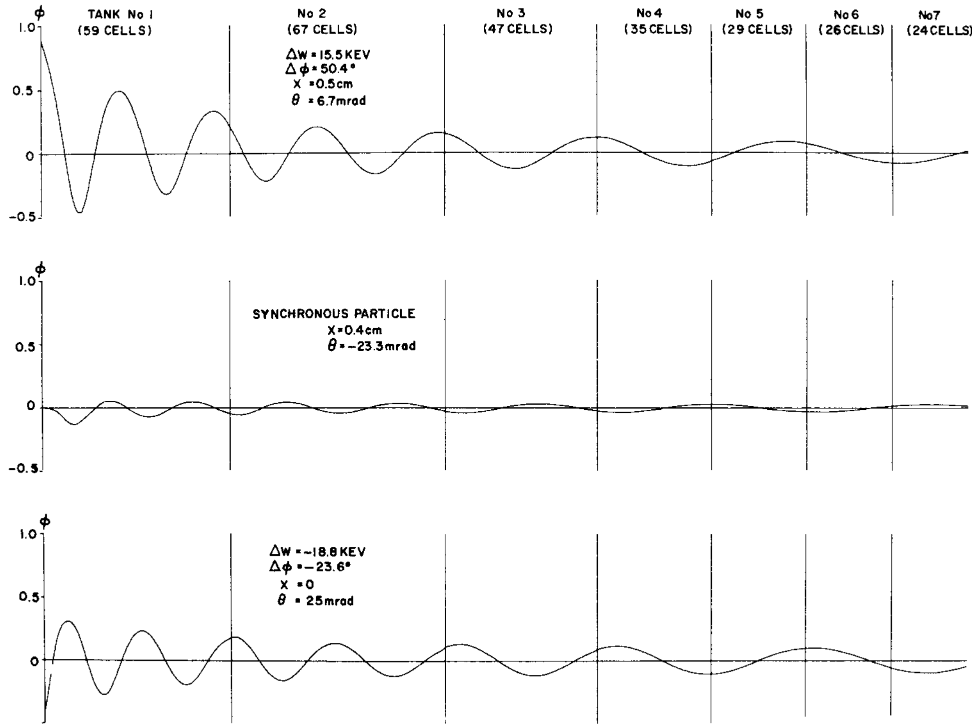
TANK No I
(59 CELLS)
No 2
(67 CELLS)
No 3
(47 CELLS)
No 4
(35 CELLS)
No 5
(29 CELLS)
No 6
(26 CELLS)
No7
(24 CELLS)
ΔW = 15.5 KEV
Δφ = 50.4°
X = 0.5cm
θ = 6.7mrad
SYNCHRONOUS PARTICLE
X = 0.4cm
θ = −23.3mrad
ΔW = −18.8 KEV
Δφ = −23.6°
X = 0
θ = 25mrad

FIG. 2

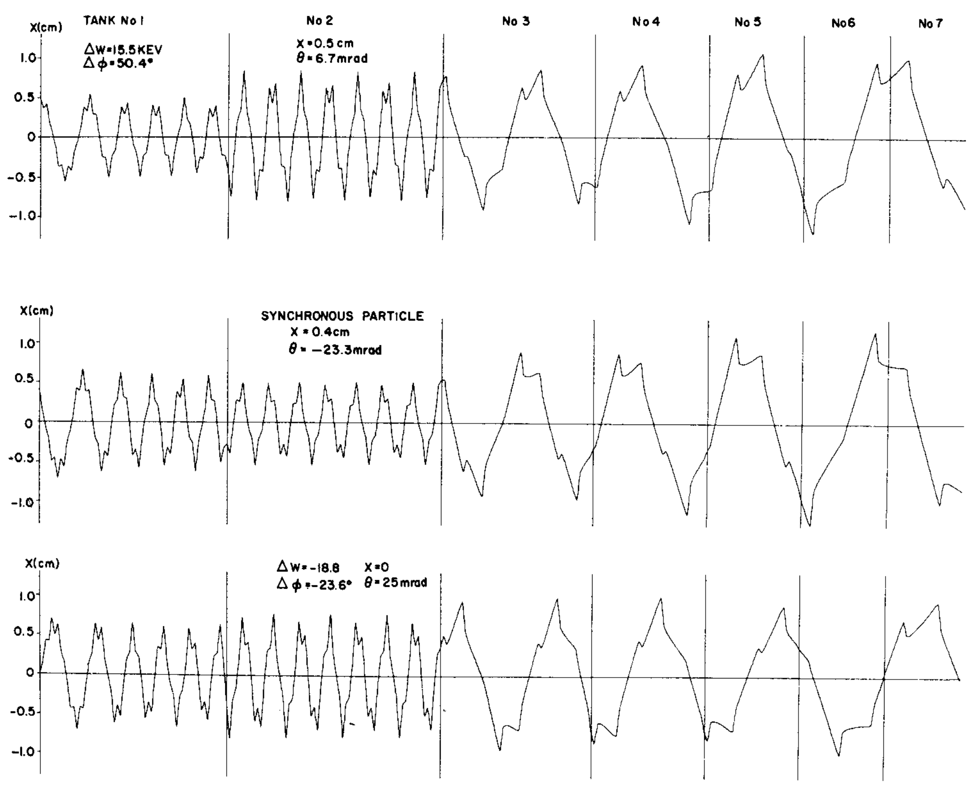
X(cm)
TANK No I
ΔW=15.5KEV
Δφ=50.4°
No 2
X = 0.5 cm
θ = 6.7mrad
No 3
No 4
No 5
No6
No 7
SYNCHRONOUS PARTICLE
X = 0.4cm
θ = −23.3mrad
ΔW = −18.8
X = 0
Δφ = −23.6°
θ = 25mrad

FIG. 3

In the ($\Delta\varphi$) - ($\Delta\gamma$) space, particles coming out of the drift tube section with the final energy of 187 MeV are all contained in the tilted ellipse shown in Fig. 6. Several dots are those particles with ($\Delta\varphi$) $\gtrsim$ 1.0 or ($\Delta\varphi$) $\lesssim$ -.6 at the injection. Since their "effective" transverse acceptance is very small (0 - 3 π cm-mrad), the total intensity of these stray particles is less than 5×10^{-5} of the main output current (i.e., the ellipse in Fig. 6). The other ellipse in Fig. 6 is the particle bunch assumed in calculating particle motions in the iris section.

Fig. 7 shows the emittance from the drift tube section in x - x' space. The area is very close to π cm-mrad. (A) and (B) correspond, respectively, to the doublet configuration (A) and (B) shown in Fig. 4.

In the low energy region ($\beta \lesssim$ .2) where the rate of the change of β is large, the accuracy of the calculation is not too good. As has been already suggested previously[(3)], it may be necessary to integrate the equations of motion point by point. This, however, requires a detailed information on electric and magnetic fields at every point. When there is no bore hole, the fields of the shaped drift tube[(4)] are accurately known but the effect of the bore is difficult to evaluate. It is hoped that the calculation at MURA and at Los Alamos ("MESSY MESH")[(5)] will eventually give the fields with the desired accuracy.

Matching between Drift Tube and Iris Sections

It is clear from Fig. 6 that a long drift space between the two sections must be avoided in order to keep the resulting phase spread as small as possible. Although β here is large (.55), the spread could be as large as $\pm 2.6^{\circ}/m$ (for $\Delta\gamma = \pm .8 \times 10^{-3}$) so that a drift space longer than $\sim$ 4m will definitely cause trouble. This is particularly serious for a linac injector of a large synchrotron where a small momentum spread ($\Delta p/p$) is essential in getting a high capture efficiency. On the other hand, it is highly desirable to have a bending magnet (to guide the beam for the energy measurement), one or two slit boxes (for measuring the beam shape in x and y-space), and one or two current transformers. If, in addition to these, two or three sets of quadrupole triplet are required to match the beam shape in both x and y-direction simultaneously, a 4m long space does not seem to be sufficient. Two schemes have been tried to solve this problem.

The first is to employ a matching scheme in the longitudinal phase space that is based on the principle of the strong focusing[(6)]. A tank operated at the stable synchronous phase $\varphi = \varphi_s$ acts as a "focusing lens" and a tank operated at $\varphi = -\varphi_s$ as a "defocusing lens". The drift space

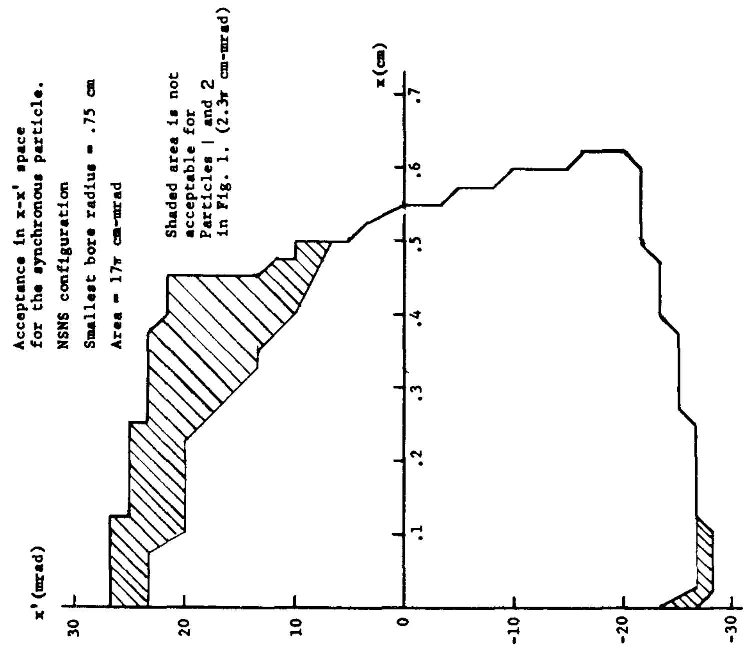
Acceptance in x-x' space
for the synchronous particle.
NSNS configuration
Smallest bore radius = .75 cm
Area = 17π cm-mrad
Shaded area is not
acceptable for
Particles 1 and 2
in Fig. 1. (2.3π cm-mrad)
x'(mrad)
x(cm)
30
20
10
0
-10
-20
-30
.1
.2
.3
.4
.5
.6
.7

FIG. 5

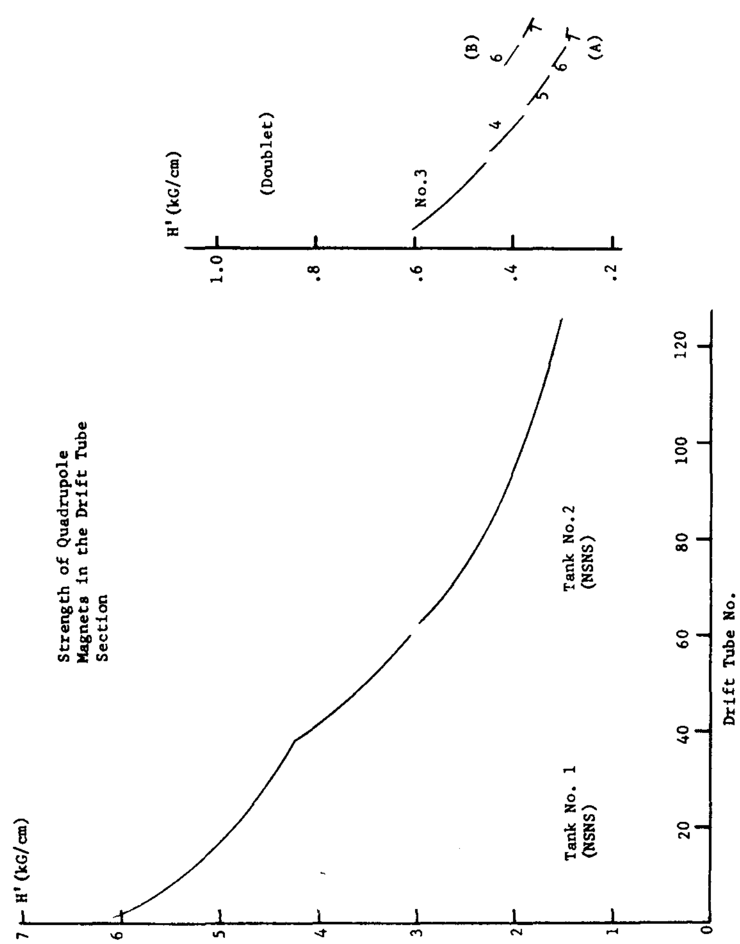
Strength of Quadrupole
Magnets in the Drift Tube
Section
H' (kG/cm)
7
6
5
4
3
2
1
0
20
40
60
80
100
120
Tank No. 1
(NSNS)
Tank No.2
(NSNS)
Drift Tube No.
H' (kG/cm)
(Doublet)
1.0
.8
.6
.4
.2
No.3
4
5
6
6
(B)
(A)

FIG. 4

then will be an element of the matching optical system and does not necessarily have to be short. The system will be composed of 10 – 15 regular tanks with reasonable values of rf level. Based on the linear theory, we found that we can achieve an almost 100% matching even with a 10 m drift space. However, numerical calculation has shown that the non-linear effects are so serious as to make the scheme rather impractical. It may be possible to design an elaborate system composed of very special tanks just for the matching purpose but this does not seem too easy.

The second choice is to eliminate the matching magnets for two transverse directions entirely, or use only one doublet. This means that the focusing system in the drift tube section (at least in the last two tanks or so) must be joined continuously to that in the iris section. Otherwise, one doublet or triplet cannot give a perfect matching in the both directions simultaneously. Since a doublet is used after each even-numbered tank in the iris section, one doublet just before the first tank together with a properly spaced doublet system in Tanks No. 6 and No. 7 will achieve the desirable matching. With 3.5 m space between two sections, we have found that the matching efficiency could be as large as 95% in both directions. It will be even better if two or three auxiliary magnets are placed in the last few drift tubes to make adjustments.

Iris Section

Various systems of quadrupole doublets and triplets to be used for the focusing in the high-energy section have been investigated in order to get the necessary information for the choice of multiplet type [7]. The doublet system used in the calculation and its acceptance shape are shown in Fig. 8 while the strength of magnets and the maximum (theoretical) excursions are given in Fig. 9. If the transverse phase space area is 10π cm-mrad at the injection (.75 MeV), the area at 187 MeV is $.6\pi$ cm-mrad. One advantage of doublet system is the relatively large space available between two magnets for a given drift space (1 m in this design). The maximum x or y excursion at 500 MeV is 68% of the excursion at 187 MeV. This is due to the increase of the momentum and the decrease of (Courant-Snyder) β_{max} [8]. It is, of course, possible to take longer magnet repeat lengths gradually as the energy increases. This, however, will increase the value of β_{max} and, for β_{max} (500 MeV)/ β_{max} (187 MeV) = 1.75, the maximum excursion stays constant. If, on the other hand, the focusing system is changed at some points, some kind of matching device will be required.

A good matching at the entrance is essential in achieving the smallest possible x_{max} or y_{max}. Different systems require different acceptance shapes so that a particular point in x-x' space leads to an entirely different maximum excursion. In connection with the matching, a device that can

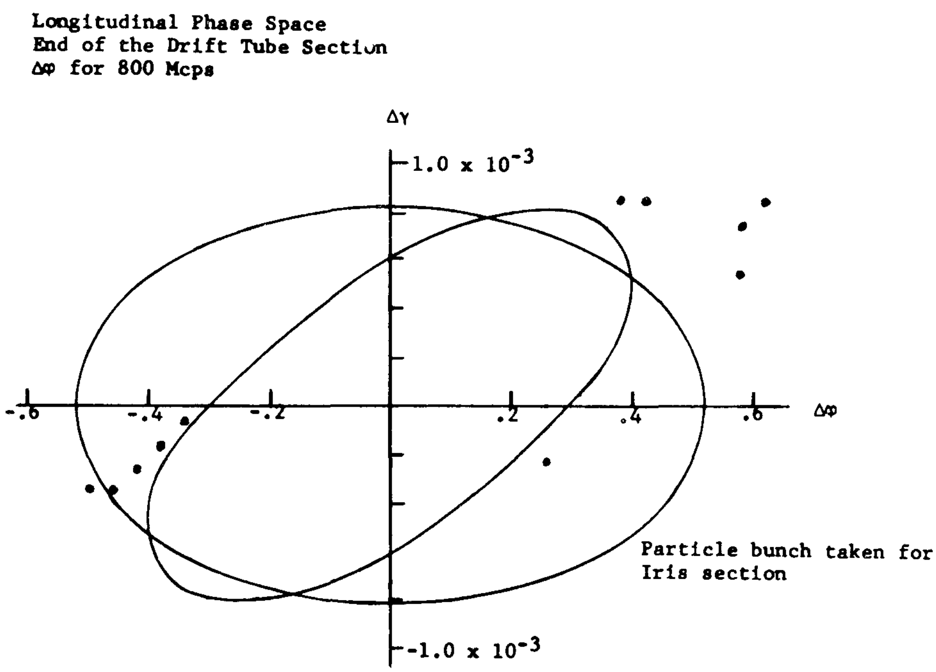
Longitudinal Phase Space
End of the Drift Tube Section
Δφ for 800 Mcps
ΔY
1.0 x 10-3
-1.0 x 10-3
-.6
-.4
-.2
.2
.4
.6
Δφ
Particle bunch taken for
Iris section

FIG. 6

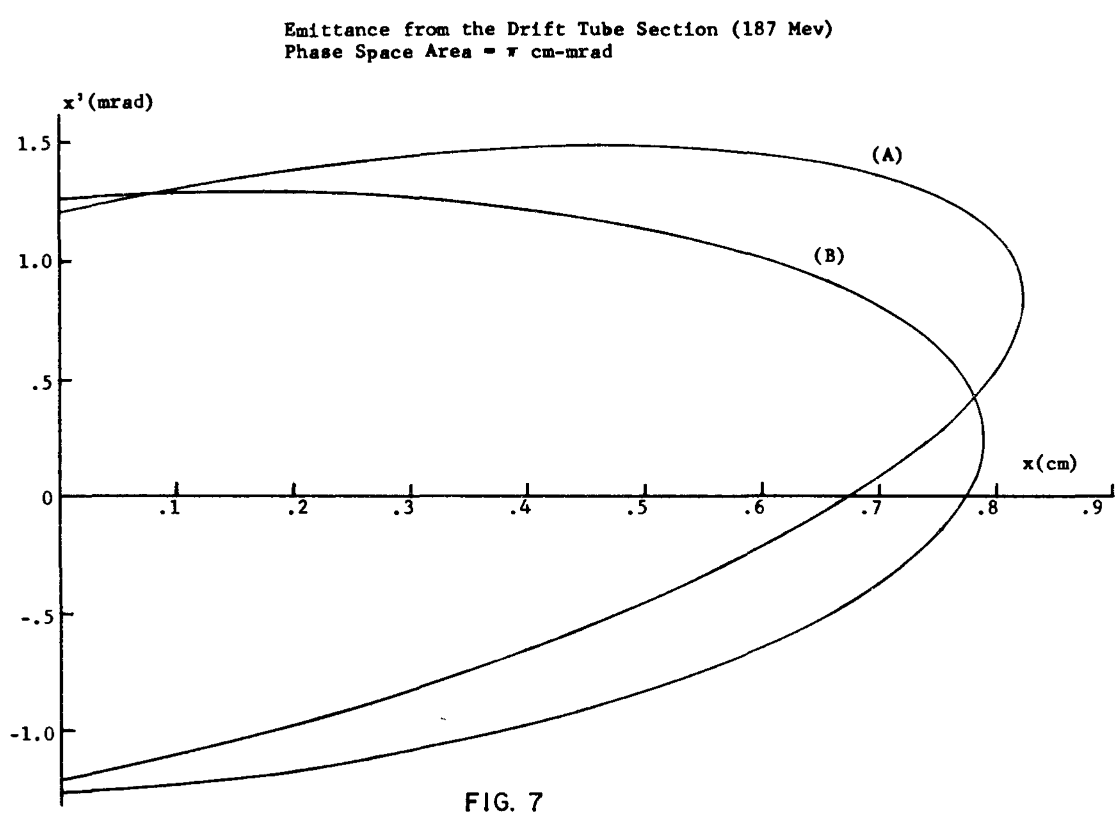
Emittance from the Drift Tube Section (187 Mev)
Phase Space Area = π cm-mrad
x'(mrad)
1.5
1.0
.5
0
-.5
-1.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
x(cm)
(A)
(B)

FIG. 7

accurately measure the beam shape in x-x' and y-y' space will be extremely useful.

The theoretical maximum excursions in Fig. 9 are computed for synchronous particles. For non-synchronous particles, the acceptance shape is slightly different from those given in Fig. 8. This slight mismatching would, in general, give larger excursions for non-synchronous particles. The largest deviation recorded from the theoretical value is .25 cm. Of course, only those particles which are near the boundary of the acceptance ellipse in Fig. 8 deviate from the theoretical maximum excursion. For example, particles within an ellipse of size .63π cm-mrad in x-x' space are not lost if, in ($\Delta\varphi$) - ($\Delta\gamma$) space, they are inside of the (larger) ellipse in Fig. 6. On the other hand, it seems very difficult to keep the iris aperture smaller than 1 cm (radius) and still not lose particles. One obvious solution would be shorter tanks at the beginning to reduce the magnet repeat length. Since a smaller iris aperture gives a smaller power loss for the same structure, further studies of this point may be called for in the future.

The choice of the doublet arrangement instead of a triplet was made because of the better over-all quality in focusing the high energy beam.[7] Triplet arrangements (+) (-) (+) (rf tanks) (+) (-) (+) are not symmetric in x and y directions so that the best operating point in one direction is very close to the stability boundary in another direction. This situation is shown in Fig. 10(*) where g_s is related to H' by

$$H' \text{ (kG/cm)} = .313\,(\gamma\beta)\, g_s^{\,2}$$

and the parameter Ω for each curve is

$$\Omega = \left[\frac{\pi e E_o \sin|\varphi|}{m_p c^2 \lambda\,(\gamma\beta)^3}\right]^{1/2} \quad ; \quad \frac{d\gamma}{dz} = \frac{e E_o}{m_p c^2}\cos\varphi$$

Symmetric arrangements (+) (-) (+) (rf tanks) (-) (+) (-) give a very narrow stability range with large values of β_{max} as can be seen from Fig. 11(*). There is a possibility of using an upper stability region

---

(*) Figs. 10 and 11 are taken from Ref. (7). Tank length is 2.5 m instead of 3 m here and β_{max} is slightly smaller than for 3 m tanks.

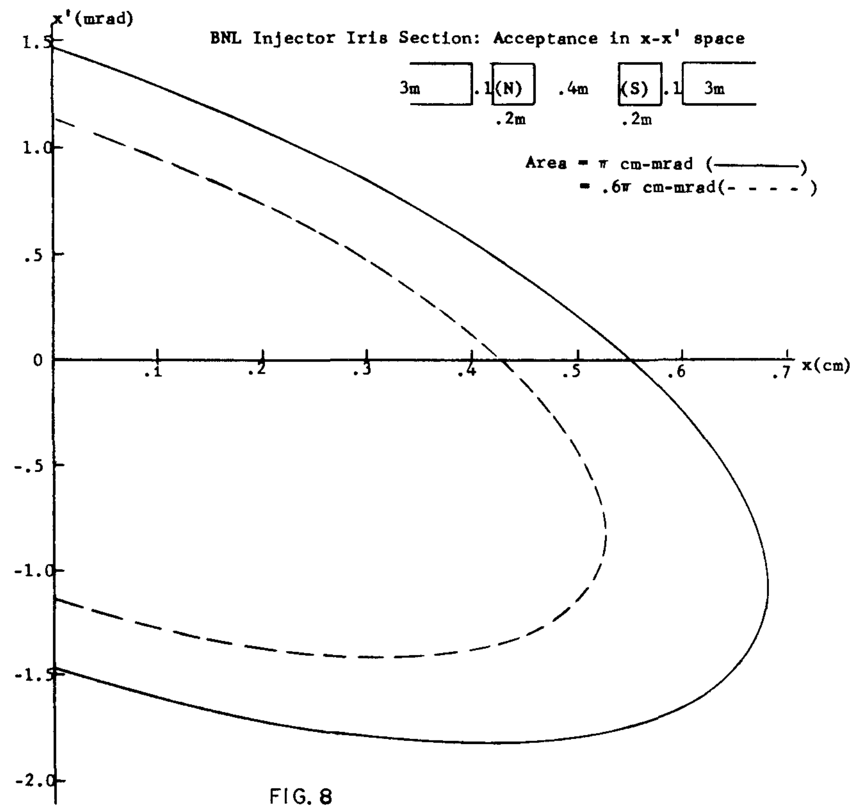
x'(mrad)
BNL Injector Iris Section: Acceptance in x-x' space
3m
.1
(N)
.2m
.4m
(S)
.2m
.1
3m
Area = π cm-mrad (———)
= .6π cm-mrad(- - - -)
1.5
1.0
.5
0
-.5
-1.0
-1.5
-2.0
.1
.2
.3
.4
.5
.6
.7
x(cm)

FIG. 8

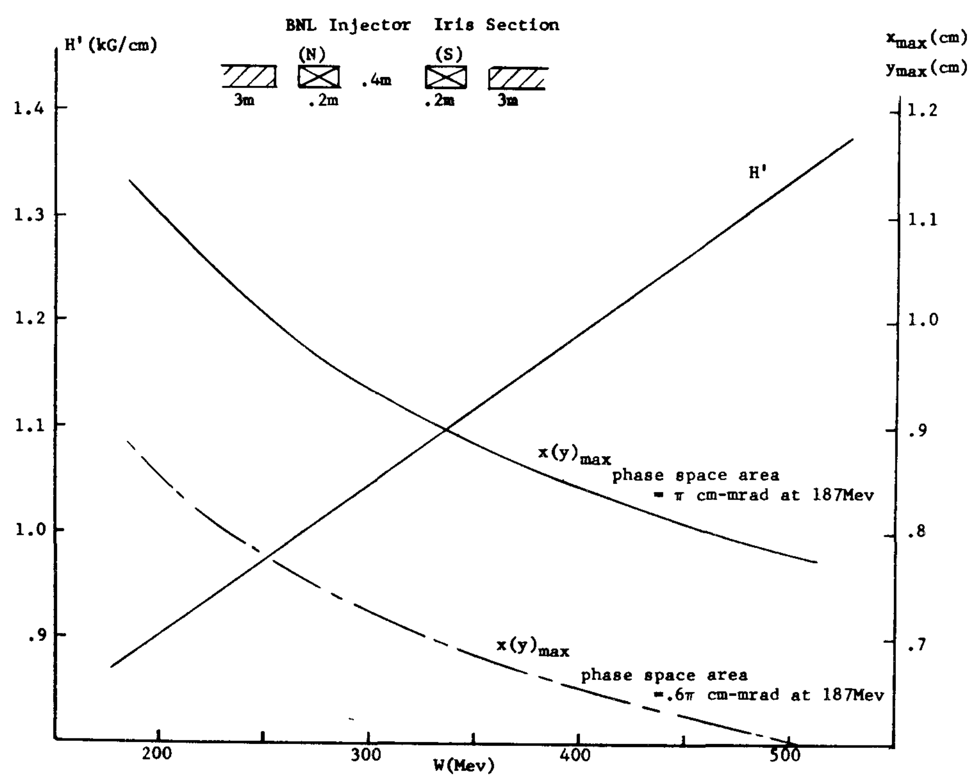
BNL Injector Iris Section
H'(kG/cm)
(N)
(S)
3m
.2m
.4m
.2m
3m
x_{max}(cm)
y_{max}(cm)
1.4
1.3
1.2
1.1
1.0
.9
1.2
1.1
1.0
.9
.8
.7
H'
$x(y)_{max}$
phase space area
= π cm-mrad at 187Mev
$x(y)_{max}$
phase space area
=.6π cm-mrad at 187Mev
200
300
400
500
W(Mev)

FIG. 9

where β_{max} are much smaller. However, aside from large magnetic fields required in the high energy region, non-synchronous particles may spend some time near the stability boundary or even cross the boundary into the stopband and get a large radial excursion. Unless an extensive numerical calculation is done, it does not seem safe to design the focusing system operating in upper stability regions.

The only remaining question is whether tolerance requirements for doublets are too difficult to achieve in the proton linac, as they appear to be in the Stanford two-mile electron linac[9], so that one is forced to take triplets. This has also been investigated in a semi-analytic fashion in Ref. (7). The method used there is outlined by R. L. Gluckstern.[10] As has been discussed in Ref. (9) and Ref. (10), the advantage of triplets is the possibility of "bench alignment" which makes the effect of magnet displacements entirely negligible (maximum tolerances are order of 100 mils). For doublets, bench alignment offers no improvement as far as the tolerance of "skew" (rotation about transverse axes) is concerned. On the other hand, it does not seem at all difficult to achieve the required tolerances for doublets (as well as for triplets) with the help of aligning monuments.[10] [11] A computer program has also been used to check the validity of the semi-analytic results. All together 50 random sets of misaligned doublets have been investigated at 400 MeV, 580 MeV, and 750 MeV. The displacements in x and y direction (individually) are larger than the analytic results in 5 cases at 400 MeV, 7 cases at 580 MeV, and 2 cases at 750 MeV. However, the radial displacements

$$(\sqrt{x^2 + y^2})$$

are always smaller than the expected values. Since these 50 runs are performed for a particular particle, we will have to repeat the similar calculation for many different particles. Nevertheless, we feel that the tolerance requirements for doublets are not too stringent to achieve.

Loss of Particles due to Failures of rf Tanks and Focusing Magnets[12]

The computer program for particle motions in the iris section can be used to find out whether the loss of particles is, in general, localized or extended over a long distance when certain tanks or a pair of focusing magnets fail.

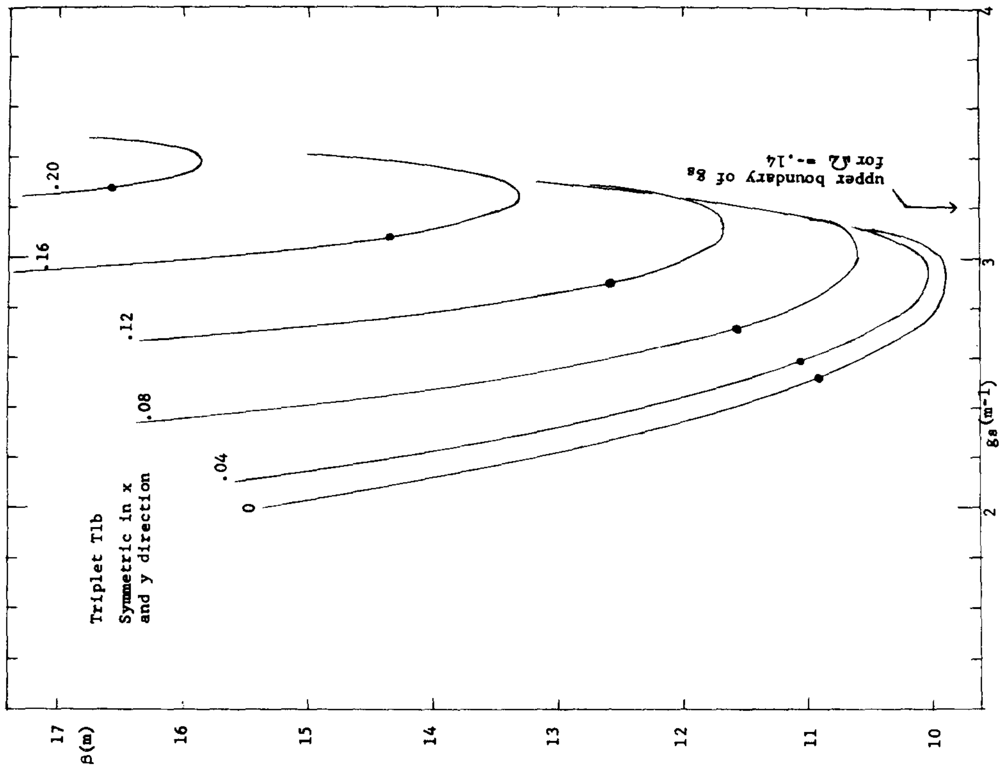
Triplet T1b
Symmetric in x and y direction
upper boundary of g_s for Ω = -.14
β(m)
g_s (m^{-1})
0
.04
.08
.12
.16
.20
17
16
15
14
13
12
11
10
2
3
4

FIG. 11

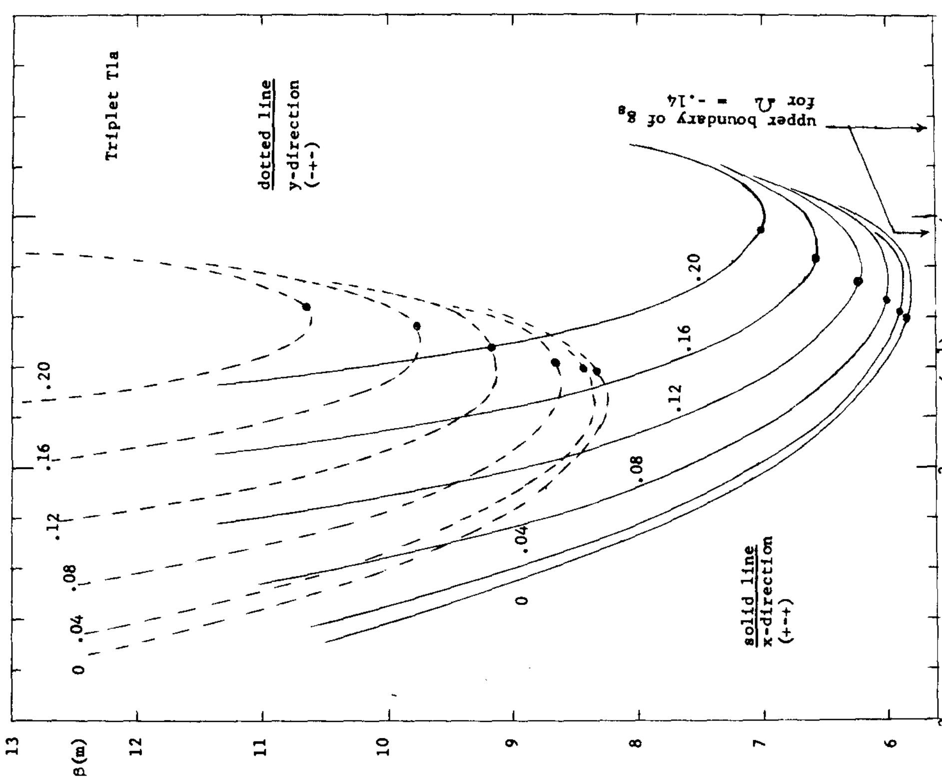
Triplet T1a
dotted line
y-direction
(-+-)
solid line
x-direction
(+-+)
upper boundary of g_s for Ω = -.14
β(m)
g_s (m^{-1})
0
.04
.08
.12
.16
.20
13
12
11
10
9
8
7
6
2
3
4

FIG. 10

If an amplifier tube fails, two consecutive tanks will be de-excited and there will be no increase in particle energy. The energy gain through two tanks are, in general, greater than the size of the "fish" (in $\Delta\gamma$ direction) so that all particles will become longitudinally unstable after passing de-excited tanks. The energy of such particles fluctuates around a certain value that is very close to what they had just before becoming unstable. For all practical purposes, the problem is then reduced to the transportation of particles with a constant momentum (in z direction) through a focusing system that is optimized for particles with increasing momenta. This means that we can predict where particles will be lost from the stability diagram of the focusing system. A detailed numerical calculation has confirmed this but it has also shown that particles are lost in a relatively long area ($\sim$ 100 m). Particles with 400 MeV or more energy are not expected to be lost at all. It is not possible to retain the bunch inside the longitudinal stability region by simply increasing the rf level in nearby tanks[13].

Since two quadrupole magnets will be getting power from the same source, the problem in case of magnet failures is to find the increase of the transverse amplitudes (A_x, A_y) when one pair of doublet is missing. At the "optical" centers, (x, x') and (y, y') will satisfy

$$x^2 + \beta_{max}\, x'^2 = A_x^2 \; ; \; y^2 + \beta_{max}^{\;2}\, y'^2 = A_y^2$$

with $\quad A_x \leq \sqrt{\beta_{max} W_x} \quad , \; A_y \leq \sqrt{\beta_{max} W_y}$.

W_x and W_y are the phase space area divided by π in x and y direction, respectively. Because of the missing pair, these ellipses will be distorted to other forms (without changing the size):

$$ax^2 + bx'^2 + 2hxx' = 1 \quad , \quad a'y^2 + b'y'^2 + 2h'yy' = 1$$

where $1/\sqrt{ab-h^2} = A_x^2/\beta_{max}$ and $1/\sqrt{a'b' - h'^2} = A_y^2/\beta_{max}$.

If these distorted ellipses are entirely inside the ellipse

$$x^2 + \beta_{max}\, x'^2 = R^2 \text{ or } y^2 + \beta_{max}^{\;2}\, y'^2 = R^2$$

with R = quadrupole aperture, there will be no loss of particles. For R = 2 cm and $A_x = A_y = 1.5$ cm, the fractional loss in both x and y directions changes from 30% at 200 MeV to 20% at 500 – 700 MeV. For R = 1.5 cm and $A_x = A_y = 1.0$ cm, corresponding figures are 40% and 30%, respectively. The important feature here is that most of these particles are lost in a relatively localized region ($\sim$ 15 m) because they can travel the distance which is at most one-half of the transverse wave length (29m) before getting the maximum excursions. Again it is not possible to adjust the strength of adjacent doublets (*) such that the loss would be decreased substantially. Preliminary results of numerical calculations show the recovery of only 5% or less at all energies.

VISSCHER: What is the radius of the aperture in your iris section calculations?

OHNUMA: The aperture radius really does not affect the calculation. It simply gives a criterion when you are concerned about the beam loss. Theoretically, the size of the beam would be about 1.5 cm or less. In calculating the misalignment effects, we used 2 cm radius.

VISSCHER: The value π cm-mrad of the transverse phase-space area you are using must have been based on some aperture radius.

OHNUMA: No. That is what we get out of the drift tube section independent of the iris section. The acceptance of the iris section is of course much larger. The calculation is a continuous one for both sections.

LAPOSTOLLE: About the possibility of a correction of the missing gap and drift spaces between tanks, especially in the Alvarez section, I think that, for transverse motions, it is possible to compensate with a slight correction of the quadrupole magnets. I wonder whether, for the longitudinal motion, it would not be possible also to compensate by a slight displacement of the accelerating gaps in order to change the phase when the bunch crosses the gap and modify the phase-focusing condition. Did you think about the idea of trying to compensate for any disturbance due to spaces between tanks, possibly by some modification at the end of the tank?

(*) Here it is assumed that the strength of two magnets of a doublet can be changed only simultaneously.

OHNUMA: We have not studied this problem in any detail in the Alvarez section. I believe that in the actual operation one must always make fine adjustments of the focusing magnets. As to the compensation for the longitudinal motion, it seems very difficult to adjust unless we can accurately measure the position of the bunch relative to the rf field in tanks. In the iris section, unless you change the strength of each magnet independently, even the transverse adjustment is not easy. You must remember that each doublet is 7 m apart. If one tank is out of operation, the bunch is completely out of the longitudinal stability region and there is no way of recovering it into the bucket. Of course, the focusing system still could transport this low energy beam down to the end without any beam loss. It depends on the energy acceptance of your focusing system.

KNAPP: You have two drift tubes close to one another that have magnets in them and then you have a large series of drift tubes with no magnet in them. Could you describe the distribution of magnets in the last four tanks of the Alvarez section?

OHNUMA: In Tank No. 3, there are 14 pairs for 46 full drift tubes; in No. 4, 6 pairs for 34; in No. 5, 4 pairs for 28; in No. 6, 3 pairs for 25; in No. 7, 3 pairs for 23 full drift tubes. The distance between the center of a focusing magnet to the next one, the magnet repeat length, ranges from 0.89 m to 7.3 m. The value of β_{max} changes gradually from 1.45 m to 12.5 m.

TENG: About the longitudinal phase space matching between the drift tube section and the iris section, have you considered the use of dispersive magnets? The distortion of the bunch in the longitudinal phase space due to the drift space can be compensated by such magnets.

OHNUMA: No, we have not thought about it. I don't know how wise it is to construct a special section or system just for this particular purpose. If we can use some regular sections as an option, it will be all right. Besides, such a system would require additional transverse matching systems and might not be desirable to have for the overall matching purpose.

TENG: You might have to bend the beam so that the drift tube section and the iris section are at an angle to one another. As far as the transverse phase space matching is concerned, you can always do that by a number of quadrupole magnets.

WHEELER: The figure of 10 m, which we have been using, was a somewhat arbitrary number which would allow us to have ample space between the sections. We have not looked carefully at how far down we can bring this distance. I expect that it can be brought down quite a lot further, far enough so that it will satisfy beam dynamics requirements.

LEISS: Isn't it true of course that the reason for a short distance is removed if you do put a phase compression system such as Teng was talking about?

WHEELER: Yes, but I would rather try for a shorter distance than to try to put in a major magnet system which would be expensive, because of a longer tunnel length and require additional complex equipment and be consuming in manpower.

WALKINSHAW: Have you looked at the possible advantages of coming down in frequency to 600 Mc or even to 400 Mc where, I think, this problem would be less serious?

WHEELER: It would be nice to come down to 600 or 400 Mc for the purpose of solving this problem but the cost is going up proportionately. I think we are caught here in an attempt to keep the cost down but still meet the dynamics requirements. Based on our numerical calculations, we feel that we can operate at 800 Mc.

REFERENCES

1. Ohnuma, S., Minutes of the Conference on Proton Linear Accelerators at Yale University, October 1963, p. 14.
2. Ohnuma, S., ibid., p. 273.
3. Comment by R. Beringer in Ref. 1 above.
4. Gluckstern, R. L., Internal Reports Y-1 and Y-4, April and September, 1961, Yale University.
5. For example, see Edwards, T. W., MURA Report No. 622, June 15, 1961.
6. Teng, L. C., 1961 International Conference on High Energy Accelerators, Brookhaven National Laboratory, September 6-12, 1961, p. 212.
7. Ohnuma, S., Internal Report Y-10, March 1964, Yale University.
8. Courant, E. D., and Snyder, H. S., Annals of Physics, 3, 1 (1958).

9. Helm, R. H., Technical Report SLAC-15, March 1963, Stanford Linear Accelerator Center.

10. Gluckstern, R. L., Internal Report Y-7, January 24, 1964, Yale University.

11. Courant, E. D., Internal Report IA-3, EDC-43 (1961), Brookhaven National Laboratory.

12. Ohnuma, S., Technical Note S0-5, April 14, 1964, Accelerator Design Study, Yale University.

13. Wheeler, G. W., and Ludlam, T. W., Minutes of the Conference on Proton Linear Accelerators at Yale University, October 1963, p. 29.