NUMERICAL STUDY OF PARTICLE DYNAMICS IN A HIGH-ENERGY PROTON LINEAR ACCELERATOR*

M. Jakobson** and W. M. Visscher
Los Alamos Scientific Laboratory, University of California
Los Alamos, New Mexico

I. Introduction

We report here on the current status of a beam-dynamical calculation which has been evolving at Los Alamos for nearly a year. This paper should be considered as an addendum to and partial summary of a Los Alamos internal memo[1] which was written at the end of April. That report is considerably more complete and detailed than this one; we shall emphasize here the improvements and extensions which have been made since then.

At that time, most of our calculations had been made for a fictitious accelerator with plane symmetry, although some had utilized cylindrical symmetry in the accelerator sections with the beam constrained to move in only one of the transverse planes. Plane symmetry exaggerates the transverse defocusing and radial-phase coupling by a factor of two over the cylindrical case; it was therefore thought that conclusions valid for the plane problem would be pessimistic ones to apply to the cylindrical case. Our subsequent experience with the latter has borne this out.

The principal advance which has been made since our earlier report is the incorporation into the computer code of the other transverse dimension enabling us to calculate the real three-dimensional orbits of the protons as contrasted to 2 or 2-1/2 dimensions before. By 2-1/2 dimensions we mean three-dimensional fields with orbits constrained to two dimensions. We now take proper account of such things as x-y interaction and the effects of random rotations of the quads about the longitudinal axis. The addition of the third dimension has, unfortunately, nearly doubled the running time on the computer, but machine time has not been a serious problem as yet.

Another change which has been made is in the part of the code which designs the accelerator sections. Instead of designing them with a constant rf gradient, as before, we now design them with a constant power

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**Permanent address: Department of Physics, Montana State University, Missoula, Montana.

consumption, for an obvious practical reason. The number of cells per section is, as before, a constant.

II. Method

We give here only a very sketchy description of the method used in the calculation; the interested reader can find much more detail in reference 1.

The π-mode rf field effective in accelerating the beam is assumed to be only that standing wave made up of components which have phase velocities equal to $\pm$ the synchronous particle velocity, and the impulse received by each particle in crossing a cell is approximated by that received by a particle moving across the cell in a straight line parallel the axis at constant speed. We call this the impulse approximation. The justification for these procedures is discussed in reference 1, and seems to be quite solid for the proton energies with which we are concerned.

The longitudinal and radial impulses calculated in this way are

$$I_z = \frac{\pi E_0}{2\omega} I_0(\kappa r) \cos\varphi \tag{1}$$
$$I_r = -\frac{\pi E_0}{2\omega} I_1(\kappa r) \sin\varphi ,$$

where I_0 and I_1 are Bessel functions of imaginary argument, E_0 is the maximum amplitude of the synchronous component of the rf field, $\kappa = \pi / L\gamma$, and φ is the phase of the proton entering the gap relative to the time at which the E field is a maximum. Our convention is to take the synchronous phase <u>negative</u>; thus Eqs. (1) predict a radial defocusing of the synchronous particle as is required by Earnshaw's theorem.

The acceleration received by each particle in each cell is computed by Eqs. (1), and the bunch is thus transported to the end of the section. At this point it enters a magnetic quadrupole lens system, which can be a singlet, doublet, or triplet. On the assumption that the transverse velocity is small compared to the longitudinal velocity, the x and v_x coordinates of a particle, on passing through a quad, are changed to

$$x' = \frac{v_x}{\Omega} \begin{matrix}\sin\\ \sinh\end{matrix} (\Omega t) + x \begin{matrix}\cos\\ \cosh\end{matrix} (\Omega t) \tag{2}$$
$$v_x' = v_x \begin{matrix}\cos\\ \cosh\end{matrix} (\Omega t) \mp \Omega x \begin{matrix}\sin\\ \sinh\end{matrix} (\Omega t) ,$$

where t is the time spent in the quad, and the upper (lower) signs and symbols refer to a focusing (defocusing) magnet. Similar equations are obtained for y' and v_y', with the upper and lower signs and symbols interchanged. Ω is the natural transverse oscillation frequency of a particle passing through the magnet, and is given by

$$(\Omega / \omega)^2 = 3.22 \left(\frac{\lambda}{2\pi}\right)^2 \frac{v_z}{\gamma c} H' \text{ (kilogauss/cm)} \tag{3}$$

for protons.

The bunch is transported through the quadrupoles by Eqs. (2), and allowed to drift through any drift spaces which may be present. Then the computer designs the next section and the process is repeated.

III. Calculations

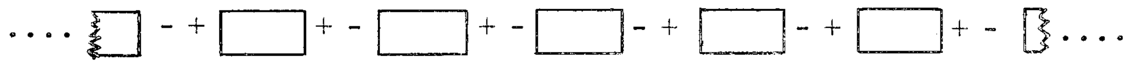
Perhaps the most efficient way to explain the features of the code is to display the input parameters and the form of the output graphs. Figure 2 shows the input parameters for the current version of our 7094 Fortran code. This problem simulates an accelerator of 50 sections, each 40 cells long, separated by magnet systems of length equal to 4 cell lengths. The magnet systems can be singlet, doublet, or triplet; the 4 cell lengths include the internal drift spaces for the doublet and the triplet. As is illustrated in Fig. 1, this setup allows no space between the sections and the lens system for installation of plumbing or probes; we therefore have made provision for some extra drift space before the beginning of the magnet system. This extra drift space input also allows study of the effect of the necessary gap in the accelerator at the frequency transition point, and of a gap for a possible intermediate energy station.

The magnet systems have length proportional to β (line 1); but the quads themselves are, say 10 cm (line 5) long for the whole length of the machine. It takes two quads to make a doublet; four (10-20-10 cm) to make a triplet. This means that the effective strength of the lens increases with the energy if the quad strength stays constant. Line 5 and line 6 of Fig. 2 contain the rest of the focusing system parameters, contained in

$$H' = H_0' \ (1 + c\,E^n)/(1 + c\,E_0^n) \ . \tag{4}$$

c in Eq. (4) is the "quad coefficient", the synchronous energy E is raised to the "quad exponent" n, and H_0' is the field gradient (in kG/cm) of the quads at the beginning of the machine. We have not made an exhaustive study of variations in c and n; what we have done, however, indicates that the optimum is near c = n = 0.

In the 7th line are two remaining parameters which complete the description of the focusing system. The orientation of the first quad encountered by the beam is specified by saying whether it is converging or diverging in x. The repeat length is the periodicity of the lens system. For example, a doublet system with 4 section repeat length would be, schematically,



A repeat length between 6 and 12 sections seems best for triplets, and an infinite repeat length does well for doublets. Short repeat lengths give rise to unstable intervals in the stable range of quad strength.

More parameters to determine the geometry are in line 3 of Fig. 2. Synchronous input energy and design phase are self-explanatory. Phase shift and design beam load are numbers which describe the energy flow along the section, from cell to cell. We refer to reference 1 for discussion of them; here we only say that these and other effects which cause variation in phase velocity of the rf in time and position seem to cause no serious trouble. We have studied machines with constant cell length within a section, rather than phase velocity = "synchronous" particle velocity everywhere, which is what the code usually requires. Constant cell length has no noticeable deleterious effect for 40 cells per section; for much longer sections, however, it reduces the phase acceptance.

The hole radius has no effect on the fields, since we use only the lowest Fourier component; its only function is to provide criterion for deciding whether a given proton is still in the machine. We throw out particles when their radii exceed R_O, and only then. Particles often spill out of the phase-energy bucket and stay in the machine to the end due to the action of the quads. Our results indicate, in fact, that the rf can be turned off in a couple of sections near the start, resulting in the loss of the entire beam from the bucket, but the entire beam will be transported to the target. Also, the rf can be turned off entirely beginning at any point and the beam will coast out, or a few sections can be run with reversed rf and all subsequent ones with rf off. In this latter case a 600 MeV beam, say, can be made to come out at the 800 MeV end of the machine with the microstructure wholly debunched and a spread of 20 MeV. The drift length is shorter if a higher energy spread is allowable, and vice versa. Thus a variable energy machine could be designed with a maximum energy of 800 MeV (no debunching) and a completely debunched beam available from 700 MeV on down. Reference 1 contains a more quantitative discussion of this point.

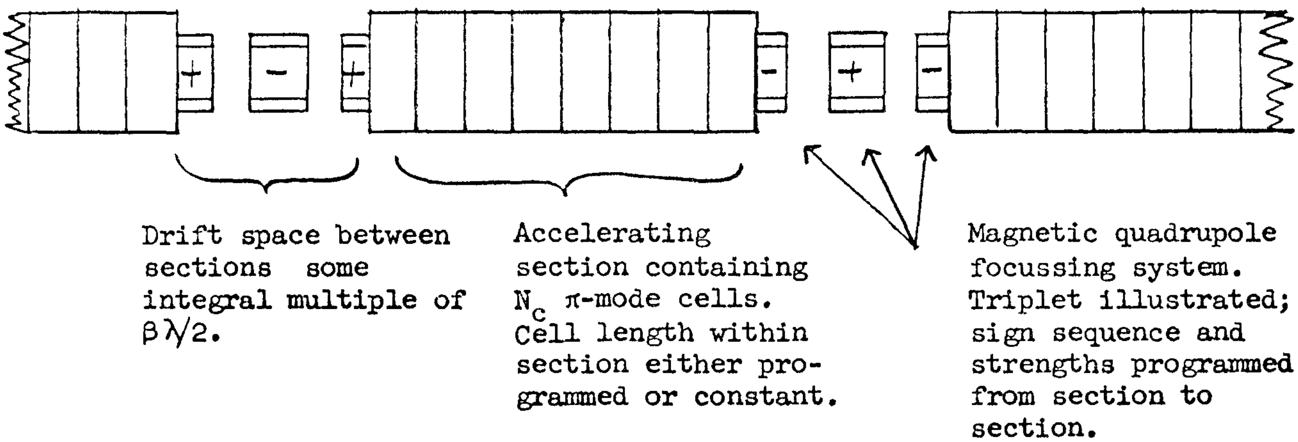


Figure 1. Schematic diagram of accelerator structure.

50 SECTIONS 40 CELLS PER SECTION 200 PROTONS IN BUNCH MAGNET SYSTEMS 4 CELLS LONG

DRIFT SPACE OF LENGTH 2 CELL LENGTHS AFTER THE 100TH SECTION REPEATED EVERY 1TH SECTION

INPUT ENERGY= 1.765E 02 DESIGN PHASE= -3.000E 01 PHASE SHIFT= 0. DESIGN BEAM LOAD= -0.

HOLE RADIUS= 2.000E-02 SECTION POWER= 5.000E-01 RANDOM START AT 1.900E 01

INITIAL QUAD GRADIENT= 3.000E 00 QUAD LENGTH= 1.000E-01 FREE SPACE WAVELENGTH= 3.750E-01

QUAD COEFFICIENT= -0. QUAD EXPONENT= -0

TRIPLET LENS SYSTEMS FIRST QUAD DIVERGING IN X 12 SECTION REPEAT LENGTH

SECTIONWISE AMPLITUDE ERROR= -0. CELLWISE AMPLITUDE ERROR= -0. SECTIONWISE PHASE ERROR= -0.

QUAD FIELD ERROR= -0. QUAD DISPLACEMENT= 10.000E-04 SECTION TWIST= -0. QUAD ROTATION= -0.

RF AMPLITUDE MULTIPLIED BY 0. FOR 0 SECTIONS STARTING WITH NUMBER 100

ENERGY LIMITS 1.765E 02, 1.765E 02 PHASE LIMITS -6.000E 01, 3.000E 01

X LIMITS -2.000E-02, 2.000E-02 Y LIMITS -2.000E-02, 2.000E-02

VR LIMITS -4.000E-03, 2.000E-03 VTH LIMITS -0, , -0.

Figure 2. Input Parameters for Computer Code.

Section power (megawatts) is in the input because rf gradient isn't. The sections are designed to consume a given amount of power with a 20 mA proton current and a ZT^2 given by an empirical formula for the cloverleaf structure (Knapp, LASL) .

$$ZT^2 = 65\,\beta/(\beta + 0.76)\ \mathrm{M}\Omega\ /\mathrm{m}\ . \qquad (5)$$

We chose 1/2 Megawatt as the section power because the power supplies envisioned at LASL will deliver ~ 1 Megawatt and splitting between two sections is feasible and will give sections of reasonable lengths and rf gradient; namely, 40 cells (4 meters up to 6 meters at 800 MeV) and 1.35 at 175 down to 1.20 MV/m synchronous particle energy gain at 800 MeV. The code designs each section with a synchronous phase velocity and converges on a given section power iteratively.

Random start has nothing to do with the machine or beam dynamics; it merely tells where to start reading the random number table. Random numbers are used for generating the seven "standard" kinds of random errors (uniformly distributed) listed in lines 8 and 9 (we have studied the effects of a number of other errors, besides those listed here), and also for generating the proton bunch. The bunch parameters are those in the last 3 lines of Fig. 2. It is characterized by upper and lower energy limits, phase limits, x and y, v_r and v_θ limits. All the particle coordinates but phase are chosen by the random number generator.

IV. Results

Our code should be able to given an answer to the question, "Are doublets or triplets better at focusing the beam in the high-energy linac?" The answer, we think, is that triplets are. Triplets give an acceptance zone in transverse phase space about 50% bigger than doublets in the example shown on Fig. 3. π milliradian-cm corresponds to the area of the small rectangle; we see therefore that either doublets or triplets are easily capable of accepting the transverse output thought to emanate from the drift tube section. Random errors of 1 mm in displacement of the lens system parallel to the axis cut the acceptance area by about 30% for each case. Radial beam confinement as reflected by the rms radii for the bunch seems about the same for triplets as for doublets; this is shown for triplets on the bottom curve on Fig. 4. Our tentative conclusion is that on the basis of simplicity and lower cost, doublets are preferable, unless doublet tilt or independent quad alignment tolerances prove to be excessively stringent, or very large transverse acceptance turns out to be necessary.

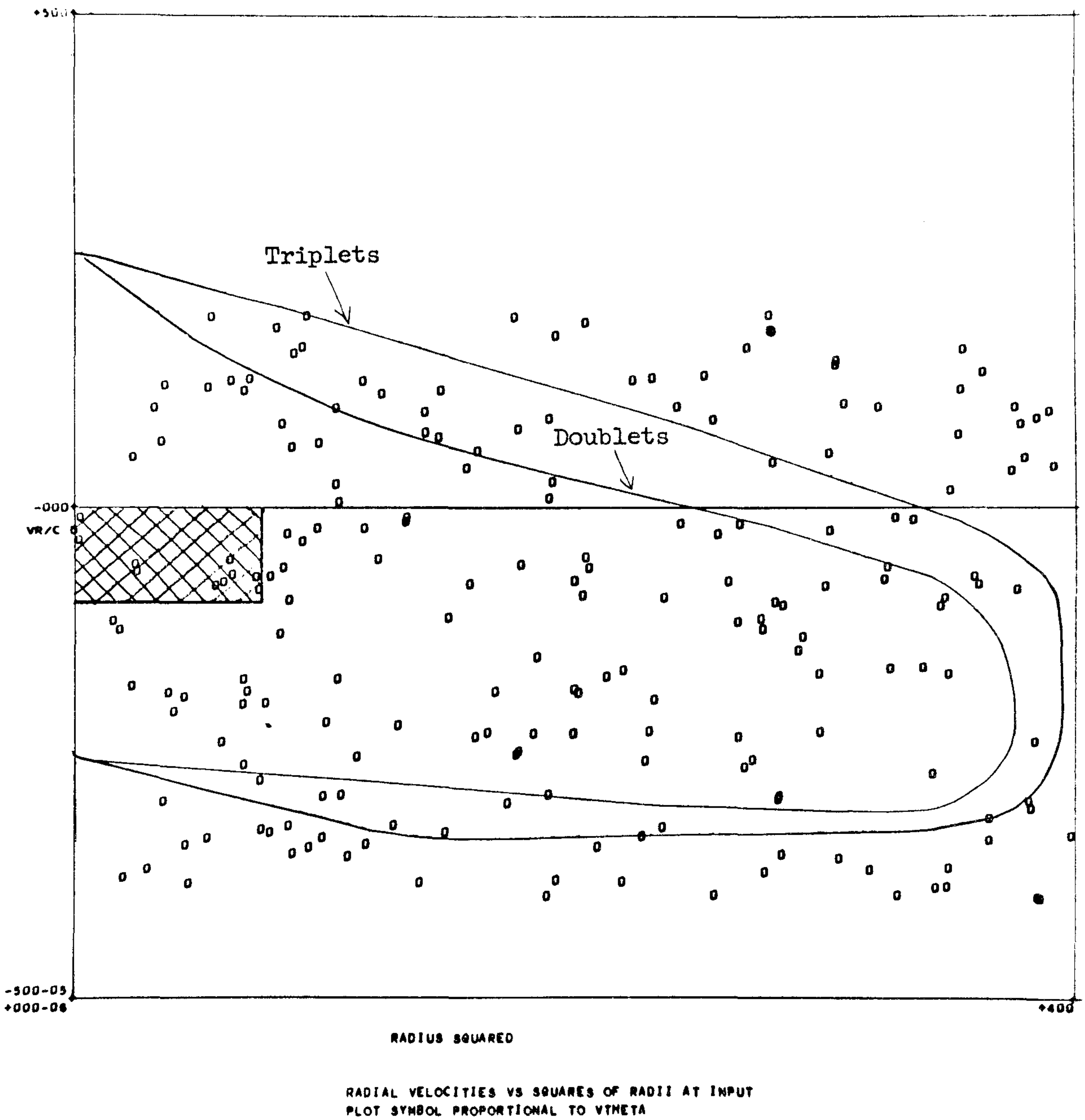


Fig. 3: Approximate acceptance envelopes in transverse sphase space for doublet and triplet focussing systems, both without errors. Crosshatched rectangle encloses π milliradian-centimeter beam area.

As an example of how the effect of a particular kind of error may be evaluated, we take a run for which the elements of the triplets are independently and randomly rotated about the longitudinal axis, up to a maximum angle of $\pm 0.5^{\circ}$. Figure 5 shows the transverse input, a rectangle in x, y space translated to r^2, v_r. Figures 6a, 6b, **6**c are a series of phase-energy plots of the bunch as it progresses down the machine. Figure 7a, finally, is the summary plot, showing the rms radius and phase deviation from synchronous of the bunch, radius and phase of a particular particle, and the total number of surviving protons, all as a function of section number. Especially interesting here is the rms radius, which should be compared with Figs. 7b, 7c, which show the same for 0 and 1° quad rotation, respectively. See also Fig. 8, which is the summary graph for doublets with 1° quad rotations.

The effects of the other kinds of errors which we have studied (and are enumerated and discussed in reference 1) can be summarized by saying that no obviously impossible tolerances are imposed by beam dynamical requirements on the geometrical and electromagnetic parameters which we have varied. For example, sectionwise rf amplitude and phase tolerances are of the order of 2% and 2°. Tank axes can be a millimeter out of alignment (for 2 cm radius aperture), while the allowable spread in indiviudal quadrupole field gradients is about 1%. Bench alignment tolerances for the triplet lens systems are a few mils; we have not yet determined them for doublets.

LAPOSTOLLE: You mentioned the possibility of debunching the beam in the last sections, especially when you use your machine for a slightly lower energy. I think that for some experiments it might be interesting not to debunch but to use an rf structure, very short bunches. Would it be possible to keep the rf structure even at a lower energy.

VISSCHER: Yes, well it depends on what energy you want. It is not possible to get the rf structure out if we let it drift all the way from, let's say, 300 MeV, because then the energy spread in the bunch will itself be sufficient to debunch, I think.

WALKINSHAW: You could keep your rf on and change the respective phasing along the tanks if you recalibrate your phasing.

LAPOSTOLLE: That was my original question.

VISSCHER: Yes, but it wouldn't be something simple like just turning off the rf.

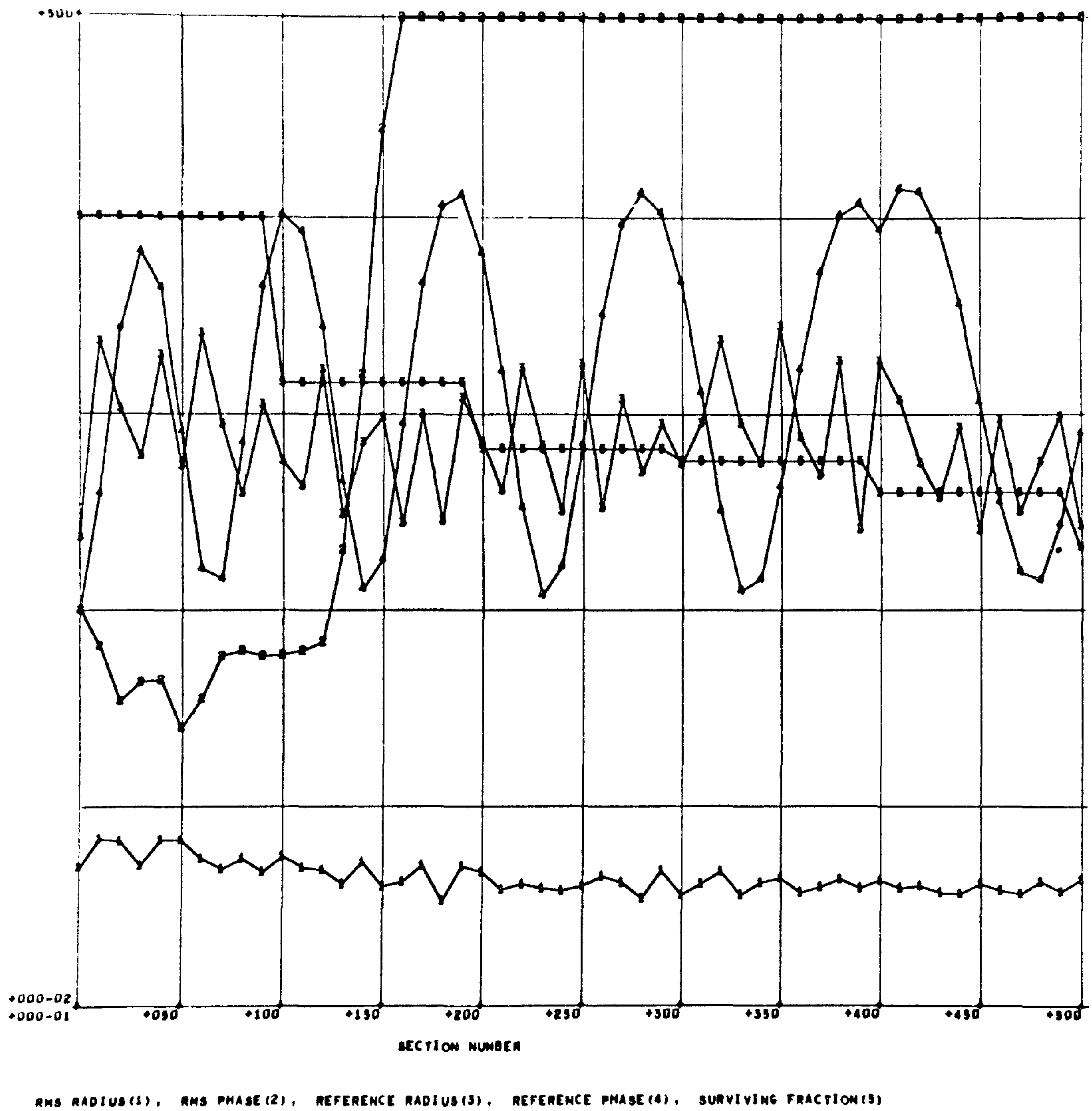


Fig. 4: Summary graph for triplet system. Particles are rejected for excessively large radius every 10 sections; the r.m.s. phase goes off scale because protons are lost from the phase bucket without blowing up radially. The "reference particle" is always that proton whose initial phase is smallest among the surviving particles. Quad displacement error for this run was 1 mm.

TENG: It would be possible only if the sections were short enough.

VISSCHER: Maybe it wouldn't be possible because the section length and the cell length wouldn't at all correspond with the beta of the particle that is drifting through.

WALKINSHAW: I don't think that matters.

TENG: Well it doesn't matter if it is short enough, but are they?

VISSCHER: In order for an accelerator section to be of any use in retaining the bunch structure, the number of cells in the section has to be less than about

$$N < \frac{\gamma(\gamma - 1)(\gamma + 1)}{4\,\Delta\gamma},$$

where $\Delta\gamma$ is the difference (mass units) between the synchronous energy and the beam energy. Thus, for example, in order to use a 700 MeV section to retain rf structure in a 600 MeV beam, the section length would be limited to 27 cells. (Otherwise the phase change of the beam within the section will exceed 180°.) For an energy difference of 300 MeV, say, this restriction becomes impossibly stringent. The beam could be coasted out with a bunched structure by inserting special rf bunchers between sections, but I don't think the accelerating sections themselves could be used for this prupose.

WALKINSHAW: Could you give a figure for the growth in transverse phase space area (with misalignment errors) relative to the phase space area of the synchronous particle? That is the increase in the transverse phase space area when errors are introduced.

VISSCHER: We have not yet made extensive studies of the transverse motions in three dimensions, but from those we have done combined with some extrapolations from the two-dimensional runs, I would expect the transverse phase space area of the beam to increase by less than a factor of two from 200 to 800 MeV with reasonable errors in quad alignment and rotation and sec tion tilt.

REFERENCES

1. Informal report, "A Numerical Study of Radial and Phase Stability in a High-Energy Proton Linear Accelerator," Los Alamos, 30 April 1964.

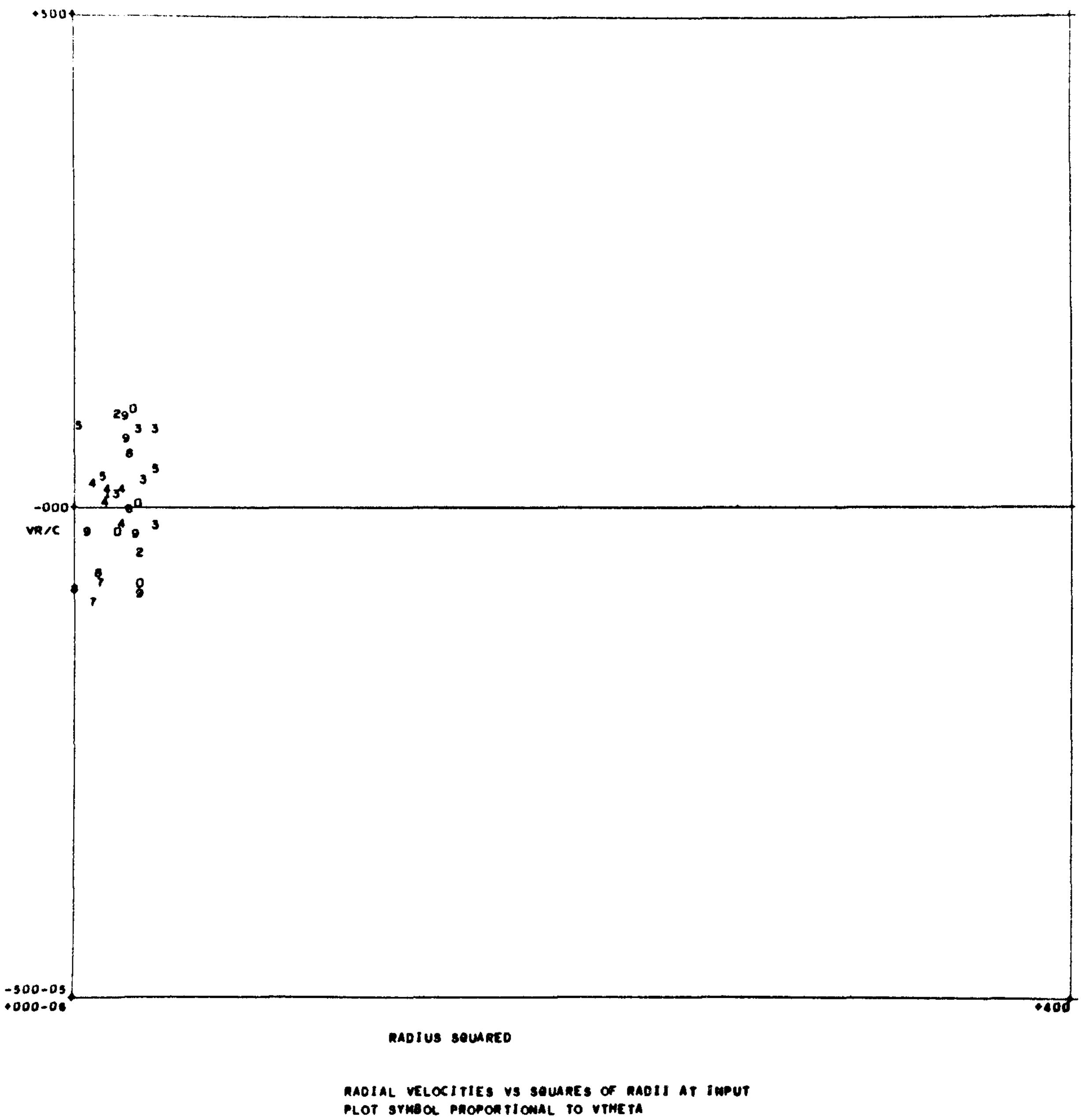


Fig. 5: Transverse input for quad rotation run. Area corresponds to 4 milliradian-cm. A plot symbol n means that $n/10 \leq v_\theta/v_{\theta\ max} < (n+1)/10$.

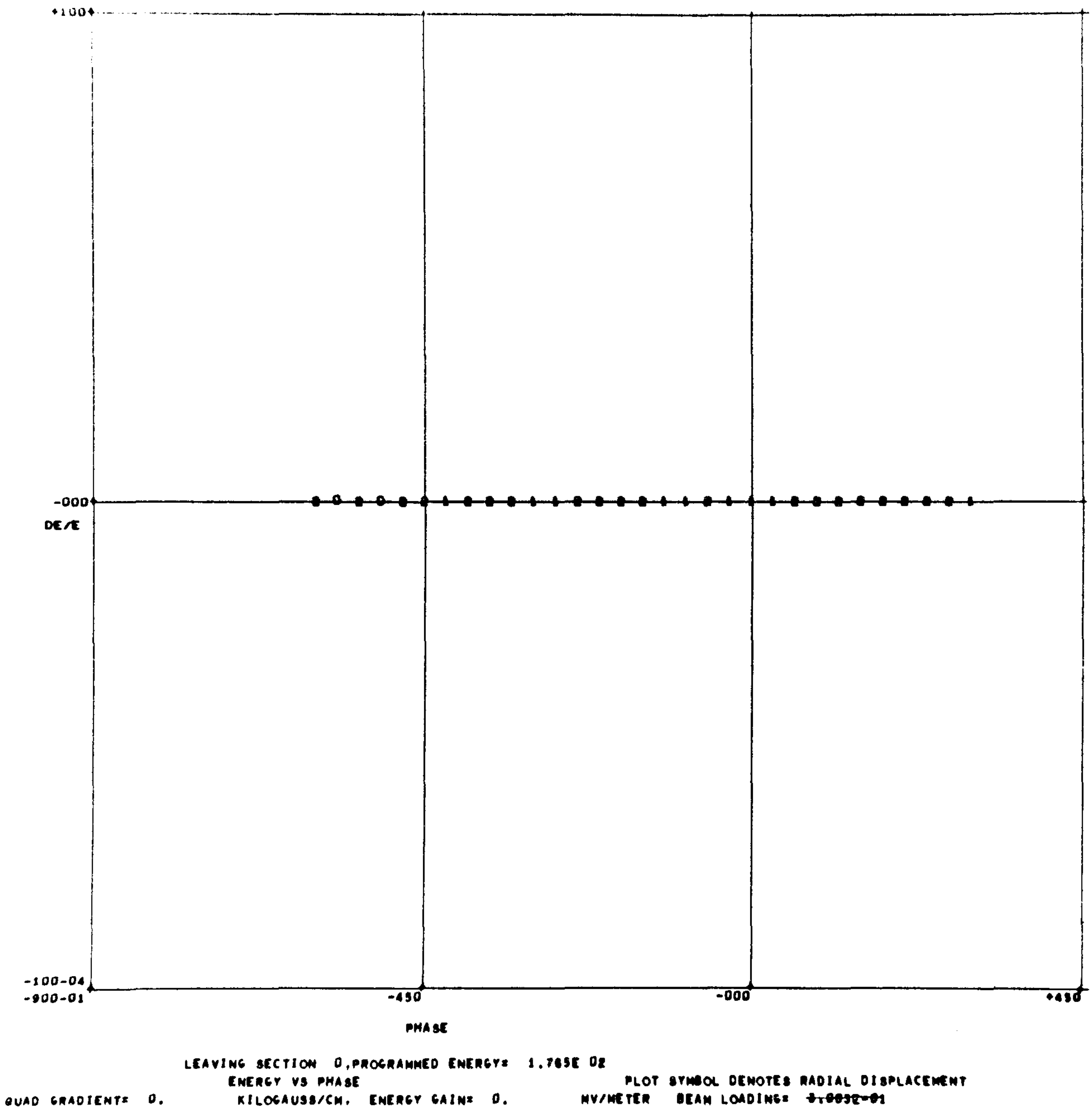


Fig. 6a: Phase-energy input for quad rotation run. Phase interval is 3 times synchronous phase; energy interval is zero, but the points will fill most of the range of the ordinate after the first section, since the phase oscillation wavelength is less than 3 sections for near-synchronous particles. A plot symbol n means the proton has $n/10 \leq r/R_0 < (n + 1)/10$.

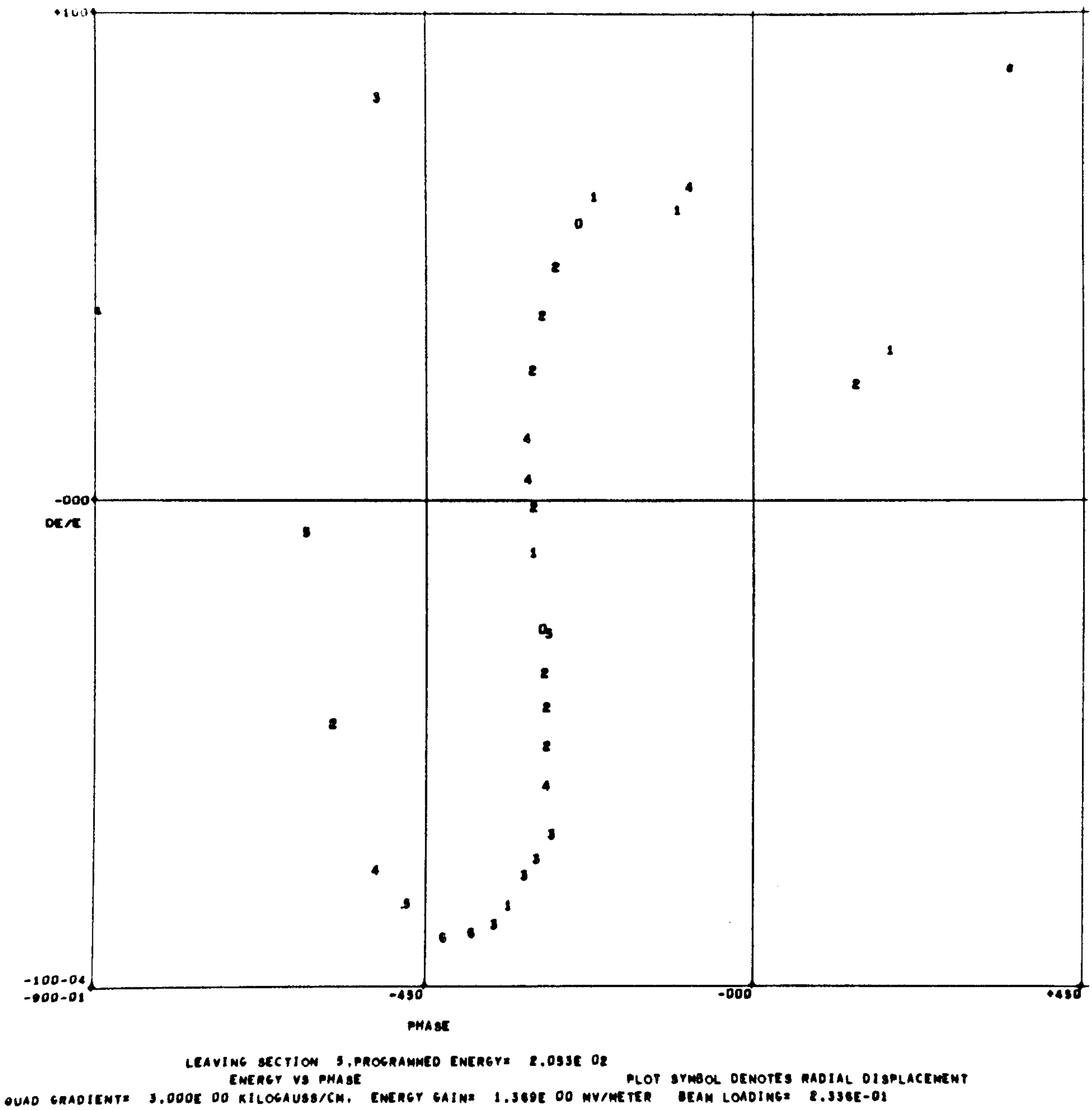


Fig. 6b: The same as Fig. 6a, but after 5 sections. The points would lie on a smooth curve were it not for the radial-phase interaction which causes the phase development to depend on the (initially random) transverse position. At this point the pattern has already rotated more than $1\frac{3}{4}$ times.

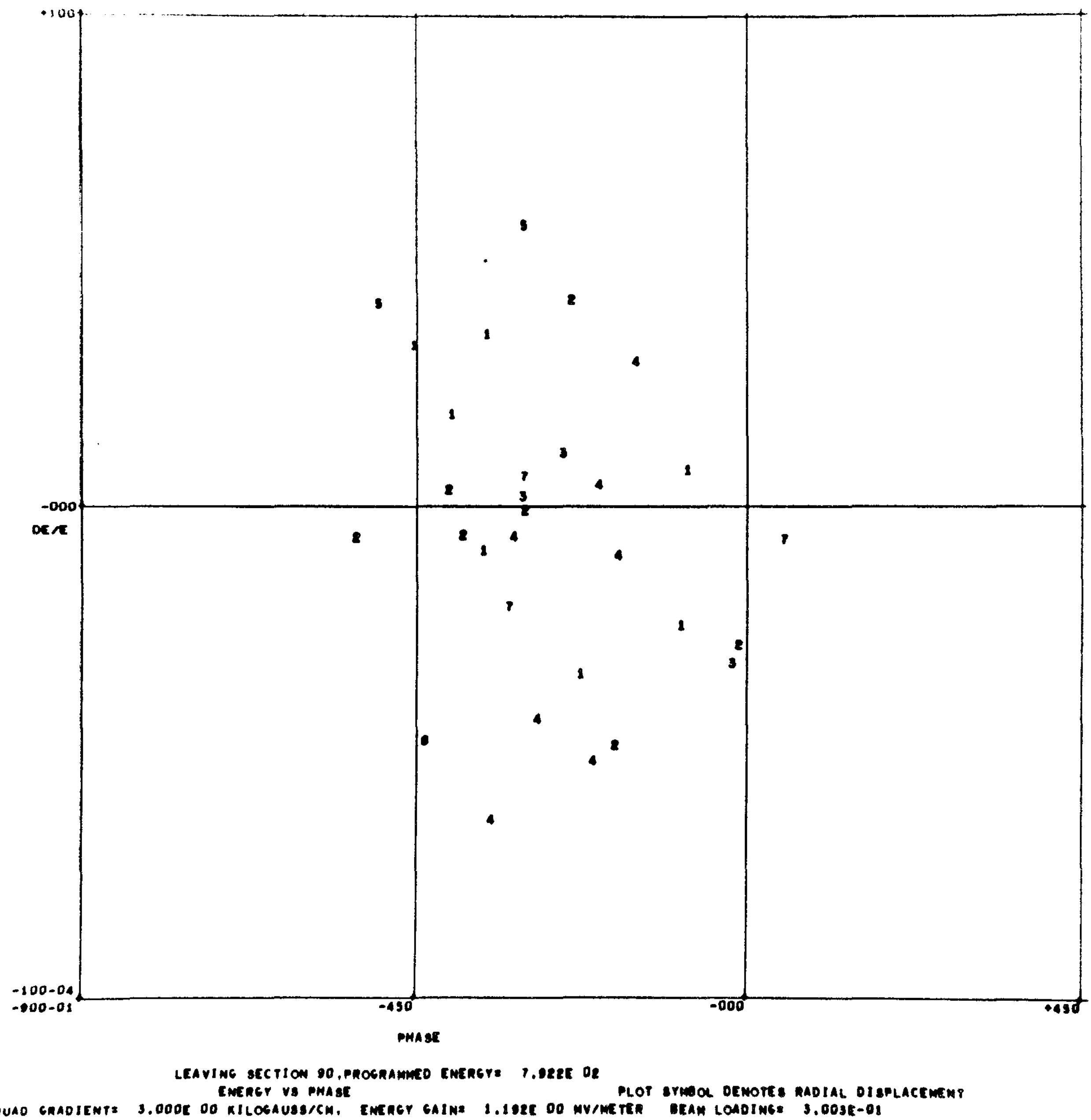


Fig. 6c: Same as Fig. 6b, but after 90 sections. Phase compression is noticeable; the wrapped-up line of the input is still recognizable.

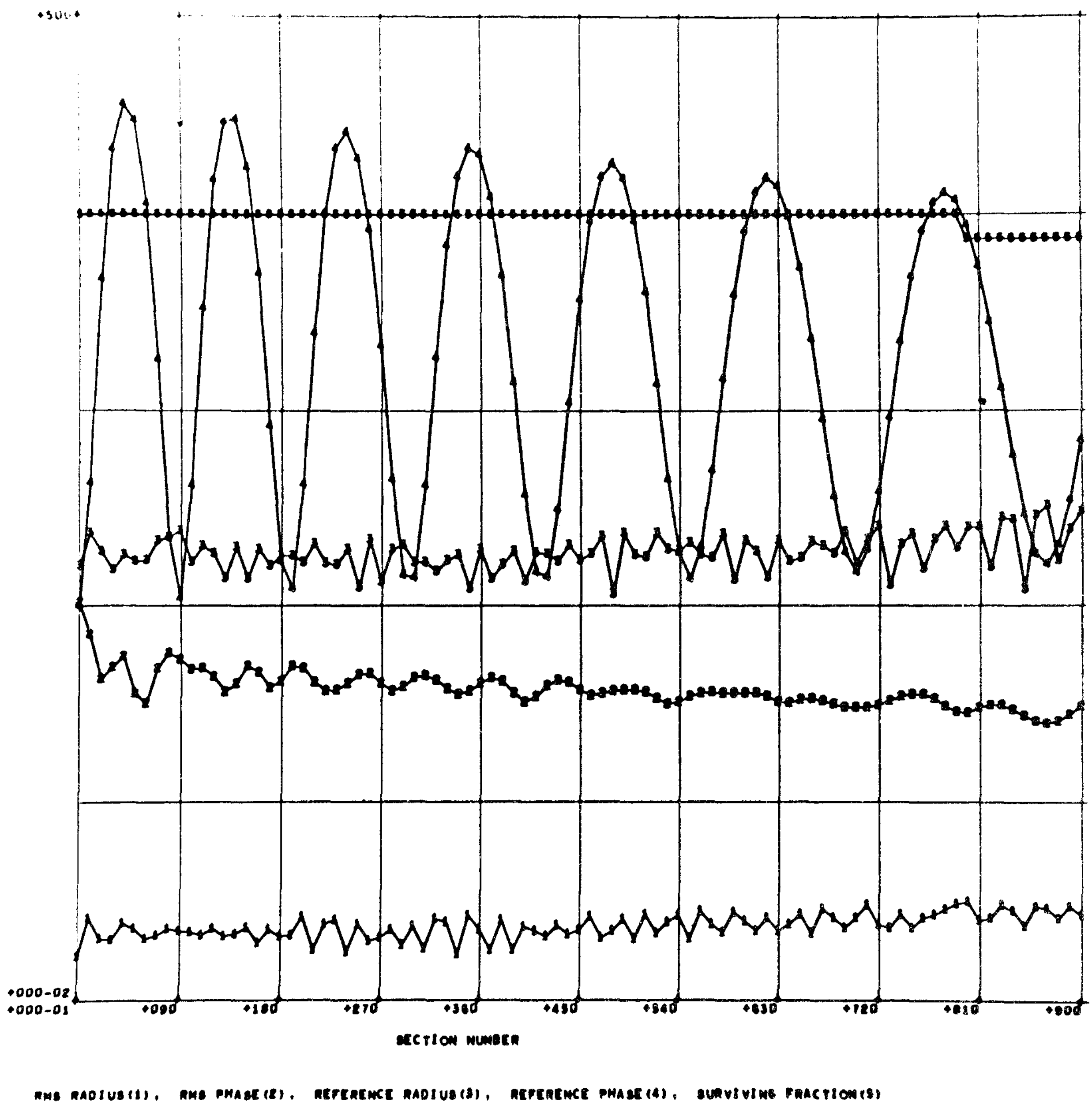


Fig. 7a: Summary plot for run with quad rotation errors of 0.5°. The r.m.s. radius shows a slight buildup, and a particle is lost radially at section 80.

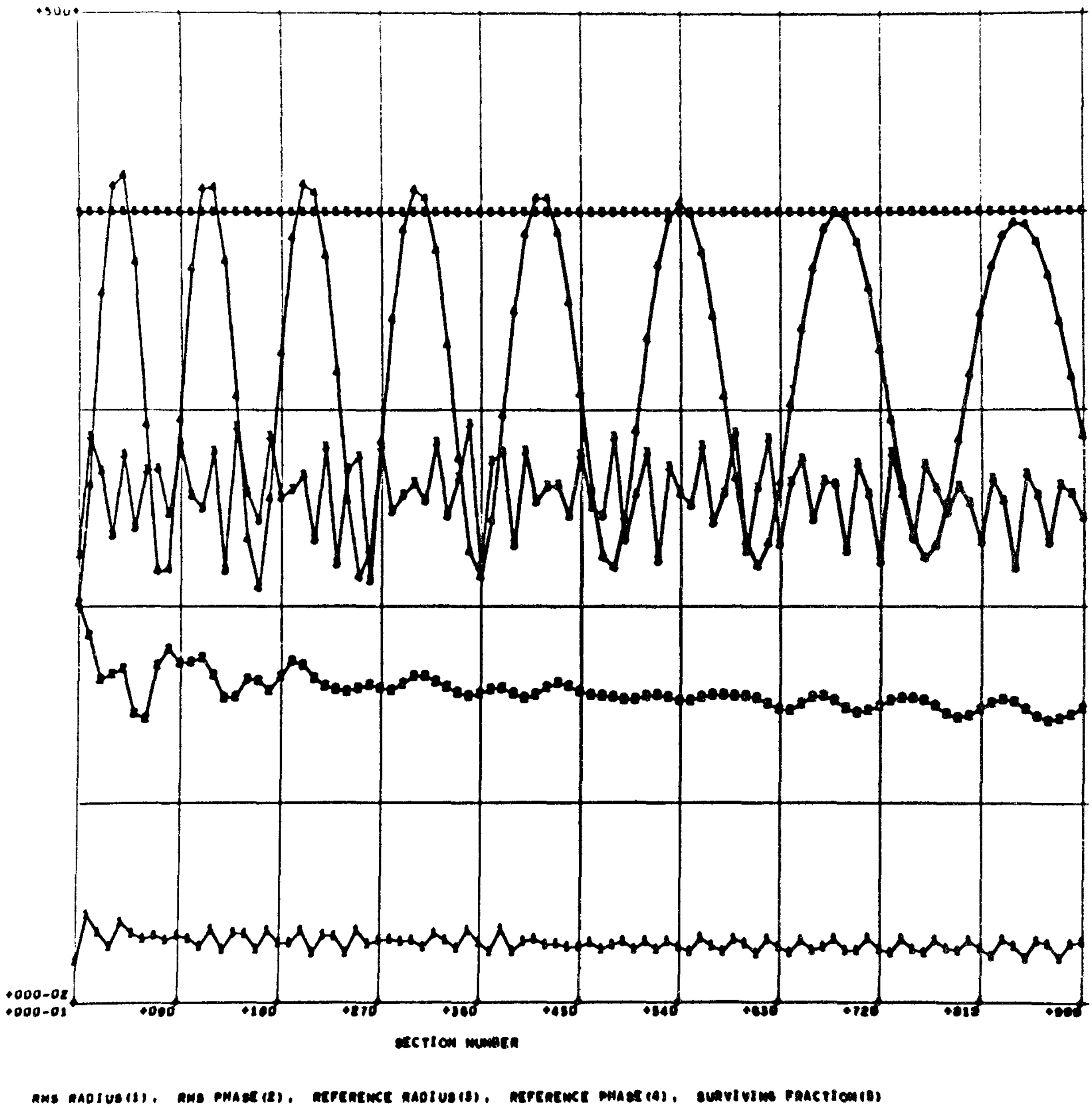


Fig. 7b: Same as Fig. 7a, but with no errors. R.M.S. radius is quite flat.

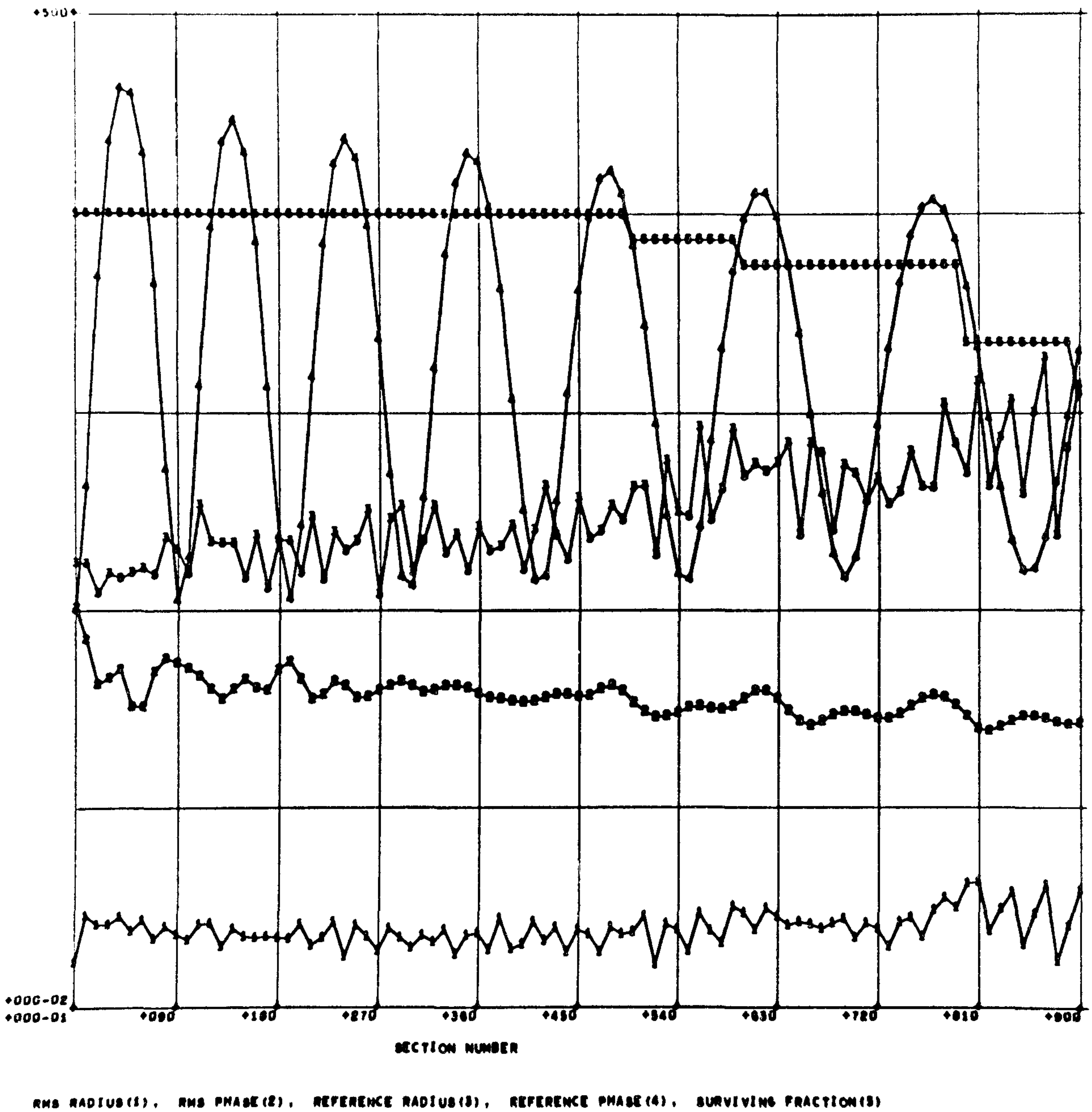


Fig. 7c: Same as Fig. 7a, but with 1° rather than 0.5° quad rotation errors.

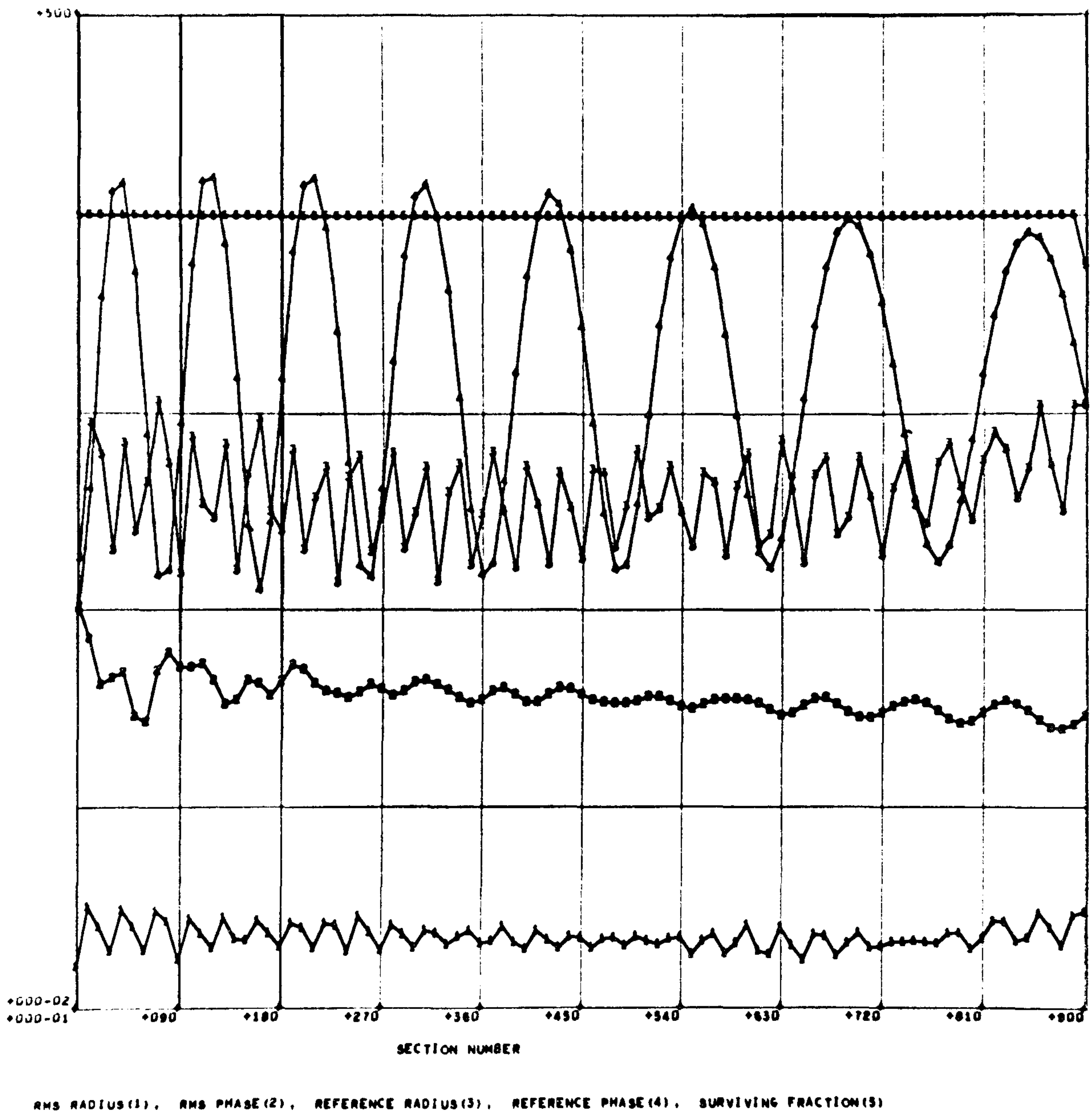


Fig. 8: Summary plot for doublets with quad rotation errors of 1°,