THOUGHTS ON MODE DISTRIBUTION IN RF MANIFOLDS

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I first reported on this idea of an rf manifold at the Yale meeting. I want to add to these ideas some further work we have done regarding the mode structure of an rf manifold system. For the benefit of those who are not familiar with it, I will describe such a system. If we have a long linear accelerator cavity which we want to excite, we parallel it with a transmission line or wave guide. At the ends we short circuit the transmission line so that there are an integral number of half-wave lengths along its length at the operating frequency of the linac. This defines a number of E_{max} points along the length of the transmission line. It is permissible to connect to these E_{max} points, but all other points are forbidden.

Now the linac may be divided up into N parts, and let each section be connected to an adjacent E_{max} point on the transmission line so that there are n half-wave lengths between load points. Thus the whole system is $nN\ \lambda_g/2$ in length.

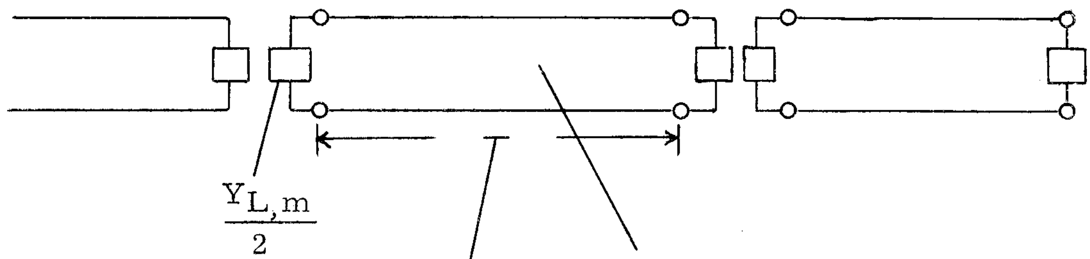
We have let the section of linac be a π-mode rf cavity. See Fig. 1. This figure shows the $\omega - \beta\ell$ diagram for such a cavity and the equivalent circuit for one of the resonances. L_m, C_m, G_m and M_m are the circuit parameters defined at the input loop to the cavity.

Since the cavity must be coupled to the transmission line (or rf manifold), we will arrange the length such that the value of M_m is tuned out and the parallel circuit appears at the E_{max} or load point. This is a very tight coupling to the manifold. The next several modes to the desired one are so close in frequency to ω_o that the value of M_m is effectively tuned out for these also.

We have let these values of

$$Y_{L,m} = G_m (1 + j\, 2\, \delta_m Q_m) \quad \text{where} \quad \delta_m \equiv 1 - \frac{\omega^2}{\omega_m^2} .$$

Now we represent the rf manifold in this way.



$$\beta \ell = \frac{2n}{\lambda g} \ell = n\pi \quad \text{on frequency}$$

The loaded manifold is now a loaded transmission line, and we will let it have the parameters α_L, β_L, and it has a characteristic admittance Y_{oL}. The equation governing the values of α_L, β_L as a function of α and β is the following.

$$\cosh \nu_L \ell \equiv \cosh \nu\ell + \frac{Y_L}{2\,Y_o} \sinh \nu\ell,$$

where $\nu_L = \alpha_L + j\,\beta_L$ and $\nu = \alpha + j\,\beta$. These lead to the equations

$$\cosh \alpha_L \ell \cdot \cos \beta_L \ell = \cosh \alpha\ell \cdot \cos \beta\ell + \frac{G_m}{2\,Y_o}\left[\sinh \alpha\ell \cdot \cos \beta\ell - 2\,Q_m\,\delta_m \cosh \alpha\ell \cdot \sin \beta\ell\right]$$

$$\sinh \alpha_L \ell \cdot \sin \beta_L \ell = \sinh \alpha\ell \cdot \sin \beta\ell + \frac{G_m}{2\,Y_o}\left[\cosh \alpha\ell \cdot \sin \beta\ell + 2\,Q_m\,\delta_m \sinh \alpha\ell \cdot \cos \beta\ell\right]$$

In order for the transmission line to have the properties we want for a manifold, we let the following be true.

$$\alpha\ell << 1 \text{ so that } \cosh \alpha\ell \cong 1 \text{ and } \sinh \alpha\ell \cong \alpha\ell .$$

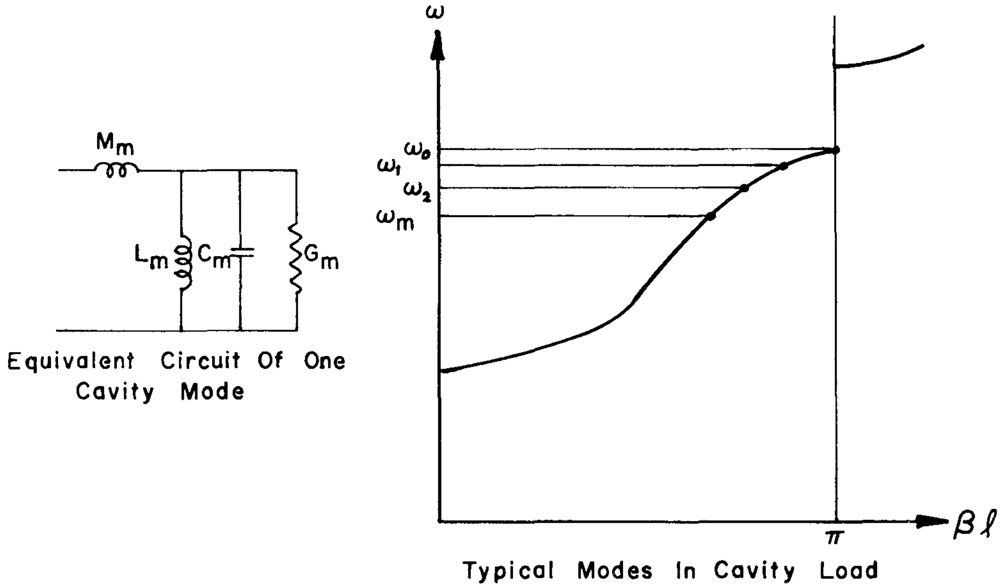

Mm
Lm
Cm
Gm
Equivalent Circuit Of One Cavity Mode
ω
ω₀
ω₁
ω₂
ωm
π
βℓ
Typical Modes In Cavity Load

FIG. 1

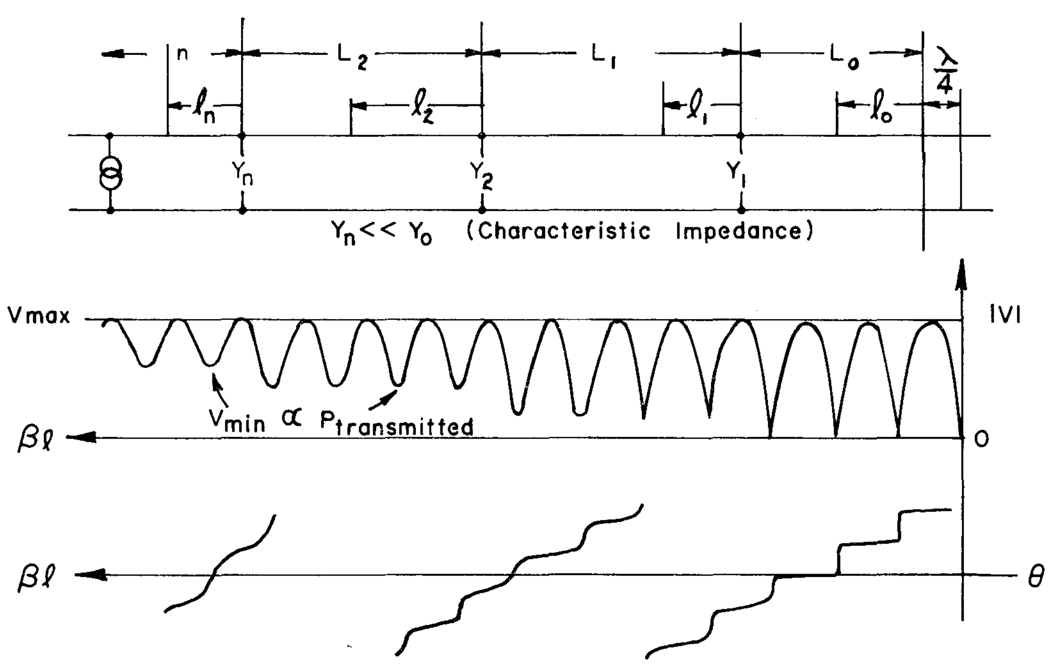

n
L2
L1
Lo
λ/4
ℓn
ℓ2
ℓ1
ℓo
Yn
Y2
Y1
Yn << Yo (Characteristic Impedance)
Vmax
IVI
Vmin ∝ Ptransmitted
βℓ
0
βℓ
θ

FIG. 2

Also we require that $\beta = n\pi(1 + \delta_o)$ where δ_o is a small quantity. Then

$$\sin\beta \cong \pm\,\delta_o n\pi$$

$$\cos\beta \cong \pm\left[1 - \frac{1}{2}(\delta_o n\pi)^2\right].$$

The above equations then reduce to

$$\cosh\alpha_L\ell \cdot \cos\beta_L\ell = \left[1 - \frac{(\delta_o n\pi)^2}{2}\right]\left\{1 + \frac{G_m}{2\,Y_o}\cdot\alpha\ell\right\} - \delta_o n\pi\left\{\frac{G_m}{2\,Y_o}\;2\,\delta_m Q_m\right\}$$

$$\sinh\alpha_L\ell \cdot \sin\beta_L\ell = \delta_o n\pi\left\{\alpha\ell + \frac{G_m}{2\,Y_o}\right\} + \left[1 - \frac{(\delta_o n\pi)^2}{2}\right]\,2\,\delta_m Q_m \frac{G_m}{2\,Y_o}.$$

Solving these equations simultaneously will give values for α_L and β_L, the propagation coefficients for the loaded manifold.

I have not solved them. There are two cases of interest, that are easy to solve.

Let $\omega = \omega_o$ so that $\delta_o = 0$. Then the two equations become

$$\cosh\alpha_L\ell \cdot \cos\beta_L\ell \cong 1 + \frac{G_o}{2\,Y_o}\alpha\ell$$

$$\sinh\alpha_L\ell \cdot \sin\beta_L\ell \cong 0.$$

The solutions are $\beta_L\ell = n\pi$ and $\alpha_L\ell \cong \sqrt{\frac{G_o}{Y_o}\alpha\ell}$.

Since $\beta_L\ell = n\pi$, the voltage at each E_{max} point differs from that at each other in phase by zero or π, and in amplitude by a factor

$$e^{\sqrt{\frac{G_o}{Y_o}\,\alpha \ell}\, M}$$

If $\frac{G_o}{Y_o}$ is a small number which we desire for other reasons and if the α of the transmission line is low which we naturally try to achieve, then the amplitudes are very nearly the same unless we allow the value of ℓ to be too large. Another way of putting this is that if we have a preconceived value that is acceptable,

$$e^{\sqrt{\frac{G_o}{Y_o}\,\alpha \ell}\, N}$$

will be limited to some value.

A generator can be introduced at any E_{max} point on the manifold, but the nearer the generator is to the loads the smaller the argument above will be, and the more tightly coupled the loads will be to the generator. If more than one generator is added, they are effectively in parallel (making sure that the sign of the phase is correct), and, if they act as current generators, will share the load depending on their drive. Figure 2 shows a section of a manifold with several loads and one generator.

I might explain that since the E_{max} are all locked together, and since $P_{transmitted} = \frac{V_{max} V_{min}}{2 Z_o}$, then the value of V_{min} is proportional to the power transmitted at a given point on the manifold. If there is a plane of symmetry or end wall that no power flows past, $V_{min} = 0$, and V_{min} grows larger as one moves past each load point.

In practice each cavity will have its phase servoed to a reference line from the same master oscillator that drives the amplifiers. Then if the cavities do not wander too far from their correct resonant frequency, each cavity will have its fields locked in phase and in amplitude to each other cavity. If the cavities are not quite at resonance, the amplifiers must supply the reactive energy necessary to keep the cavities in phase. They will be able to do this because

1) the various load cavities should tend to average out the reactive energy required, and

2) in order to overdrive the linac in order to obtain fast rise time, the amplifiers must have considerable surplus of energy available, which can go to supply reactive energy.

An amplitude servo will be required to increase the drive so that beam loading can be supplied by the amplifier tubes. Probably it will be desirable to servo the phase of the drive line to the phase of the anode line, to minimize the losses in the amplifier tubes.

The immediate worry one has when one first sees this scheme is, "what about the other modes"? By going back to these original equations and looking at what happens near $\omega = \omega_o$ or $\delta_m = \delta_o$, the equations become

$$\cos \beta_L \ell \cong 1 - \frac{(\delta_o n \pi)^2}{2} \left[1 + \frac{2}{n\pi} \frac{G_o Q_o}{Y_o} \right]$$

$$\alpha_L \ell \cdot \sin \beta_L \ell \cong \delta_o n \pi \left[\alpha \ell + \frac{G_o}{Y_o} + \frac{2}{n\pi} \frac{G_o Q_o}{Y_o} \right] .$$

The solutions to these equations are

$$\beta_L \ell \cong \delta_o n \pi \left[1 + \frac{2}{n\pi} \frac{G_o Q_o}{Y_o} \right]^{1/2}$$

$$\alpha_L \ell \cong \alpha \ell + \frac{G_o}{Y_o} + \frac{2}{n\pi} \frac{G_o Q_o}{Y_o} .$$

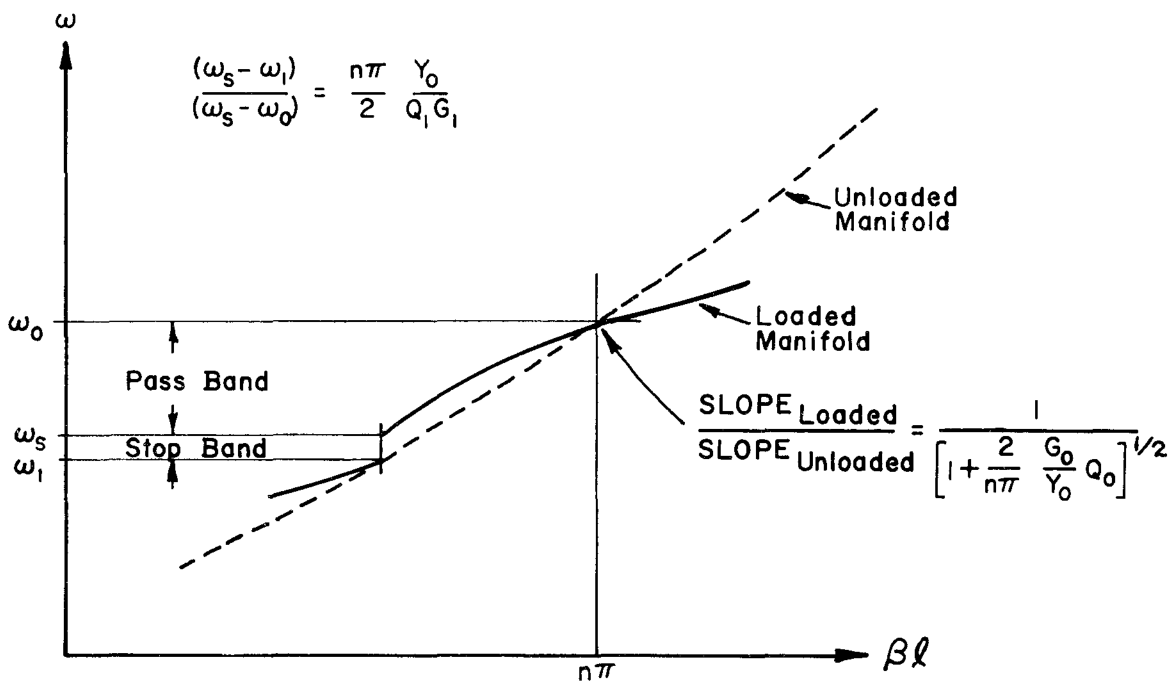
See Fig. 3. Note the slope of the $\omega - \beta \ell$ curve and that there is a stop band adjacent to ω_1, the next cavity mode.

Analogous to the modes along the $\omega - \beta \ell$ curve for the π cavity (see Fig. 1), there will be modes along this $\omega - \beta \ell$ curve if the value Nn is too large. The limits of the frequency where this first permissible point can be found will vary between

$$\frac{1}{nN} < \delta_A < \frac{1}{nN \left[1 + \frac{2}{n\pi} \frac{Q_o G_o}{Y_o} \right]^{1/2}} \; ; \quad \delta_A = \frac{\omega_A - \omega_o}{\omega_A}$$

depending where it falls on the $\omega - \beta \ell$ curve.

Next consider a long cavity in the π mode. Panofsky's formula says that the mode next to the operating frequency is given by $\delta_I = \frac{\lambda^2}{8 M^2 \ell_o^2}$ where $\ell_o = \beta_{particle} \frac{\lambda}{2}$. The total length



ω
$\frac{(\omega_s - \omega_1)}{(\omega_s - \omega_0)} = \frac{n\pi}{2} \frac{Y_0}{Q_1 G_1}$
Unloaded Manifold
Loaded Manifold
ω_0
Pass Band
ω_s
Stop Band
ω_1
$\frac{\text{SLOPE}_{\text{Loaded}}}{\text{SLOPE}_{\text{Unloaded}}} = \frac{1}{\left[1 + \frac{2}{n\pi} \frac{G_0}{Y_0} Q_0\right]^{1/2}}$
nπ
βℓ

FIG. 3

$\ell_T = M \ell_c$ and also equals $nN \frac{\lambda_g}{2}$, so that it can be written as $\delta_I = \frac{1}{2 n^2 N^2} \frac{\lambda^2}{\lambda_g^2}$. If the manifold is a coaxial line as it might well be at 200 Mc, then $\frac{\lambda}{\lambda_g} = 1$, and $\delta_I = \frac{1}{2 n^2 N^2}$. This is to be compared with δ_A which varies as the first power of nN.

Also note that if $N = 1$, it gives the value of ω_1 in Fig. 3.