A NEW INTERPRETATION OF STRUCTURE COMPENSATION

G. Dôme and P. Lapostolle
European Organization for Nuclear Research
Geneva, Switzerland

Summary

In a periodic structure, there are always two basic waves for each frequency. Inside a passband, these basic waves are the two travelling waves which can propagate in both directions down the structure. At the edge of a passband (0 or π mode), the two travelling waves become identical, and another basic solution must be found. This second solution, instead of being periodic, varies linearly along the structure. One of the basic waves is thus periodic, while the second one is non-periodic, and they have opposite parities with respect to a symmetry plane of the structure.

If we consider the adjacent edges of two neighbouring passbands, the two periodic waves at these two cut-off frequencies have also different parities. When the structure is progressively deformed until the two adjacent cut-off frequencies merge, that is when the structure is being compensated, the non-periodic wave at one frequency tends towards the periodic wave at the other frequency: therefore, in a compensated structure, both basic waves are periodic along the structure at the crossing point of the two passbands.

Assume several cells of a terminated structure are detuned in such a way that the overall resonant frequency remains equal to the cut-off frequency of the perfect structure; reactive power flows out of the detuned cells, and such a non-zero flux can arise only from a combination of the two basic waves at the given frequency. The field between two detuned cells thus varies linearly in general, whereas it remains periodic in a compensated structure.

Finally, measured field variations along the axis of locally compensated non-uniform Alvarez structures are compared with theoretical estimates obtained by matching the maximum current on the outer wall of two adjacent Alvarez cells.

Possible electromagnetic waves in a periodic structure

In a periodic structure with geometrical period L along the z-axis, the most general field solution may be represented as a linear combination of two linearly independent solutions $(\vec{E}_1, \vec{H}_1)$ and $(\vec{E}_2, \vec{H}_2)$. Because of the periodicity of the structure, the vector functions $\vec{E}_1(z + L)$, $\vec{H}_1(z + L)$ and $\vec{E}_2(z + L)$, $\vec{H}_2(z + L)$ are also solutions of the wave-equation, and are thus derived from the former solutions by the linear relations

$$\left\| \begin{matrix} \vec{E}_1(z + L) \\ \vec{E}_2(z + L) \end{matrix} \right\| = A \left\| \begin{matrix} \vec{E}_1(z) \\ \vec{E}_2(z) \end{matrix} \right\| \qquad \left\| \begin{matrix} \vec{H}_1(z + L) \\ \vec{H}_2(z + L) \end{matrix} \right\| = A \left\| \begin{matrix} \vec{H}_1(z) \\ \vec{H}_2(z) \end{matrix} \right\| \tag{1}$$

where the 2 x 2 matrix A is independent of the coordinates.

By a suitable choice of the basic solutions $(\vec{E}_1, \vec{H}_1)$ and $(\vec{E}_2, \vec{H}_2)$, the matrix A may be put into Jordan's canonical form. If we denote the two eigenvalues of A by λ_1, λ_2, we must distinguish between three cases :

case 1 : Unequal eigenvalues, thus linearly independent eigenvectors.
The canonical form of A is $\left\| \begin{matrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{matrix} \right\|$

Case 2 : Equal eigenvalues λ with one eigenvector.
The canonical form of A is $\begin{Vmatrix} \lambda & 0 \\ \zeta & \lambda \end{Vmatrix}$
where ζ is any constant different from zero.

Case 3 : Equal eigenvalues λ with two independent eigenvectors.
The canonical form of A is $\begin{Vmatrix} \lambda & 0 \\ 0 & \lambda \end{Vmatrix}$
This case thus corresponds to the particular value $\zeta = 0$.

The determinant of A is easily shown [1] to be unity. Therefore $\lambda_1 \lambda_2 = 1$ in all cases, and we may take

$$\lambda_1 = e^{-j\theta} \qquad \lambda_2 = e^{j\theta} \tag{2}$$

where θ may be complex.

We now look for the implications of eq.(1) in the three cases.

Case 1 . $\lambda_1 \neq \lambda_2$, i.e. $e^{-j\theta} \neq e^{j\theta}$ or $\theta \neq n\pi$ (n, any integer).
From (1) and (2) we get

$$\begin{cases} \vec{E}_1(z + L) = e^{-j\theta}\, \vec{E}_1(z) \\ \vec{E}_2(z + L) = e^{j\theta}\, \vec{E}_2(z) \end{cases}$$

$$\begin{cases} \vec{H}_1(z + L) = e^{-j\theta}\, \vec{H}_1(z) \\ \vec{H}_2(z + L) = e^{j\theta}\, \vec{H}_2(z) \end{cases} \tag{3}$$

The two basic solutions $(\vec{E}_1,\vec{H}_1)$ and $(\vec{E}_2,\vec{H}_2)$ thus appear as waves travelling respectively in the positive and in the negative z-direction, both having a (complex) phase shift θ per cell along the structure.

Case 2 . $\lambda_1 = \lambda_2 = \lambda$ with $\lambda^2 = 1$.
We have $\lambda = 1$ when $\theta = 0$ and $\lambda = -1$ when $\theta = \pi$.
This case thus corresponds to the edge of a passband.

Since the matrix A is supposed to have only one eigenvector corresponding to λ, we get with Jordan's canonical form

$$\begin{cases} \vec{E}_1(z + L) = \lambda\vec{E}_1(z) \\ \vec{E}_2(z + L) = \lambda\vec{E}_2(z) + \zeta\, \vec{E}_1(z) \end{cases}$$

$$\begin{cases} \vec{H}_1(z + L) = \lambda\vec{H}_1(z) \\ \vec{H}_2(z + L) = \lambda\vec{H}_2(z) + \zeta\vec{H}_1(z) \end{cases} \tag{4}$$

or more generally

$$\begin{cases} \vec{E}_1(z + pL) = \lambda^p\vec{E}_1(z) \\ \vec{E}_2(z + pL) = \lambda^p \left[\vec{E}_2(z) + p\,\frac{\zeta}{\lambda}\, \vec{E}_1(z)\right] \end{cases}$$

$$\begin{cases} \vec{H}_1(z + pL) = \lambda^p\vec{H}_1(z) \\ \vec{H}_2(z + pL) = \lambda^p \left[\vec{H}_2(z) + p\,\frac{\zeta}{\lambda}\, \vec{H}_1(z)\right] \end{cases} \tag{5}$$

$$p = 0, \; \begin{matrix} 1, & 2, & 3\ldots \\ -1, & -2, & -3\ldots \end{matrix}$$

The solution $(\vec{E}_1, \vec{H}_1)$ is periodic, with period L when $\theta = 0$ and period 2L when $\theta = \pi$. It may be considered as a travelling wave having a phase shift per cell equal to 0 or π, but nothing allows it to be distinguished from a standing wave.

The second solution $(\vec{E}_2, \vec{H}_2)$ exhibits an overall linear variation along the structure, and is defined only up to an additional term proportional to the periodic solution $(\vec{E}_1,\vec{H}_1)$.

From (5) we get the complex Poynting vector along the structure :

for the periodic solution,

$$\frac{1}{2}\left[\vec{E}_1 \times \vec{H}_1^*\right]_{(z+pL)} = \frac{1}{2}\left[\vec{E}_1 \times \vec{H}_1^*\right]_{(z)} \qquad (6)$$

for the non-periodic solution,

$$\frac{1}{2}\left[\vec{E}_2 \times \vec{H}_2^*\right]_{(z+pL)} = \frac{1}{2}\left[\vec{E}_2 \times \vec{H}_2^*\right]_{(z)} + \frac{p}{\lambda}\left\{\frac{\zeta}{2}\left[\vec{E}_1 \times \vec{H}_2^*\right]_{(z)} + \frac{\zeta^*}{2}\left[\vec{E}_2 \times \vec{H}_1^*\right]_{(z)}\right\} + p^2|\zeta|^2 \cdot \frac{1}{2}\left[\vec{E}_1 \times \vec{H}_1^*\right]_{(z)} \qquad (7)$$

If we consider the region of the structure which is enclosed between two cross-sectional planes at z and (z + pL), the difference between the average electric and magnetic energy stored in this region is proportional to the total flux of the complex Poynting vector flowing into it. For the periodic solution, this flux is zero by virtue of (6): in every cell, the stored electric and magnetic energies are equal. As shown by (7), such an equality does not hold for the non-periodic solution.

Case 3. $\lambda_1 = \lambda_2 = \lambda$ with $\lambda^2 = 1$.

Since the matrix A is now supposed to have two independent eigenvectors corresponding to λ, we get with Jordan's canonical form

$$\begin{cases} \vec{E}_1(z+L) = \lambda\vec{E}_1(z) \\ \vec{E}_2(z+L) = \lambda\vec{E}_2(z) \end{cases} \qquad \begin{cases} \vec{H}_1(z+L) = \lambda\vec{H}_1(z) \\ \vec{H}_2(z+L) = \lambda\vec{H}_2(z) \end{cases} \qquad (8)$$

In this very particular case both $(\vec{E}_1, \vec{H}_1)$ and $(\vec{E}_2, \vec{H}_2)$ are periodic solutions, with period L when $\theta = 0$ and period 2L when $\theta = \pi$. They may be considered as travelling or standing waves. In the mathematical theory of Hill's differential equation [2], this case is called a case of coexistence of two periodic solutions. Since the most general solution is a linear combination of $(\vec{E}_1, \vec{H}_1)$ and $(\vec{E}_2, \vec{H}_2)$, any solution is then periodic.

In short, within a passband or a stopband, there exist two basic travelling waves defined by (3). At the edges of a passband, $\theta = 0$ or π : the waves travelling in the positive or in the negative z-directions become identical to each other and consequently also identical to standing waves. They are periodic in z, with period L when $\theta = 0$ and period 2L when $\theta = \pi$.

The second linearly independent wave, according to (5), is in general non-periodic ; instead it varies linearly along the structure. In very special cases, however, the second basic wave is also periodic down the structure.

Symmetry properties of the solutions at 0 or π mode in a symmetrical structure

Let us assume, which is the case for most accelerating structures actually used, that the unit-cell of the structure has a symmetry plane normal to its axis. If such a plane is chosen as the origin of the z-coordinate, all the planes $z = n\frac{L}{2}$ (n, any integer) are also symmetry planes of the structure.

Due to the symmetry or the structure about z = 0, it may be shown [1] that the periodic solution is either symmetrical or antisymmetrical about z = 0. The second (in general non-periodic) basic solution may then be taken respectively as antisymmetrical or symmetrical about $z = 0$.

It is also worth noticing that the parity of a periodic solution is the same about all symmetry planes of the structure in 0 mode, but in π mode this parity is different about the symmetry planes $z = nL$ and $z = (2n + 1)\frac{L}{2}$.

<u>Boundary conditions for the two basic waves at 0 or π mode in a symmetrical structure</u>

From the symmetry properties just mentioned, there are two possibilities :

a) $z = 0$ is an antisymmetry plane for $\vec{E}_1, \vec{H}_2$ and a symmetry plane for $\vec{H}_1$, $\vec{E}_2$.

Designating with a subscript t the part of the field which is transverse to the z - axis, we get at once

$$\vec{E}_{1t}(0) = 0 \qquad \vec{H}_{2t}(0) = 0$$

We do not write the conditions for the z-components of the fields, because they follow from the conditions for the transverse components by Maxwell's equations.

Taking $z = -\frac{L}{2}$ in equation (4) yields

$$\begin{cases} \vec{E}_1(\frac{L}{2}) = \lambda\vec{E}_1(-\frac{L}{2}) \\ \vec{E}_2(\frac{L}{2}) = \lambda\vec{E}_2(-\frac{L}{2}) + \zeta\,\vec{E}_1(-\frac{L}{2}) \end{cases}$$

$$\begin{cases} \vec{H}_1(\frac{L}{2}) = \lambda\vec{H}_1(-\frac{L}{2}) \\ \vec{H}_2(\frac{L}{2}) = \lambda\vec{H}_2(-\frac{L}{2}) + \zeta\vec{H}_1(-\frac{L}{2}) \end{cases}$$

In 0 mode, $\lambda = 1$. Using again the symmetry properties about $z = 0$, we obtain

$$\vec{E}_{1t}(\tfrac{L}{2}) = 0 \qquad \vec{H}_{2t}(\tfrac{L}{2}) = \frac{\zeta}{2} \cdot \vec{H}_{1t}(\tfrac{L}{2})$$

In π mode, $\lambda = -1$. Therefore

$$\vec{H}_{1t}(\tfrac{L}{2}) = 0 \qquad \vec{E}_{2t}(\tfrac{L}{2}) = -\frac{\zeta}{2}\,\vec{E}_{1t}(\tfrac{L}{2})$$

b) $z = 0$ is a symmetry plane for $\vec{E}_1$, $\vec{H}_2$ and an antisymmetry plane for $\vec{H}_1$, $\vec{E}_2$.

This case is deduced from a) by permuting $\vec{E}$ and $\vec{H}$. Whence

$$\vec{H}_{1t}(0) = 0 \qquad \vec{E}_{2t}(0) = 0$$

In 0 mode,

$$\vec{H}_{1t}(\tfrac{L}{2}) = 0 \qquad \vec{E}_{2t}(\tfrac{L}{2}) = \frac{\zeta}{2} \cdot \vec{E}_{1t}(\tfrac{L}{2})$$

In π mode,

$$\vec{E}_{1t}(\tfrac{L}{2}) = 0 \qquad \vec{H}_{2t}(\tfrac{L}{2}) = -\frac{\zeta}{2} \cdot \vec{H}_{1t}(\tfrac{L}{2})$$

These boundary conditions are summarized in table I. The periodic waves are determined by homogeneous boundary conditions, to which there corresponds a discrete spectrum of eigenfrequencies. In contrast, the non-periodic waves are determined by non-homogeneous boundary conditions which require a preliminary knowledge of the corresponding periodic wave, and also prevent the non-periodic waves to be excited in a finite ideal structure terminated by metallic end-plates.

In all cases, the flux of the complex Poynting vector across any symmetry plane of the structure (at $z = n\frac{L}{2}$) is zero for the periodic wave.

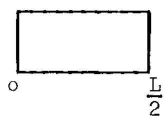
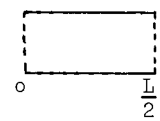
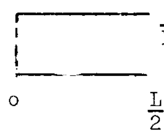
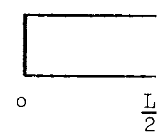
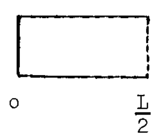
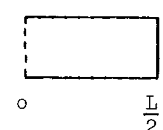
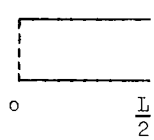
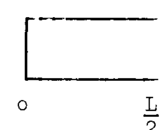
Taking for example $z = 0$ in (7), we then get for the non-periodic wave

$$\frac{1}{2}\left[\vec{E}_2 \times \vec{H}_2^*\right]_{(pL)} - \frac{1}{2}\left[\vec{E}_2 \times \vec{H}_2^*\right]_{(0)} =$$

$$\frac{p}{\lambda}\left\{\frac{\zeta}{2}\left[\vec{E}_1 \times \vec{H}_2^*\right]_{(0)} + \frac{\zeta^*}{2}\left[\vec{E}_2 \times \vec{H}_1^*\right]_{(0)}\right\} \qquad (9)$$

The flux of the complex Poynting vector across a symmetry plane of the structure thus varies linearly from cell to cell ; in other

Table I. Boundary conditions for the two basic waves at a passband-edge

a) The periodic wave is antisymmetrical about the plane z = 0	b) The periodic wave is symmetrical about the plane z = 0
0 mode	
Periodic wave 	Periodic wave 
Non-periodic wave  $\vec{H}_t = \frac{\zeta}{2} \cdot$ (Periodic field at $\frac{L}{2}$)	Non-periodic wave  $\vec{E}_t = \frac{\zeta}{2} \cdot$ (Periodic field at $\frac{L}{2}$)
π mode	
Periodic wave 	Periodic wave 
Non-periodic wave  $\vec{E}_t = -\frac{\zeta}{2} \cdot$ (Periodic field at $\frac{L}{2}$)	Non-periodic wave  $\vec{H}_t = -\frac{\zeta}{2} \cdot$ (Periodic field at $\frac{L}{2}$)
——— short circuit ($\vec{E}_t = 0$)	------ open circuit ($\vec{H}_t = 0$)

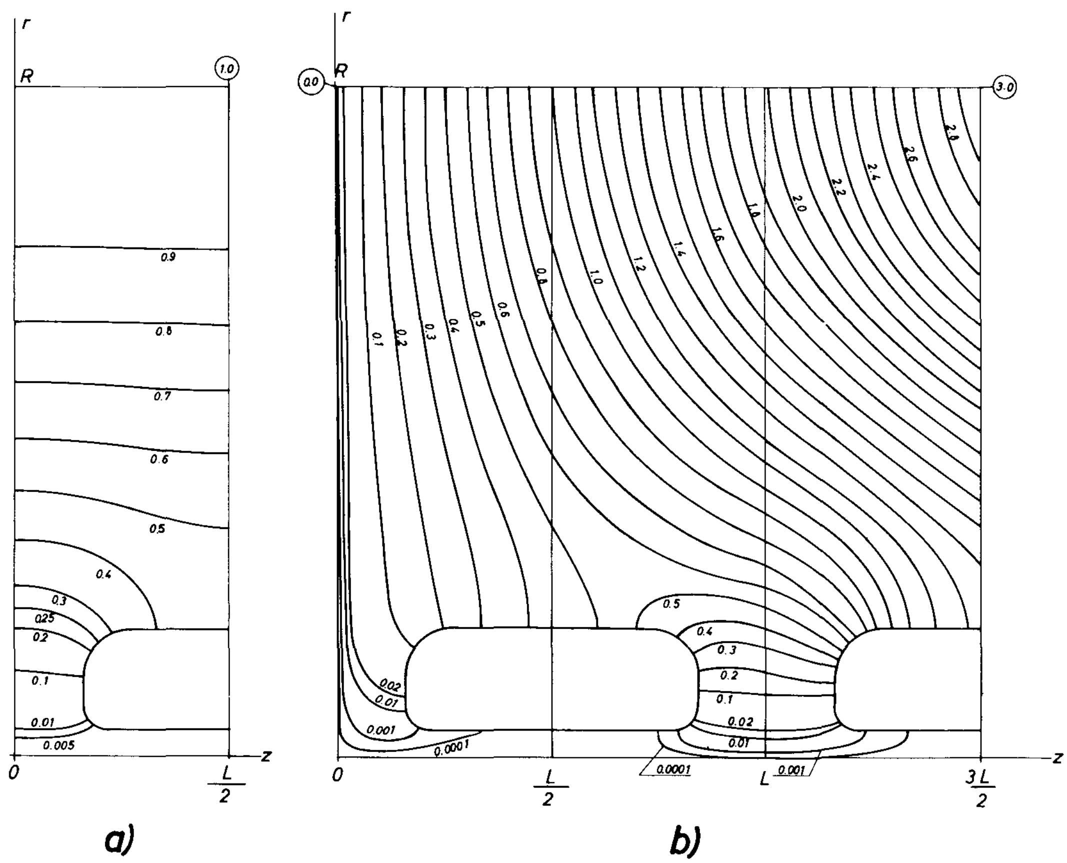


Figure 1. The two basic waves at 0 mode in an Alvarez structure without stems (Proton energy 18.737 MeV)

a) Periodic wave, antisymmetrical with respect to the plane z = 0.
b) Non-periodic wave, symmetrical with respect to the plane z = 0.

words, the difference between average electric and magnetic energy stored in a cell is the same from cell to cell, but it is not zero for the non-periodic wave. This fact together with the non-homogeneous boundary conditions show that the non-periodic wave is not resonant, but driven at the frequency of the (resonant) periodic wave. A somewhat similar idea has been put forward recently by Carter [3].

As an example, the boundary conditions of table I have been used to compute the two basic waves at 0 mode in an Alvarez structure without stems [4]. For such a E_{010} mode, the periodic wave is antisymmetrical about the plane z = 0. The electric field lines, or lines of constant rH_{φ}, are shown in figure 1 for both waves. The indicated numerical values are those of rH_{φ}, normalized to the maximum value for the periodic wave. For the non-periodic wave, they correspond to $\zeta = 2$.

Parity of two periodic waves corresponding to adjacent cut-off frequencies

Two neighbouring passbands of a structure may lie close together at π mode (figure 2a) or at 0 mode (fig. 2b). If, by continuous deformation of the structure, the two adjacent cut-off frequencies can be brought together so that the two passbands join up, the passbands are said to be confluent and the structure is called compensated.

The periodic waves corresponding to two adjacent cut-off frequencies must satisfy the boundary conditions of table I; at the same time their eigenfrequencies may become infinitely close by progressive shaping of the structure. Since the spectrum of eigenfrequencies is discrete for all homogeneous boundary conditions of table I, this means that, except for the case of degeneracy which we disregard here, the two periodic waves corresponding to adjacent cut-off frequencies must have different parities.

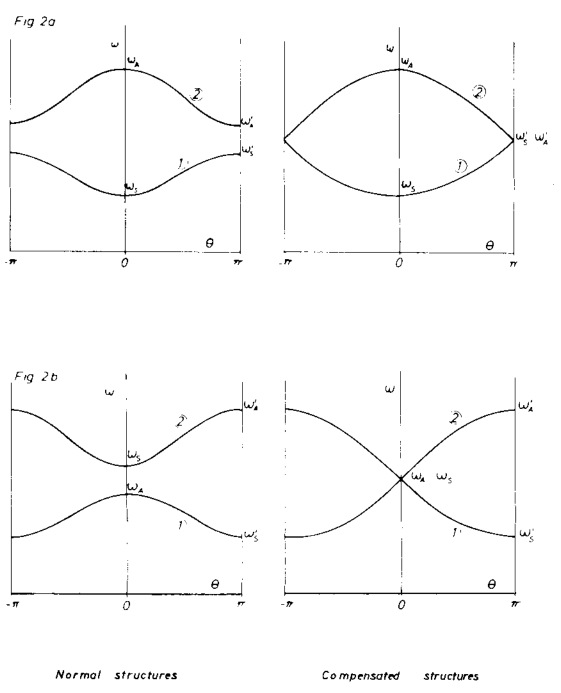


Fig. 2. Dispersion curves, showing the symmetry of the periodic waves at cut-off for two adjacent passbands.

ω_S, angular frequency of a periodic symmetrical wave
ω_A, angular frequency of a periodic antisymmetrical wave

This statement is illustrated in figure 2. Although it is the only statement which can be made about the symmetry of periodic waves at cut-off, it is particularly important. First, it implies that in an ideal terminated structure, only one of the two periodic waves corresponding to adjacent cut-off frequencies can be excited, because only one periodic wave can satisfy the boundary conditions at the end-plates. Moreover, when the stopband between two neighbouring passbands is made vanishingly small, the two periodic waves at the edges of the stopband ultimately become two periodic waves having the same frequency but opposite parities, thus representing two linearly independent periodic waves: there is coexistence of two periodic solutions, which is just case 3. We see now that this very special case arises when the structure is compensated. It is also clear that, except for the case of degeneracy, the two coexistent periodic waves have different parities.

Compensated structures

In such a case of coexistence, the two sets of boundary conditions a) and b) in table I simultaneously admit a solution at the same frequency. This means that the homogeneous problem associated with the non-homogeneous boundary conditions for a non-periodic wave now admits a non-trivial solution at the frequency of the periodic wave: the non-homogenous problem then no longer admits a solution at this frequency, and the non-periodic wave no longer exists. This is to be expected, since the two basic waves are periodic, so that every solution is also periodic. In fact, when the stopband is shrinking to zero, ζ is going to zero, and the non-periodic wave ultimately reduces to the periodic wave with opposite parity: the two basic waves thus remain continuous when passing through the compensated case.

An accelerating structure is normally designed to work with a periodic wave in 0 or π mode (this wave must be antisymmetrical with respect to the metallic end-plates of the cavity), so that the field is periodic down the structure. If some perturbation of the structure causes the second basic wave to be excited, the total field will remain periodic in a compensated structure, because this second wave is also periodic down the structure. In a non-compensated structure, on the other hand, any excitation of the second (non-periodic) wave will induce a tilt in the total field, since this non-periodic wave varies linearly along the structure.

The second basic wave is always excited when several cells of a terminated structure are detuned in such a way that the overall resonant frequency remains equal to the cut-off frequency of the perfect structure. Indeed, due to the unbalance between stored electric and magnetic energies in the detuned cells, reactive power must flow out of or into these cells. As already pointed out, the flux of the complex Poynting vector across any symmetry plane of the structure is zero for a periodic wave; such a non-zero flux can arise only from a combination of the two basic waves, which leads to the most general non-periodic solution for normal structures, and to a superposition of periodic solutions having both parities for compensated structures. The field between two detuned cells thus varies linearly in normal structures, whereas it remains periodic in a compensated structure.

An important remark should be made here. If instead of the axial electric field, we consider the integral of this field on the axis over a cell length, we get from equation (5):

$$\int_{-\frac{L}{2}+pL}^{+\frac{L}{2}+pL} \vec{E}_2(z).\,d\vec{z} = \lambda^p \left[\int_{-\frac{L}{2}}^{+\frac{L}{2}} \vec{E}_2(z)\,.\,d\vec{z} + p\frac{\zeta}{\lambda} \int_{-\frac{L}{2}}^{+\frac{L}{2}} \vec{E}_1(z)\,.\,d\vec{z} \right] \qquad (10)$$

When the periodic field $\vec{E}_1$ is symmetrical about $z = 0$, the integral $\int_{-\frac{L}{2}}^{+\frac{L}{2}} \vec{E}_1 \,.\, d\vec{z}$ vanishes, and the integral of the non-periodic field over a cell length $\int_{-\frac{L}{2}+pL}^{+\frac{L}{2}+pL} \vec{E}_2 \,.\, d\vec{z}$ is then periodic.

In drift tube structures it is essentially the integral of the field which determines the acceleration in a gap. Taking the z-origin in the middle of a gap, there are two possibilities:

a) the periodic wave is antisymmetrical about $z = 0$ (this is always the case when the structure is terminated by a metallic end-plate at $z = nL$). Then the voltage of the periodic field across a gap does not vanish, and by (10) the voltage of the non-periodic field varies linearly along the structure.

b) the periodic wave is symmetrical about $z = 0$. The voltage of the non-periodic field across a gap is then also periodic down the structure.

Although the latter possibility can never be realized in a uniform structure which is terminated by a metallic end-plate at $z = nL$, it can occur locally in a non-uniform structure, as shown in figure 3. This figure is part of the recent measurements made at CERN in order to investigate the field stability against perturbations in an accelerator tank[5]. It shows the variation of an averaged electric field $E_{max} \cdot g/L$ along a scale model of the tank 2 (10-30 MeV) of the CERN linac injector. In figure 3 this model was compensated on the average in 0 mode by using a cross-bar stem configuration with two stem diameters : .

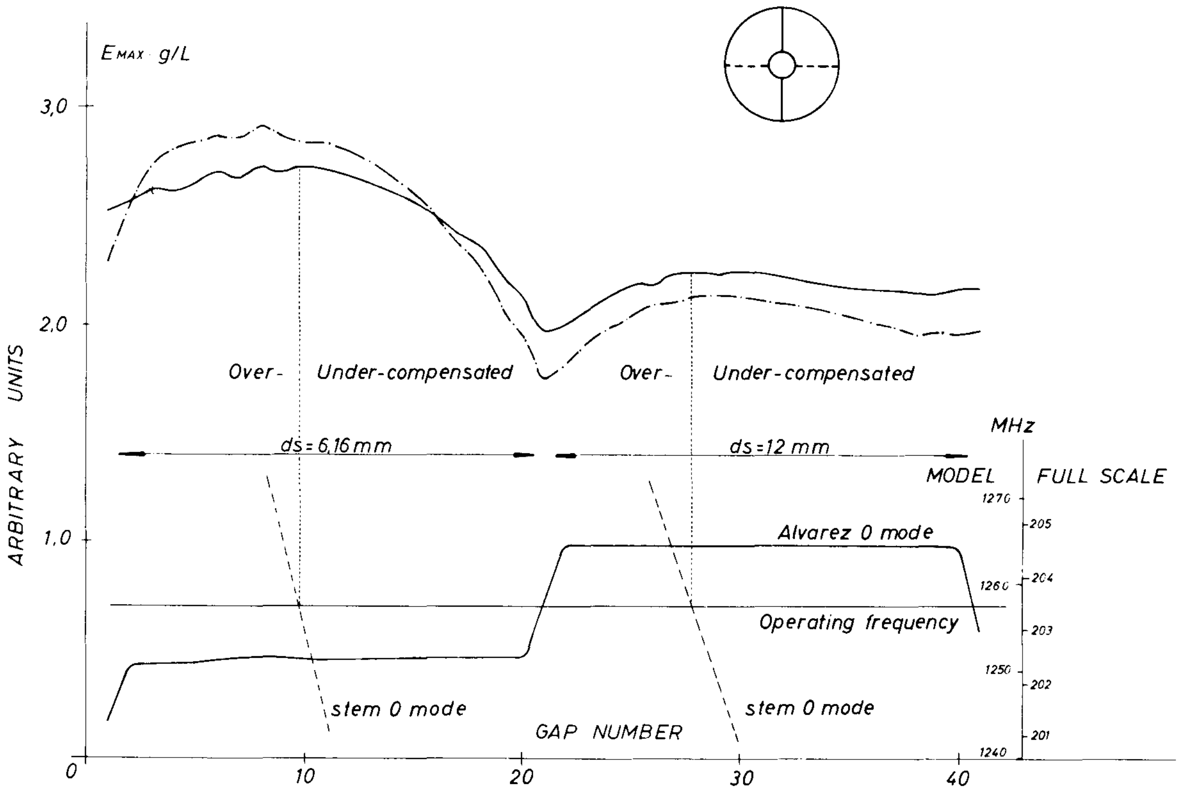


Figure 3. Electric field along the axis of a partially compensated non-uniform structure, and resonant frequencies of individual cells in the Alvarez and stem 0 modes.

——— without perturbation (frequency 1253.54 MHz)

.-.-.- with a tank perturbation of -2 MHz in cell 1

a diameter of 6.16 mm, which would compensate locally the cell no. 10, and a diameter of 12 mm, which would compensate locally the cell no. 26 (see figure 17 in reference 5). It is remarkable that the slope of the field just vanishes in the neighbourhood of these cells. Considering the structure as locally uniform, we expect from the above reasoning that the field should be flat in the cell where the operating frequency equals the 0 mode frequency of the stem resonance, since the periodic wave is then symmetrical about z = 0. At this cell the slope of the field changes sign, as does the difference between the 0 mode frequency of the stem resonance and the operating frequency. Since the same situation occurs twice along the structure, there must be a discontinuity in the field slope when the stem diameter changes abruptly.

Figure 3 also shows theoretical estimates of the local Alvarez and stem 0 modes frequencies along the tank, together with an estimate of the operating frequency. The discrepancy of 3.5 MHz between this estimate and the measured operating frequency can be attributed to manufacturing errors in our tank 2 model [5].

Non-uniform compensated structures

When a structure is not strictly periodic, it must be compensated locally everywhere in order to achieve good field stability against perturbations [5]. Once this has been done, the structure is extremely insensitive to manufacturing errors and is thus likely to produce the theoretical field which is computed for a perfect structure.

The question then arises, how to compute the theoretical field in a non-uniform Alvarez structure by matching cells which are computed individually using short-circuit boundary conditions (see table I,a). This matching of computed fields is clearly not possible everywhere on the common surface between two adjacent cells. Nevertheless, for low energy cells, matching the azimuthal magnetic field at one point of this surface almost results in a perfect matching at all other points [6]. As a working condition, but hoping for a better condition in the future, we have chosen to match the maximum value of rH_φ at the outer wall of the cells, where this quantity represents the total current flowing in the wall at the cross-sectional plane which passes through the middle of a drift tube.

This has been done for the 25 first cells of the new injector in Saclay (0.75-5 MeV) and for the tank 2 of the CERN linac [5]. The average axial field per cell turns out to be very nearly constant in both cases. The corresponding computed variations of the axial field at the centre of the gaps, referred to the average field in the first cell, and multiplied by g/L in order to deal with a quantity which varies rather little along the tank, are shown respectively in figures 4 and 5. The figures also display the same quantity as measured by a standard frequency perturbation technique, on the best locally compensated models of both tanks [5]. The absolute scale for the measured field has been determined by using the theoretical value of stored energy in all cells, obtained by numerical computations.

For tank 1 (figure 4), the splitting of the theoretical curve into three parts marked with A,B,C corresponds to a change of the drift tube bore diameter from A to B, and of the drift tube outer diameter from B to C. The agreement between theoretical and measured fields is as good as may be expected from the mechanical accuracy of the model ; in fact, the difference between the measurements made on the two types of compensated structures for tank 1 shows the magnitude of the experimental errors.

For tank 2 (figure 5), the agreement between theoretical and measured fields is still quite good if it is taken into account that the model has not been perfectly compensated and moreover that, due to the varying stem diameter, the individual cell frequencies exhibit a large variation along the tank [5] : it should be remembered that the field perturbations due to tuning errors in a compensated structure are zero only up to the first order in these tuning errors.

Conclusion

In a periodic structure at 0 or π mode, in addition to the well known periodic field solution, there exists a second basic solution which in general exhibits an overall linear variation along the structure. For a compensated structure, on the other hand, this second basic solution is also periodic. Therefore, if the second basic solution is excited by perturbations in the ideal structure, the total field will have a tilt in normal structures, whereas it will remain periodic in a compensated structure.

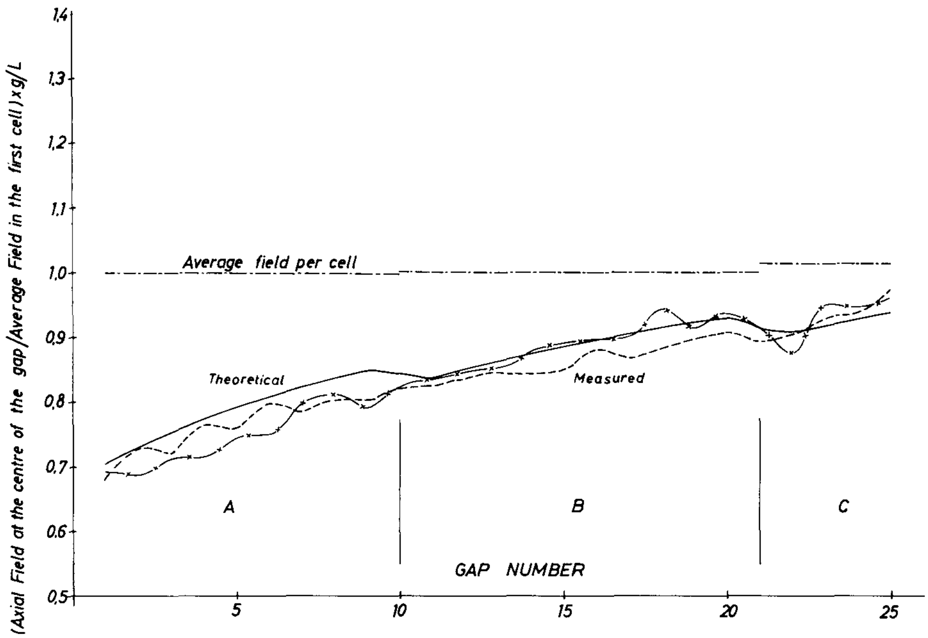


Figure 4. Comparison of theoretical and measured electric field at the centre of the gaps, for the best locally compensated tank 1 models.

——— theoretical field

-x-x-x field measured with two stems at 45° , alternating at 180° from one drift tube to the next

------ field measured with two stems at 150°, all parallel.

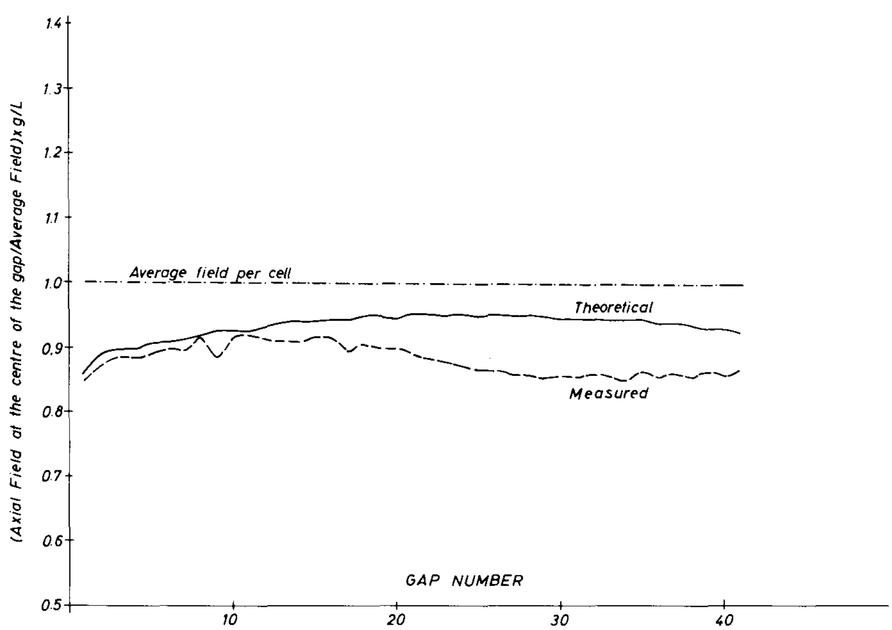


Figure 5. Comparison of theoretical and measured electric field at the centre of the gaps, for the best locally compensated tank 2 model.

Cross-bar structure, with variable stem diameter.

REFERENCES

1. G. Dôme: Some general considerations about the possible electromagnetic waves in a symmetrical lossless periodic structure, with special application to compensated structures. CERN Internal Report ISR-300/LI/68-1, January 1968.

2. W. Magnus and S. Winkler : Hill's equation Interscience Publishers, Wiley, New York, 1966, Chapter VII.

3. D. Carter : Field stabilization in periodic systems. Brookhaven National Laboratory Accelerator Dept. Internal Report AGSCD-24, September 1967.

4. G. Dôme and M. Bell : Numerical computation of field distribution in the first two passbands of a symmetrical lossless periodic structure. CERN Internal Report, to be published.

5. G. Dôme and I. White : Measurements of field stability against perturbations in non-uniform linac structures, Paper presented to this Conference. p.475

6. G. Dôme and P. Lapostolle : Sur la distribution des champs dans une longue cavité d'accélérateur de type Alvarez avec et sans compensation. CERN Internal Report ISR-300/LI/68-2, January 1968.