

ACCELERATION OF LIGHT AND HEAVY IONS WITH HELIX STRUCTURE

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ABSTRACT

The problem of linac optimization with respect to the total RF-power is solved, giving simple mathematical expressions. A new method of calculating the current distribution on the helical wave guide, the RF-losses and the maximum magnetic fields on the surface of the helix is described. Some parameters of a superconducting Proton Helac, which shall be constructed in cooperation with the Kernforschungszentrum Karlsruhe, are discussed. For normal temperature relevant parameters of a Heavy Ion Helac are presented. This accelerator is proposed for post acceleration after a tandem machine (Emperor) as part of the projects planned by the GSI (Gesellschaft für Schwerionenforschung, Darmstadt).

I. Introduction

The accelerator group of our institute at the Frankfurt University has been concerned with the helical wave guide and its application to particle acceleration. Some results were presented at the Heidelberg Conference last year, especially the design of a heavy ion helix accelerator (Helac) for uranium was discussed<sup>1</sup>. Results concerning beam dynamics of this accelerator are enclosed in the contribution of J. Klabunde et al. at this conference<sup>2</sup>.

Here we want to discuss the power-optimization of a linac; consequent design parameters of a superconducting Helac for protons and of a normal temperature Helac for heavy ions as a postaccelerator to a Tandem-Van de Graaff (Emperor) are considered and a new method of calculating the current distribution on the helix is described.

II. General linac optimization

The force acting on a particle with charge  $e$  and synchronous phase  $\varphi_s$  moving in the longitudinal direction  $z$  through the linac is given by

$$\frac{dT}{dz} = \frac{e \cdot E_0 \cdot \cos \varphi_s}{A} = \frac{e \cdot \cos \varphi_s \cdot \sqrt{\eta \cdot p}}{A} = B \sqrt{\eta \cdot p} \quad (1)$$

where  $\eta = \frac{E_0^2}{p}$  stands for shunt impedance,  $p$  for power losses per unit length,  $E_0$  for amplitude of the RF field and  $T$  for particle energy per nucleon. In general the geometric parameters of an accelerating structure vary along accelerator corresponding to the increasing particle energy, causing a variation of  $\eta$ . When optimizing the linac one has to answer the

following questions:

1. How to choose the geometrical parameters and the frequency in order to minimize the total RF power  $N$ , the linac parameters being length  $L$ , initial energy  $T_1$ , final energy  $T_2$ .
2. How to distribute the RF power losses per unit length  $p(z)$  along the accelerator achieving minimum total RF power losses  $N$ .

It turns out, that a general solution of the problem exists, which will be discussed here.

Assuming given functions  $\eta(T)$  and  $B(T)$  integration of Equ. (1) yields

$$\int_{T_1}^{T_2} \frac{dT}{B(T)\sqrt{\eta(T)}} = \int_0^L \sqrt{p(z)} dz = \text{const.} = C, \quad (2)$$

with given  $T_1, T_2, L$  the integral is constant. The problem of minimizing the total power loss

$$N = \int_0^L p(z) dz \quad (3)$$

with respect to the auxiliary condition  $C = \int_0^L \sqrt{p(z)} dz$  is an isoperimetric problem of variation calculus, which can be solved by the method of Lagrange's multiplier. The solution yields, that power losses per unit length have to be constant along accelerator, independent of the variation of shunt impedance along accelerator:

$$p = \left(\frac{C}{L}\right)^2 = \text{const.} \quad (4)$$

The total power losses  $N$  are then given by

$$N = \frac{C^2}{L} = \frac{1}{L} \left( \int_{T_1}^{T_2} \frac{dT}{B(T)\sqrt{\eta(T)}} \right)^2. \quad (5)$$

Thus minimizing  $N$  reduces to minimizing the integral

$$A = \int_{T_1}^{T_2} \frac{dT}{\sqrt{\eta}} \quad (6)$$

with  $B = \text{const.}$  Series impedance and attenuation length have no influence on optimization.

These conditions (4) and (6) represent a complete theoretical solution of the optimization problem. In practice it is impossible for many types of linac to fulfil condition (4). Calculations show for a wide range however, that deviations from the condition  $p = \text{const.}$  cause only small effects on total power losses  $N$ . In case of the Helac - consisting of many sections - the formalism derived is fully applicable. The method is used in the numerical examples of the next chapter.

### III. Parameters of a superconducting Helac

In cooperation with the KFZ Karlsruhe the Frankfurt helix group is engaged in the study of the superconducting helix, which may be applied as low energy part of a 1 GeV proton accelerator.

This project is reported by A. Citron<sup>3</sup> and J. Vetter et al.<sup>4</sup>. Here the choice of helix parameters basing on the optimization method given above

shall be described.

In figs.1,2 the shunt impedance  $\eta$  for travelling waves is plotted in the energy range  $T = 0.75 - 20$  MeV. Superconducting aspects being neglected at first, room temperature values are used.

At each frequency one has to select the largest possible  $\eta$ -values belonging to various helix radii. There are lower limits of helix radius  $a$ , pitch  $s$  and wire diameter  $d$ , due to the beam extension and the helium cooling inside the helical conductor.

On the  $\eta$ -curves the points of minimum pitch  $s = 1$  cm are marked by circles, the not usable parts with  $s < 1$  cm by dashed lines. The fat printed line indicates the available maximum shunt impedance: In the first accelerator sections the pitch is near the minimum value and the radius decreases with increasing particle energy. At higher energies pitch and radius increase.

To determine the optimum frequency the integrals  $A = \int_{T_1}^{T_2} dT / \sqrt{\eta}$ , evaluated at each frequency with the maximum shunt impedance, have to be compared. As shown by table I the minima of the optimization area  $A$  and the RF power losses  $N$  are reached at about 150 MHz.

In case of superconduction the shunt impedance is to be multiplied by a frequency dependent improvement factor<sup>5</sup>. The most favourable frequency therefore seems to be the lowest one (see last column of table I). On the other hand since the frequency jump at the transition from the helix to the cavity structure must not be too high a compromise value of 90 MHz was chosen.

If it is possible to overcome the open technical problems of the superconducting helix, this structure looks very attractive for heavy ion acceleration too.

#### IV. Approximative calculation of current distribution on helices.

The sheath theory of helical wave guides is very useful for investigations of the RF wave propagation on such structures, as long as one is not interested in the conditions on the helix wire itself or immediately around it. The latter is necessary however, if for instance informations on electrical losses and maximum field strengths are wanted.

Therefore current distributions on helices have been calculated approximately on the assumption of simplified conditions<sup>6</sup>. The helix excited by a standing RF wave is replaced by a series of ideal conducting equidistant circular rings with sinusoidal distribution of the total currents, the wave length of which is given by the sheath theory as a function of frequency and geometrical parameters. The axial wave length should be small compared with the vacuum wave length and large compared with the winding distance. Helix accelerator parameters comply with these requirements. Relating the magnetic flux at the conductor surfaces and the current density in quasi-stationary approximation one can establish integral

equations for the surface current distribution around the windings, numerically solvable by computer.

Examples of calculated current distribution for various helix parameters are shown in fig. 4, according to the arrangement and the nomenclature given in fig. 3. The following main features are apparent: At the antinodes of the total current one has large values of current density at the parts of the winding nearest to the axis. At the nodes of the total current large opposite directed surface currents occur at those parts near adjacent windings.

To calculate ohmic losses and shunt impedances of real circular wire helices Johnsen<sup>7</sup> completed the results of sheath theory by a form factor. As a first approximation he applied a constant, valid for medium ratios of wire diameter  $d$  and distance  $s$  ( $d/s = 0.3 \dots 0.7$ ). Using the method given above one can investigate the parameter dependence of that form factor. The calculated values are in good agreement with experimental results as comparisons show (fig. 5). Helix losses in relation to the Johnsen values plotted against the parameter  $x$  show an increasing tendency,  $x$  being a quantity proportional to frequency. A main reason that experimental values are higher than the calculated ones is seen in additional losses caused by surface roughness.

There is a further useful application of the ring model calculations. Knowing the surface current distribution one immediately has the tangential magnetic field strength at the conductors. Since superconduction is very sensible to magnetic field, the knowledge of the maximum field strength is indispensable for the design of a superconducting helix accelerator. In table II the ratio of the maximum magnetic field  $H_{\max}$  at standing waves to the accelerating electric field amplitude  $E_0$  of the travelling wave at different particle energies are shown for the superconducting Proton Helac. The relation of the helix total current to the electric field strength is taken from sheath theory. The parameter sets are based on a shunt impedance optimization starting from a wire diameter of 0.6 cm. For 1 MV/m of the accelerating field one gets maximum magnetic fields of 270 G in the low energy part of the accelerator and due to the very disadvantageous  $d/s$ -ratio up to 760 G in the end part. To reduce these values one succeeds by enlarging the helix dimensions, especially in the middle parts (see second column at 5 MeV) and by improving the  $d/s$ -ratio towards 0.6 ... 0.7 (see values at 20 MeV). A favourable reduction of helix dimension in the low energy part is not practicable because of the limiting minimum wire diameter and distance. The variation of the  $b/a$ -ratio in the range 2-3 has small influence. The last line of table II gives the accelerating field  $E_0$  related to a maximum magnetic field of 1000 G (standing wave).

V. Focusing device for a superconducting Proton Helac.

The figs. 6-8 give some results of beam dynamic calculations for the superconducting Proton Helac. The methods are described in <sup>2</sup>. All calculations include effects of non-linearity and coupling. A rather high acceleration rate ( $E_{\text{O trav.}} = 3.464 \text{ MV/m}$ ) is assumed, which probably may not be achievable. On the other hand decreasing the electrical field results in an increased acceptance; in any case acceptance areas turn out to be sufficient. The accelerator consists of seven helix sections with an increasing length from 44 to 200 cm. The total length is about 10 m. The magnetic quadrupole doublets have a common length of 20 cm. The drift spaces between quadrupoles and sections are 10 cm, the necessary drift length for each of the four vacuum pumps is 10 cm. In fig. 6 the initial phase acceptance is displayed. In order to achieve axial matching when the frequency increases by a factor of probably 7 at 20 MeV the area determined by the curve  $C = 0.2$  can only be used. Tracing this curve through the accelerator yields the fig. 7 at 20 MeV. Radial acceptances for different initial conditions in phase space are plotted in fig. 8. The magnetic field gradients increase from 1.73 at 0.75 MeV to 2.5 kG/cm at 20 MeV. When an aperture radius of 2 cm is assumed one achieves a normalized acceptance area for  $\Delta\phi_{\text{O}} = 0$  and  $\Delta T_{\text{O}} = 0$  of  $1.6 \pi \text{ cm}\cdot\text{mrad}$ . The common area in fig. 8 amounts to  $0.9 \pi \text{ cm}\cdot\text{mrad}$ . This values are large enough compared with an emittance area of  $0.1 \pi \text{ cm}\cdot\text{mrad}$  delivered by the injector.

The method of focusing by alternating the phase has been investigated too, some results are given in <sup>8</sup>.

VI. Normal temperature Helac as a postaccelerator for heavy ions.

The concept<sup>1</sup> of the helix accelerator is well suited for postacceleration of heavy ions, preaccelerated by a static machine. In the last weeks a Helac of this kind was proposed as a part at the program of the GSI. The optimized shunt impedance for a frequency of 108 MHz is plotted versus particle energy per nucleon. The helix radius is varied from 3 to 2 cm, the minimum pitch  $s$  being 0.6 cm. Table III shows the relevant parameters of two versions, one for bromine and one for iodine. The injector is a Van de Graaff-tandem (Emperor) with an output energy of about 100 MeV/ion

Between Emperor and first helix section a foil stripper is provided. The charge states of the ions behind the stripper are taken from<sup>9</sup>. Lower output energy after the dc-machine and smaller charge states result in an increase of the accelerator length. The final energy of the Helac is 6 MeV/N. The maximum electrical field strength does not exceed 80 kV/cm between the windings in this design. Total RF-power refers to a duty cycle of 100 %. Some information on beam dynamics is presented in <sup>2</sup>.

In fig. 10 the structure is drawn schematically. The inner conductor is supported by ceramic stems ( $\text{Al}_2\text{O}_3$ ), which are adjusted at the nodes of

of voltage between outer and inner conductor. Experiments concerning the use of metallic stems look very promising. They simplify the cooling system and seem to be necessary in case of the superconducting Helac.

The schematic pattern of fig. 11 gives a survey of the arrangement of RF-power supply and the control system. The helix is excited in the standing wave mode, the RF-power ( $100 \text{ kW}_{\text{max}}$ ) being coupled directly near one end of the helix outside of the vacuum chamber. Each section has its own RF transmitter, all of them driven by a common master oscillator. The automatic regulation system consists of three circuits. Two fast controls keep field amplitude and phase constant within 1% resp.  $1^\circ$ .

The third slow control acts on separate closed winding at one end of the helix by a servo mechanism, thus tuning the section.

#### Acknowledgement

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TABLE I

Influence of frequency on the rf power losses of a superconducting HELAC for protons (0.75 - 20 MeV)

$$N = 2 \cdot \frac{1}{L} \cdot \underbrace{\left[ \int_{T_1}^{T_2} dT \eta^{-1/2} \right]^2}_{A = \text{optimization area}} \cdot \frac{1}{[e \cdot \cos 30^\circ]^2} \cdot \frac{1}{I}$$

- N* rf power (standing waves)
- L* accelerator length
- T<sub>1</sub>, T<sub>2</sub>* initial and final proton energy (*T<sub>1</sub>* = 0.75, *T<sub>2</sub>* = 20 MeV)
- η* max. shunt impedance for copper at room temperature (helix radius ≥ 2 cm, helix pitch ≥ 1 cm)
- I* improvement factor for superconducting helix

frequency [MHz]	$A$ $\left[ \frac{\text{MeV}}{\sqrt{\text{M}\Omega/\text{m}}} \right]$	<i>N</i> · <i>L</i> [MW·m] room temp.	<i>I</i> [x 10 <sup>6</sup> ]	<i>N</i> · <i>L</i> [W·m] supercond.
50	4.1	44.8	1.55	28.9
85	3.6	34.6	0.79	43.8
100	3.5	33.6	0.65	51.0
150	3.4	30.8	0.39	80.0
200	3.8	38.4	0.27	142.0

TABLE II

The maximum magnetic field strength  $H_{max}$  (standing wave) related to the axial electric field amplitude  $E_0$  (travelling wave) for the superconducting proton HELAC at different parameters (frequency 90 MHz)

particle energy $T$	[MeV]	0.9	5.	20.
helix pitch $s$	[cm]	1.	1.7	3.7
helix radius $a$	[cm]	4.	2.3	3.8
wire diameter $d$	[cm]	0.6	0.6	1.
radius of the outer conductor $b$	[cm]	12.5	8.	12.5
$d/s$		0.6	0.6	0.2
$b/a$		3.1	2.	3.3
$\tan \psi = \frac{s}{2 \pi a}$		0.04	0.07	0.13
number of windings per quarter of wave length $N$		4	9	5
$\frac{H_{max}(stand.w.)}{E_0 (trav.w.)}$	$\left[ \frac{Oe}{MV/m} \right]$	272.	267.	377.
$E_0$ (trav.w.) for $\psi$	[MV/m]	3.7	3.7	2.7
$H_{max}=1000$ Oe (stand.w.)		3.7	2.7	3.9
		760.	630.	430.
		1.3	1.6	2.3
		2.9	2.9	2.7



TABLE III Parameters for Helix postaccelerator

	Version I (Bromine)	Version II (Iodine)
$T$ (MeV/N)	1.25 - 6.16	0.79 - 6.26
$\beta = v/c$ (%)	5.15 - 11.4	4.1 - 11.5
$(\xi/N)_{min}$	0.288	0.22
Number of sections	16	23
Length (m)	21	30
$\Delta U$ (MV)	17.0	24.8
Frequency (MHz)	108.48	108.48
Synchronous phase	30°	30°
RF - power (MW)	1.1	1.5
Focusing	Magnetic quadrupoles, singulet, $N=1$	
Field gradient (kG/cm)	0.4 - 0.9	0.7 - 1.2
Radial acceptance (cm mrad)	24.6	21.0
Normal acceptance ( $\beta \cdot A$ )	1.26	0.87
Energy resolution $\Delta T/T$	< 1%	< 0.8%
Energy resolution $\Delta T/T$ with debuncher	< 0.1%	< 0.1%
Energy variation (continuously)	1.25 - 6.15	0.79 - 6.26
Energy variation with microstructure	3.5 - 6.15	3.5 - 6.26
Radius of outer conductor (cm)	12.5	12.5
Radius of helix (cm)	2.5 - 2.0	3.0 - 2.0

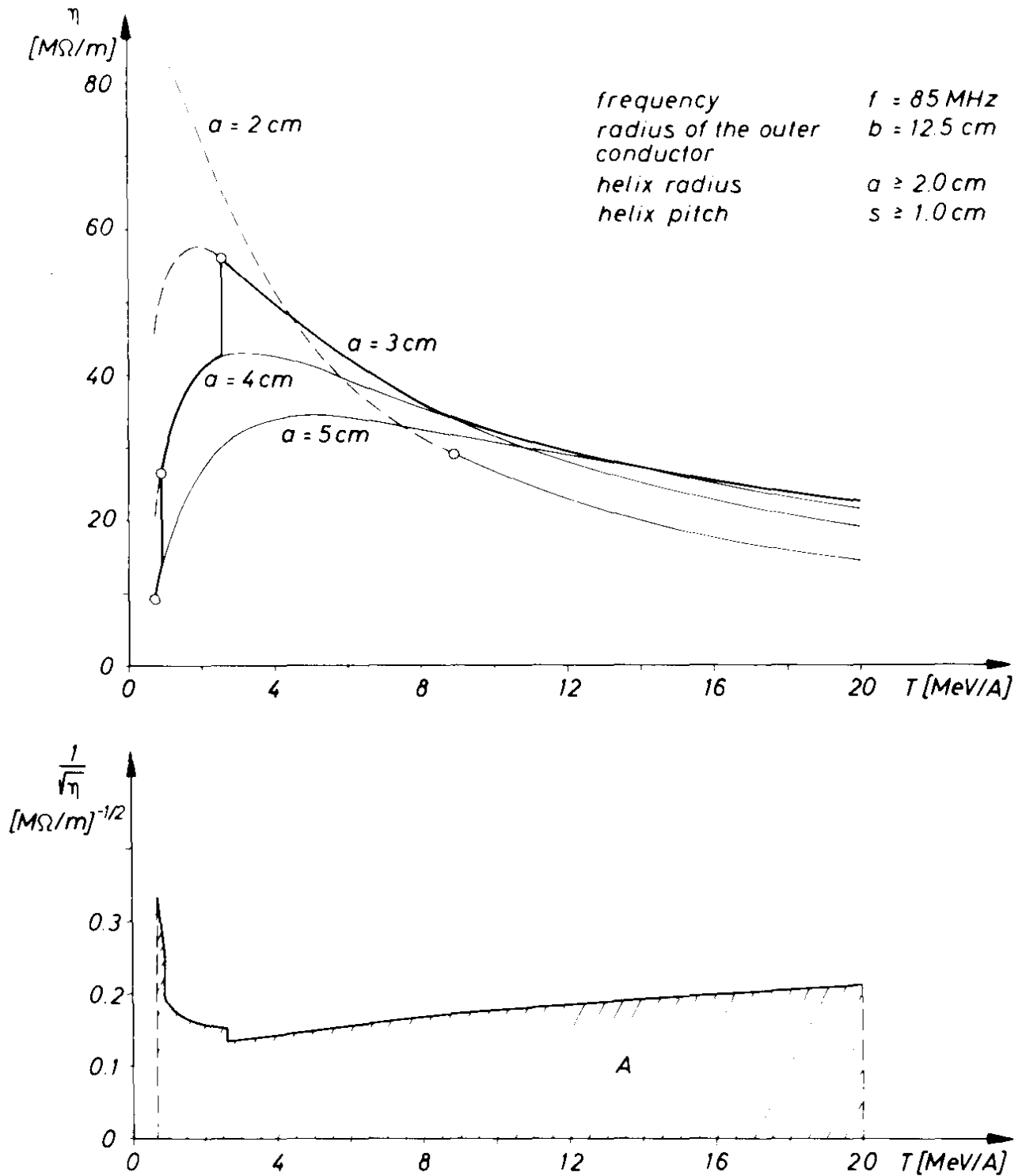
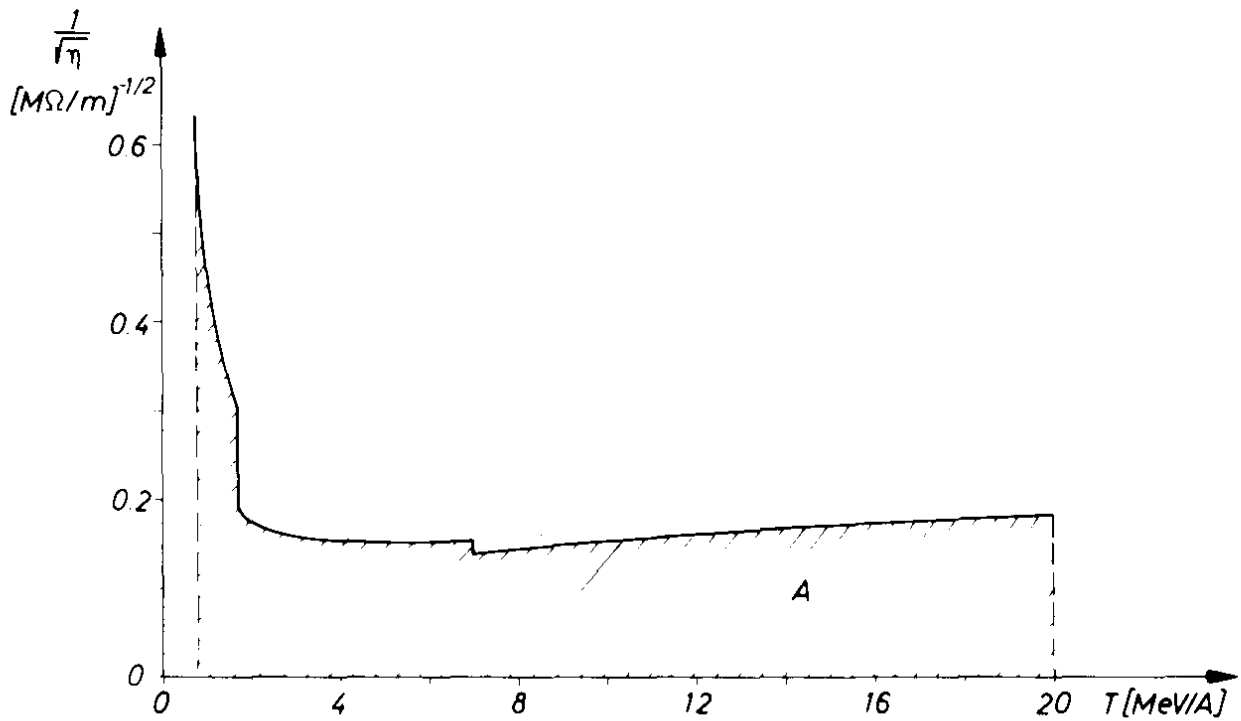
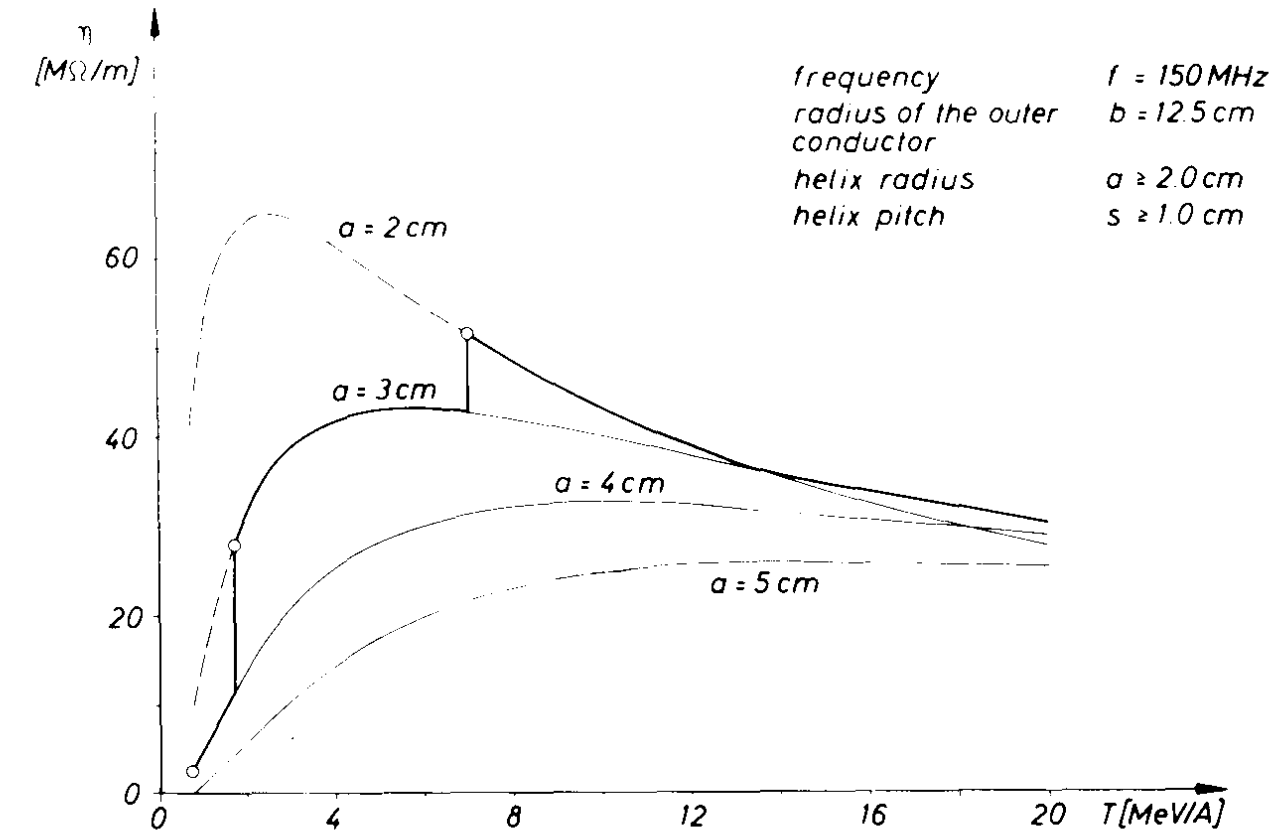


Fig. 1

Shunt impedance  $\eta$  versus particle energy  $T$  and optimization area  $A = \int dT \eta^{-1/2}$  (Optimization for superconducting proton HELAC, values for copper at room temperature)



Shunt impedance  $\eta$  versus particle energy  $T$  and optimization area  
 $A = \int dT \eta^{-1/2}$  (Optimization for superconducting proton HELAC, values for copper at room temperature)

Fig. 2

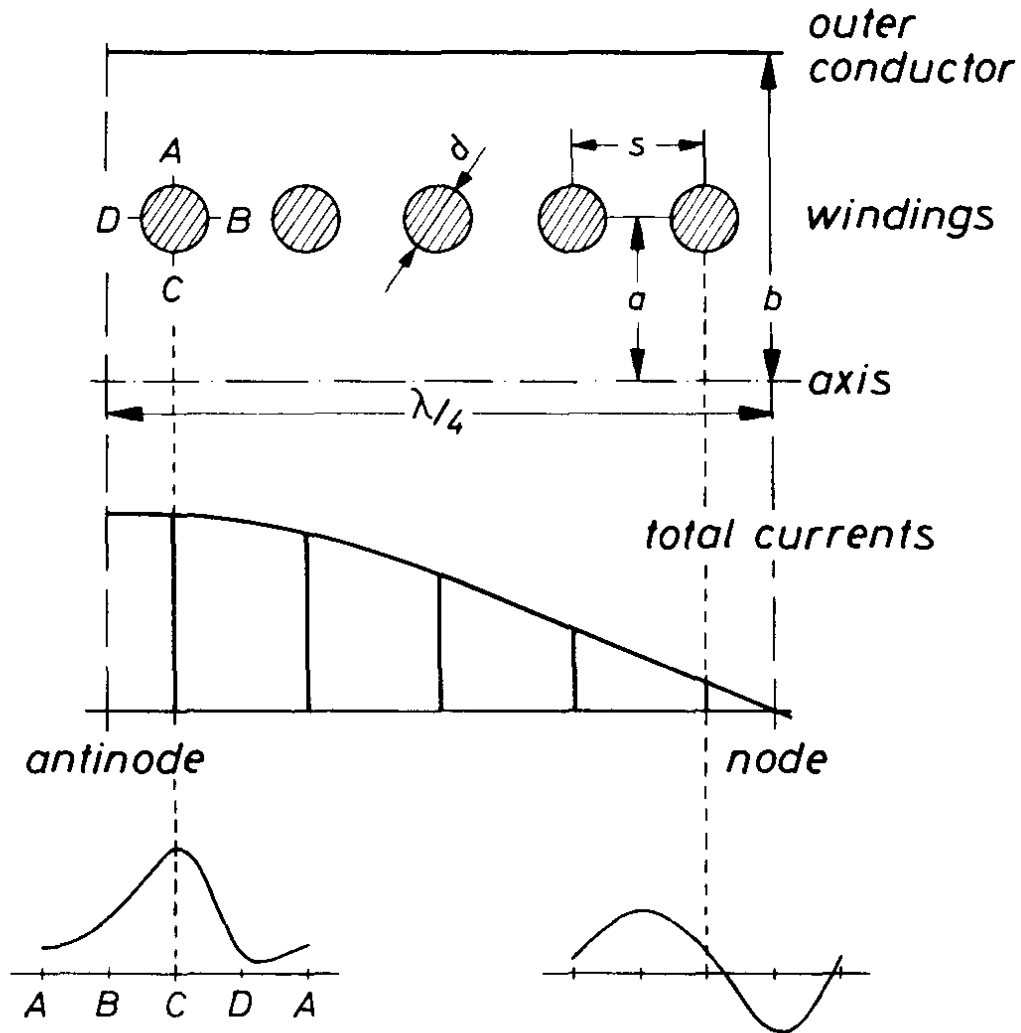


Fig. 3 Current distribution around the windings

Parameters:

$d/s =$  wire diameter/pitch

$\tan \psi = s/2\pi a = 1/2\pi \times \text{pitch}/\text{winding radius}$

$b/a =$  radius of outer conductor/winding radius

$N = \lambda/4s =$  number of windings per quarter of wave length

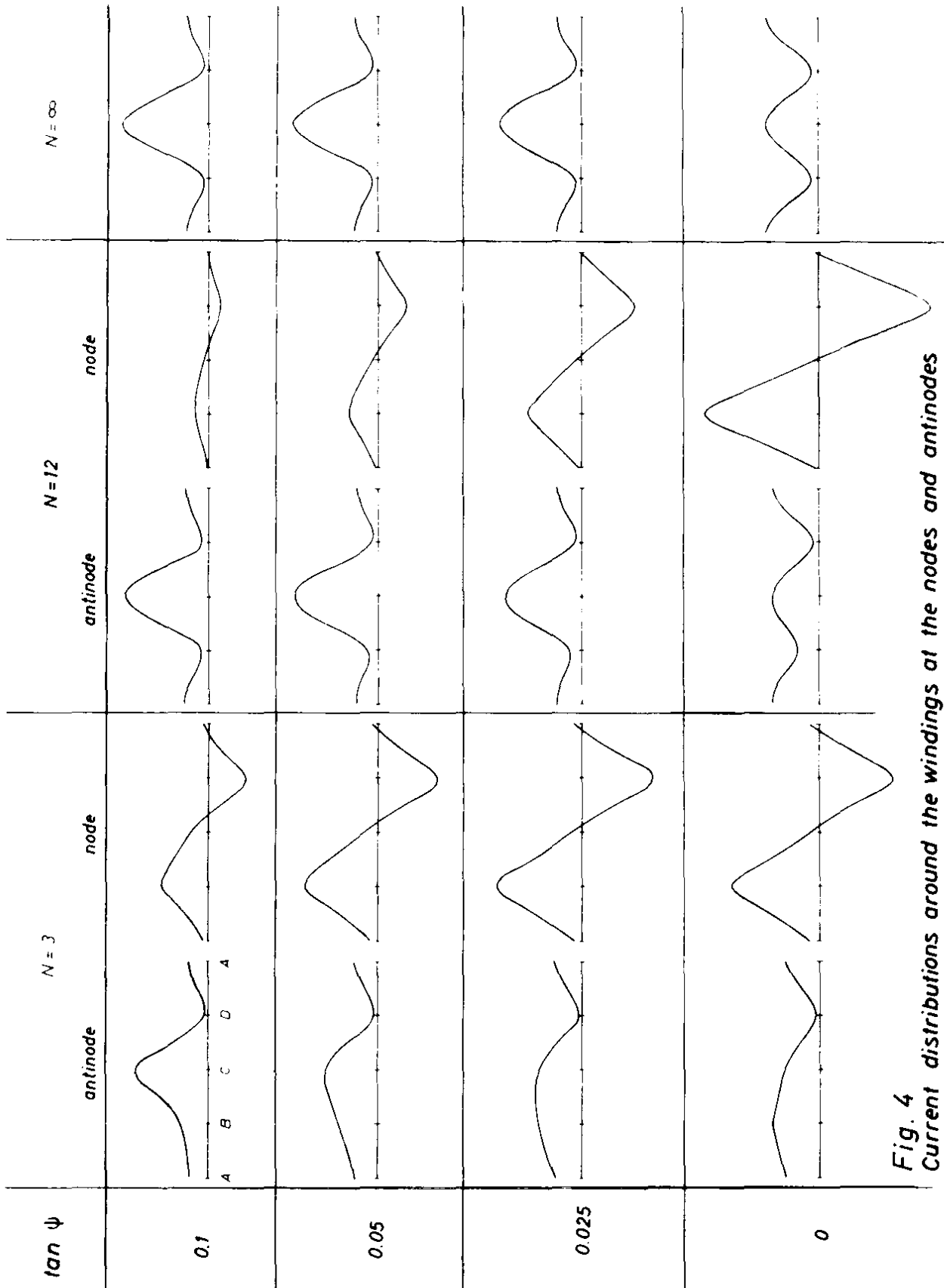
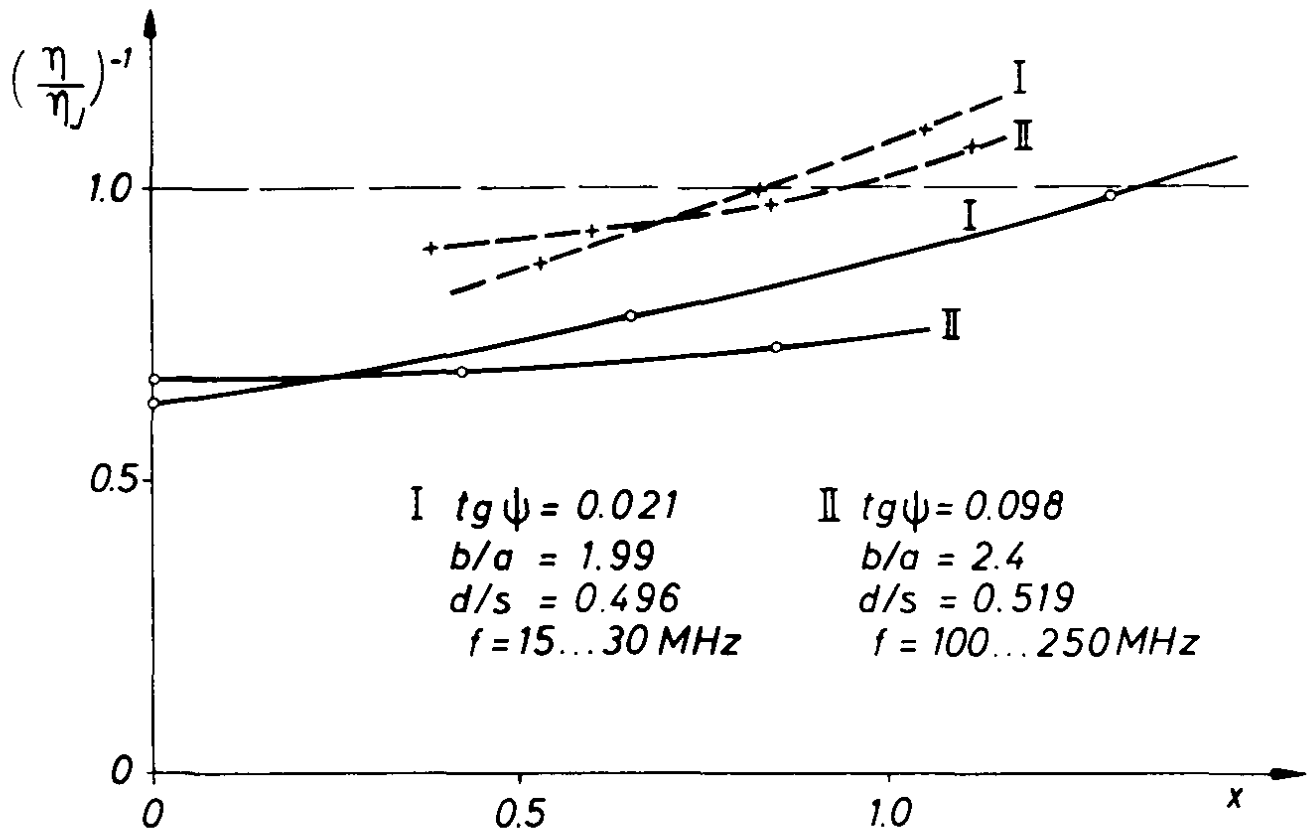
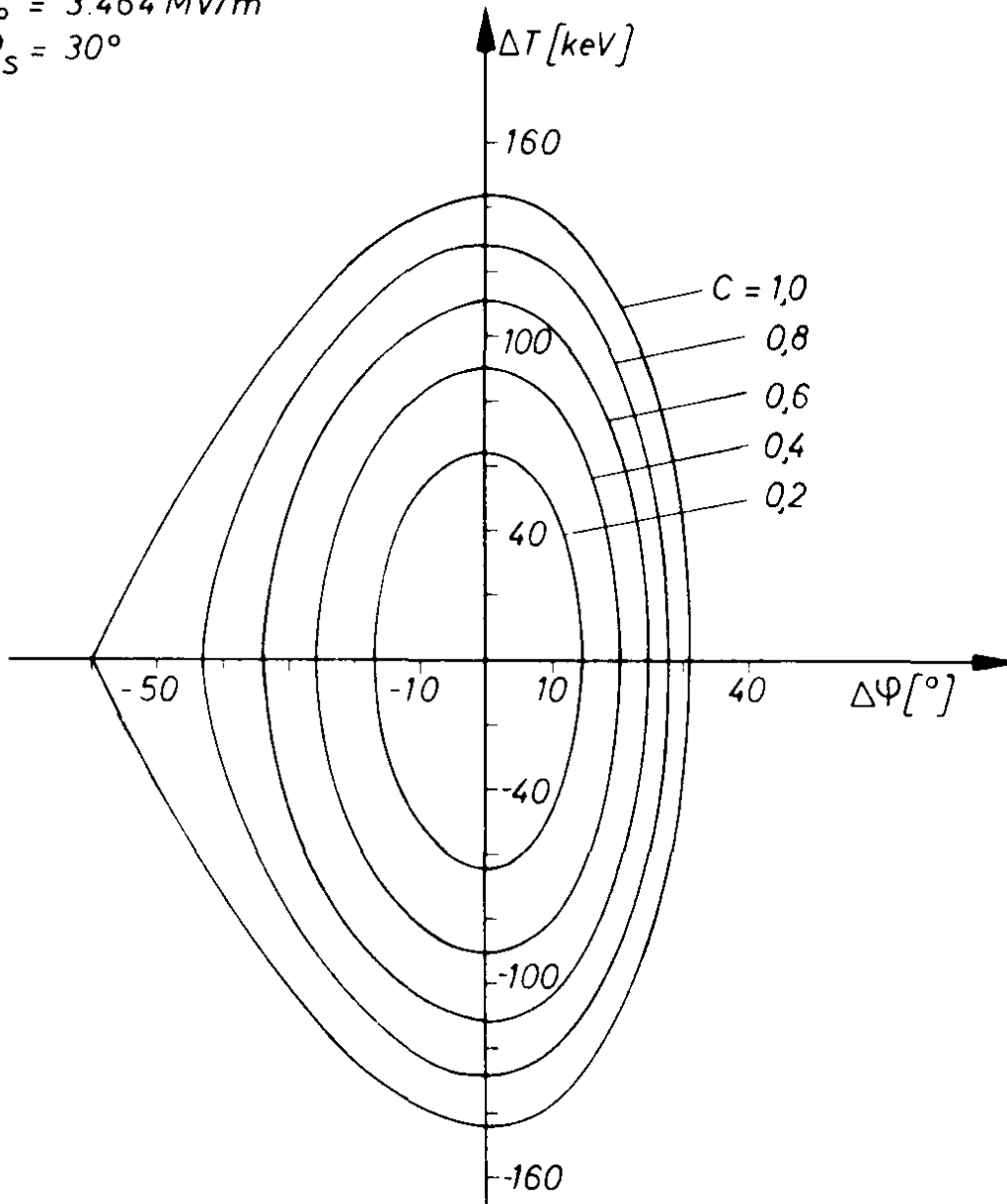


Fig. 4  
Current distributions around the windings at the nodes and antinodes  
of total current for different values of  $N$  and  $\tan \psi$ ;  $d/s=0.7$ ,  $b/a=2$ .

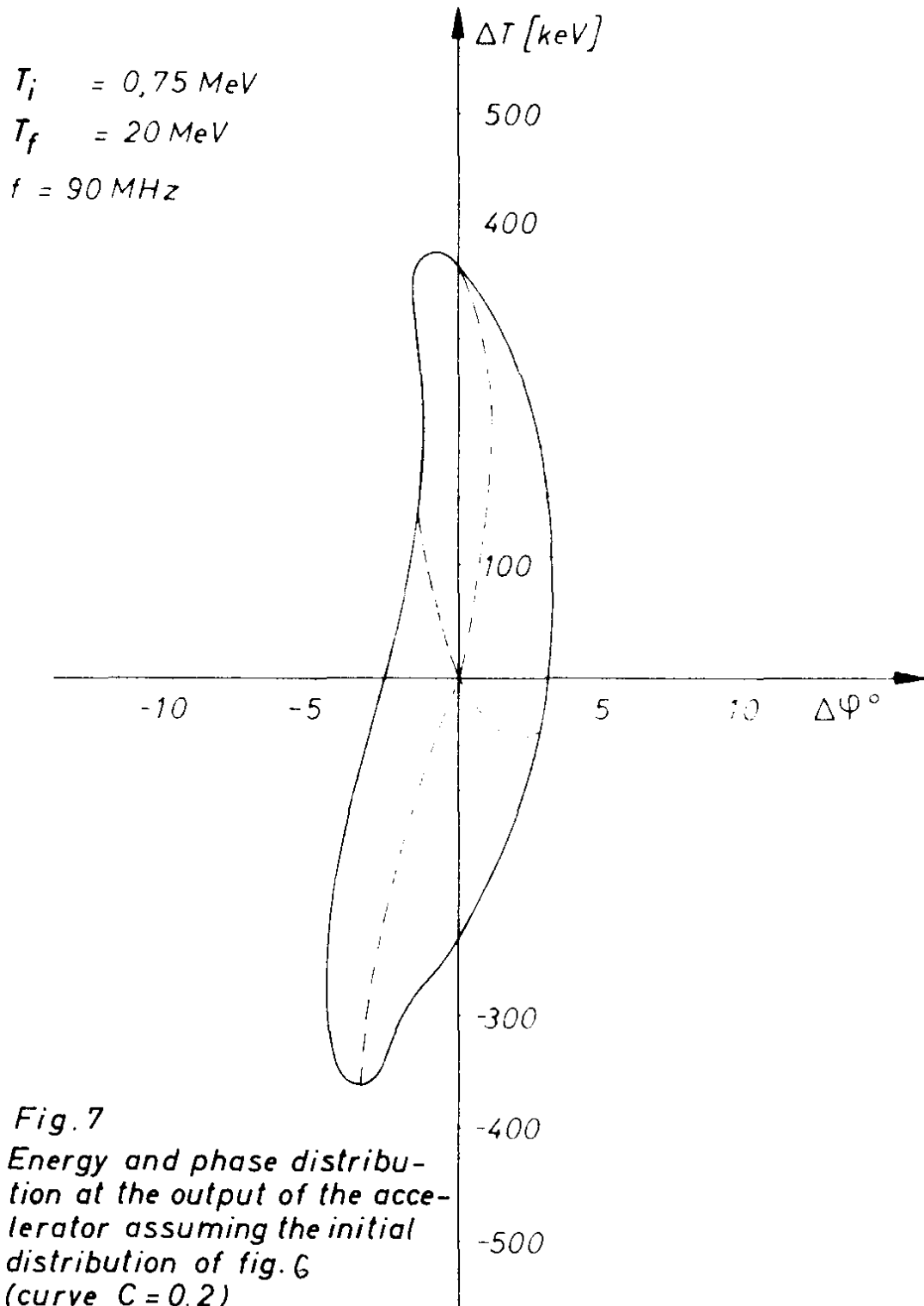


*Fig. 5*  
 Ratio of the shunt impedance  $\eta$  (---measured, —calculated) and the approximative sheath model values (Johnsen)  $\eta_J$  as a function of  $x = \gamma \cdot a \approx \frac{2\pi f}{v} \cdot a$

$T = 0.75 \text{ MeV}$   
 $f = 90 \text{ MHz}$   
 $E_0 = 3.464 \text{ MV/m}$   
 $\Psi_s = 30^\circ$



**Fig. 6** Longitudinal motion,  
energy - phase - acceptance





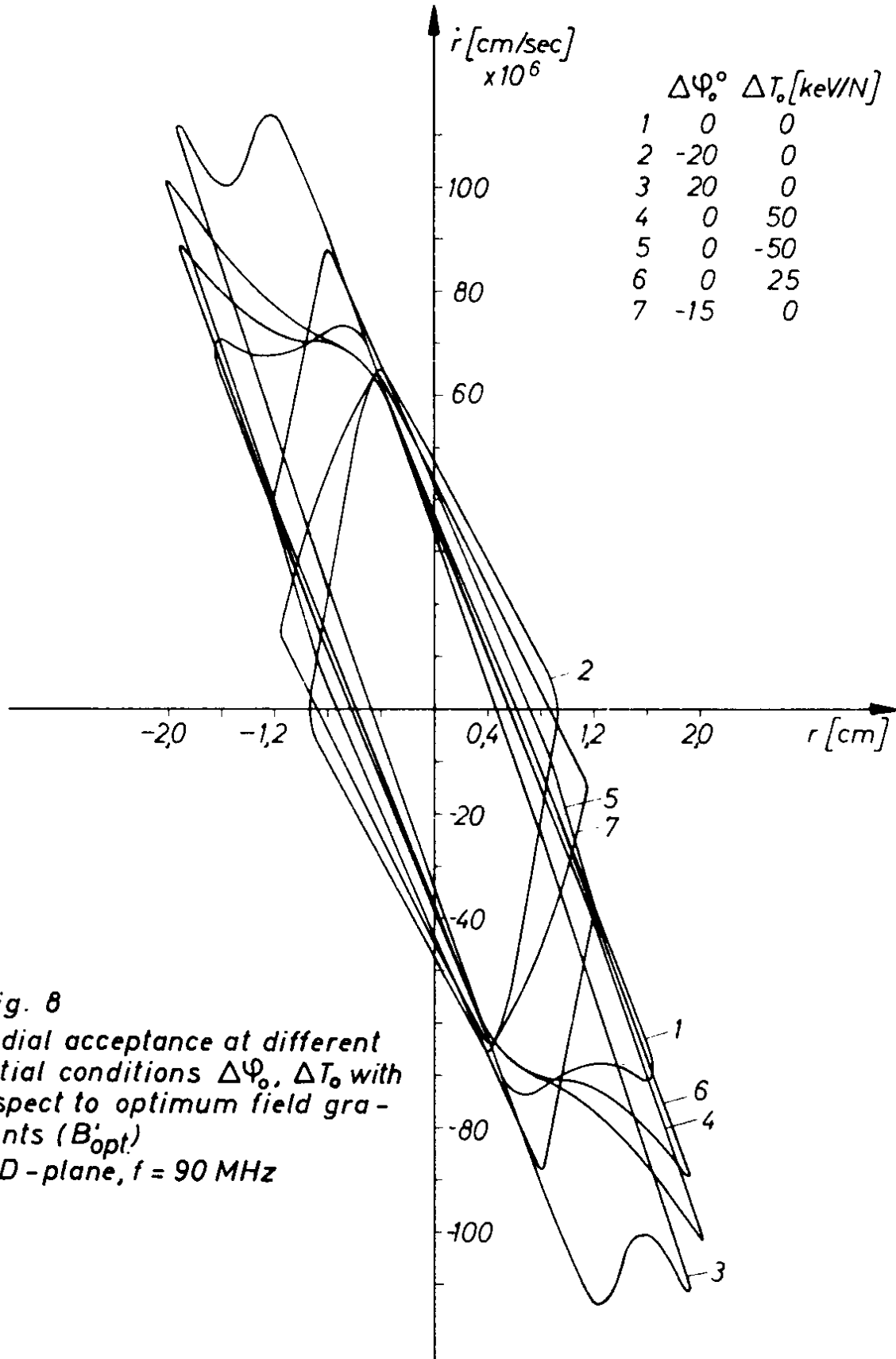


Fig. 8  
 Radial acceptance at different  
 initial conditions  $\Delta\Phi_0, \Delta T_0$  with  
 respect to optimum field gra-  
 dients ( $B'_{opt}$ )  
 HFD - plane,  $f = 90$  MHz

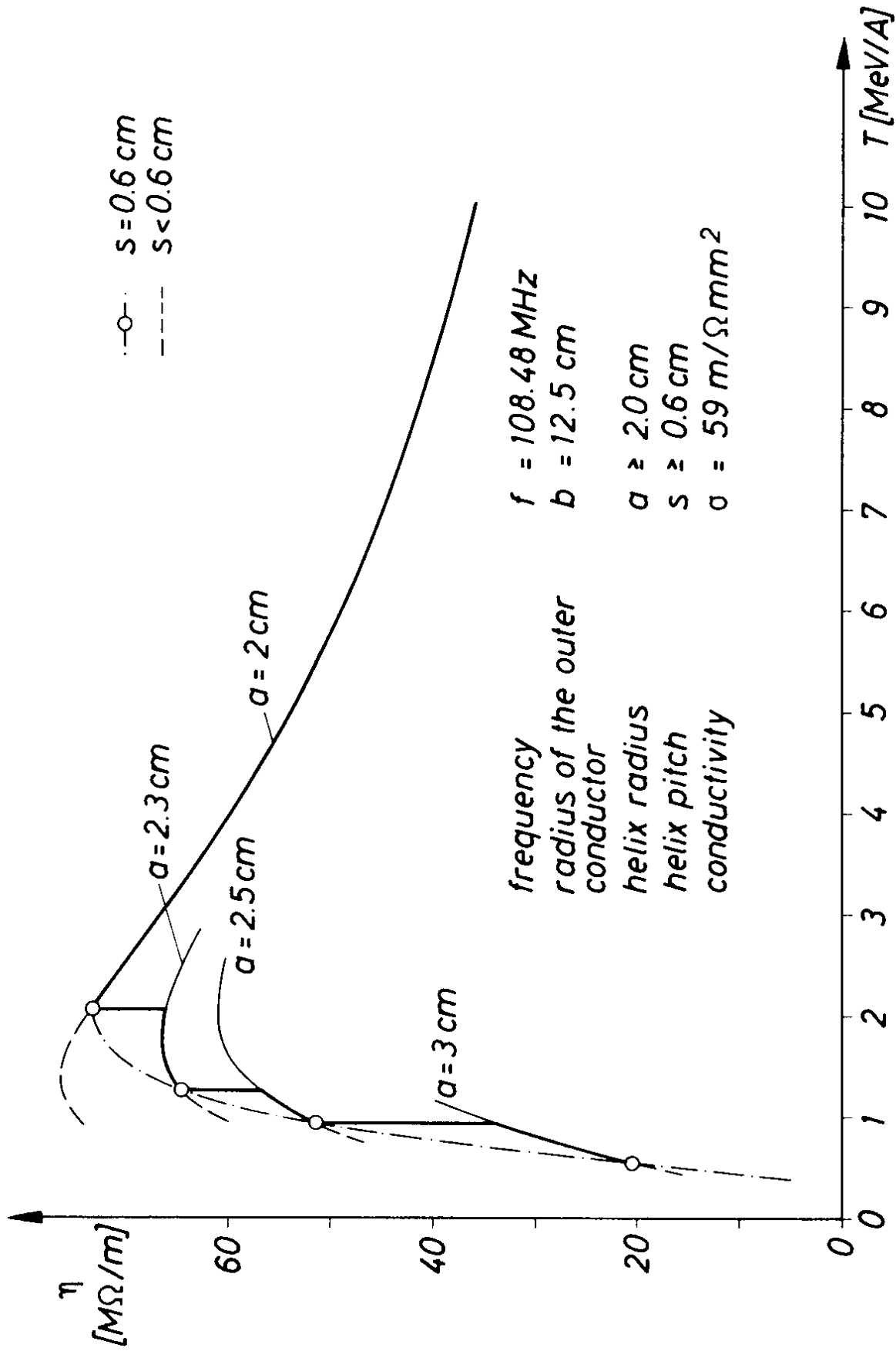


Fig. 9 Shunt impedance  $\eta$  versus particle energy per nucleon  $T$  (HELAC for heavy ions)

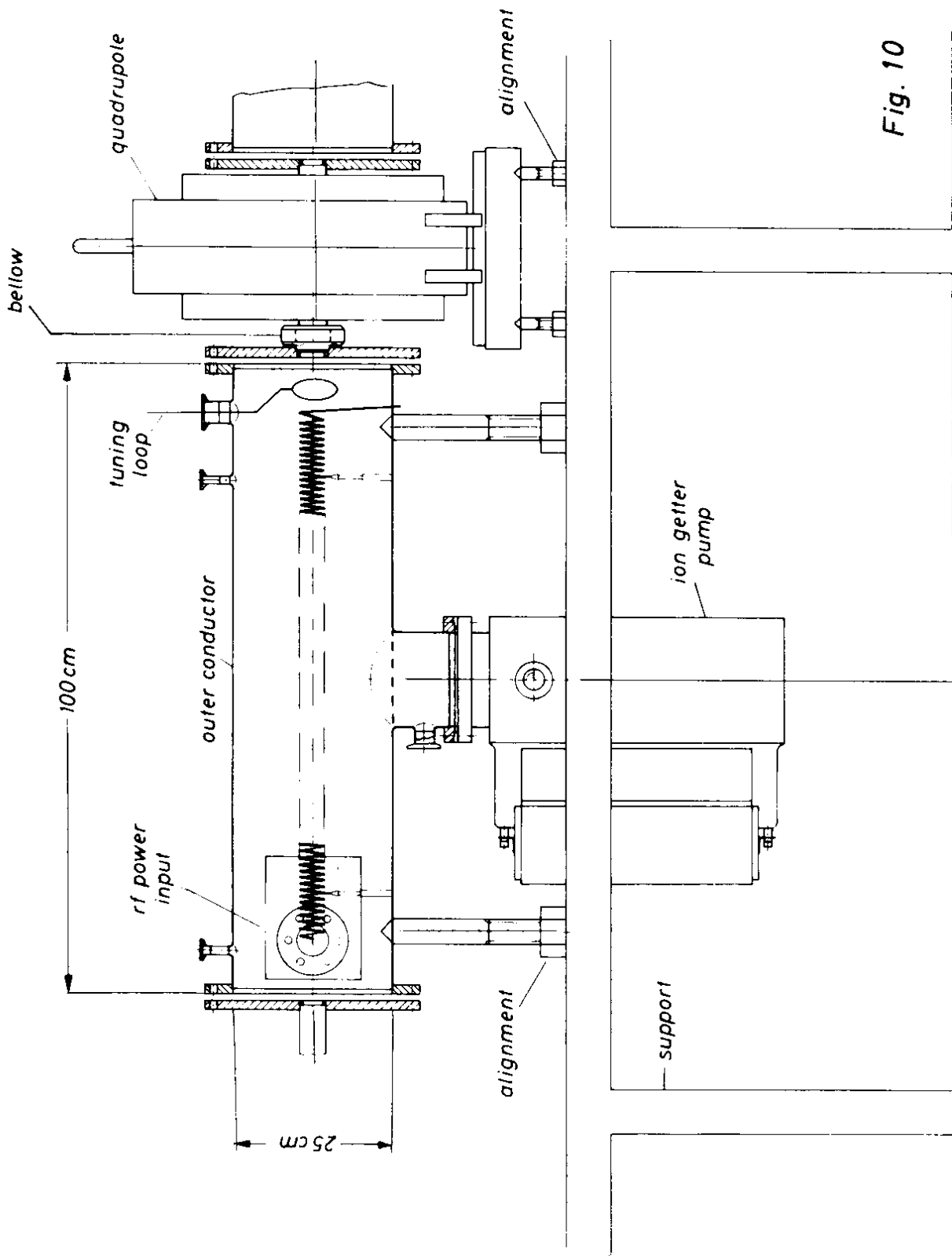
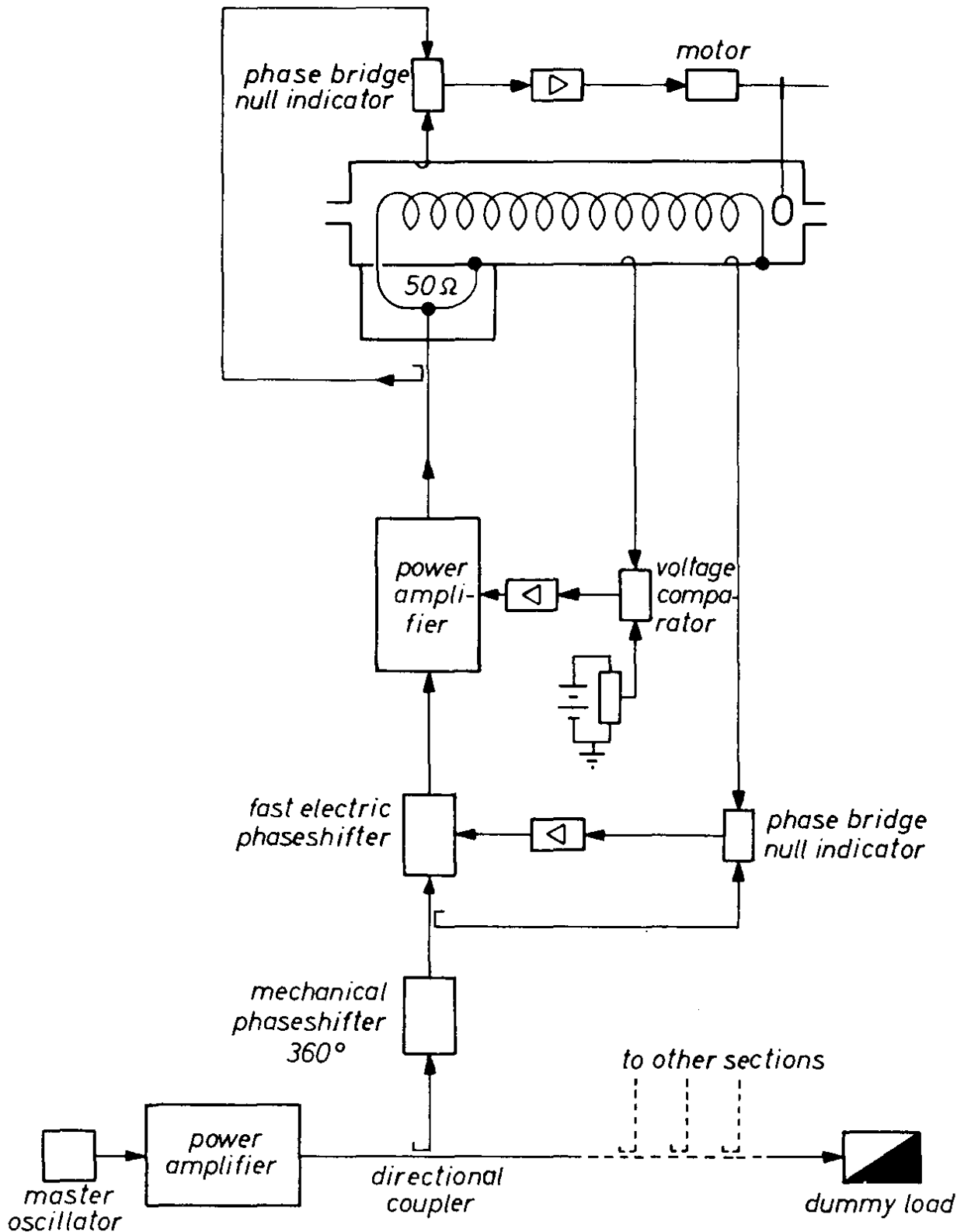


Fig. 11  
RF-chain and control loops for one section, schematic.



DISCUSSION

G. A. Loew (SLAC): Could you say a little bit about the plans you have for the technology for making these helices out of niobium or lead or other materials?

A. Citron (Karlsruhe): Tubing can be bought and then we wind them on a mandrel. So far nothing terrible has happened. So far we are not tied to any particular tolerances since we just wanted to make a helix and could put it on the right frequency.

H. A. Schwettman (Stanford): In optimizing the helix structure for a superconducting accelerator the shunt impedance is not a very important consideration. The energy gradient and the coupling between the electromagnetic fields and the mechanical vibrations are the most important factors.

H. Klein (Frankfurt): I agree with you. One possibility to overcome the mechanical difficulties could be to increase the diameter of the helix.

D. F. Nagle (LASL): In the helix accelerator for heavy ions, what was the injection energy?

H. Klein: About 1 MV per ion.

D. F. Nagle: If you change from one heavy-ion species to another one, will you change the excitation of the tanks?

H. Klein: Mostly the phases between the tanks.

J. E. Vetter (Karlsruhe): I'd like to make an additional comment on the question of how the helices are fabricated and how to adjust the tolerances. As far as we all know, that is no problem at all, but later on we will fire the tanks for the first time and adjust them and then most of the stresses will be out and then we will fire them for a second time. What we have actually done is experiments as to how these helices can be supported in such a furnace and this seems to be a severe problem. In the first attempt we made, the helix was stretched to a single wire and so it wasn't the right way. What we did then was to make small rods of tungsten and supported each winding of the helix; this had good results.

J. P. Blewett (BNL): The interesting structure that you showed in your last slide looked as though it might have quite a few other resonances besides the ones for which it was designed. Could you tell us something about the tests on this?

H. Klein: No. Just one resonance frequency for all the system, and it was designed for energy per nucleon about 1 MeV. There were no other resonances besides these.