BAND CLOSING IN AN ALVAREZ TYPE PERIODIC STRUCTURE

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ABSTRACT

Dispersion curves for azimuthally symmetric TM modes in an Alvarez type structure are studied. The method of Walkinshaw et al., as formulated in the 1969 IEEE paper of Gluckstern et al., has been reformulated for arbitrary phase advance per cell in a periodic stemless drift tube structure. In the present formulation, the dispersion curve is obtained from a two by two transcendental determinant, permitting simpler and more accurate computation than with the 2N by 2N determinant for an arbitrary N cell drift tube structure in the previous work. The new formulation has been used to study band closing by varying the drift tube radius to achieve tuning of the equivalent coupling cell. In this way, two bands have been made to cross at the π-mode with non-zero slope. The behavior of the dispersion curves for the undercoupled and overcoupled cases is also shown. Information is also obtained about higher bands. The relations of these results to those obtained with coupled circuit chains is discussed.

I. Introduction

In a previous paper,[1] the method of Walkinshaw, Sabel, and Outram[2] was extended to treat cavities consisting of stemless drift tube loaded cells with varying lengths. In the course of analysis of different geometries, it became clear that it was desirable to start from the dispersion curve for a periodic structure. Although the previous method was used to obtain results for a periodic structure, it became cumbersome and inaccurate for cavities with a large number of cells. For this reason, we here present an alternative rapidly convergent formulation for a periodic structure with arbitrary phase advance per cell.

A further modification of the previous method is the inclusion of a first harmonic term in addition to the constant term in the gap. This allows us to obtain complete dispersion curves, including modes which had not appeared before. The dispersion curves obtained here are for azimuthally symmetric TM modes, but the method can easily be adapted to other TE and TM modes in the same structure.

In the course of the present work, we shall explore the behavior of the pass bands in an Alvarez-type structure. In particular, we shall study band closing[3] which has been the basis for the development of remarkably stable structures at Brookhaven[4] and Los Alamos.[5] Although our physical geometry at band closing is unlike that used in a multi-stem,[4] tuned post,[5] or cross bar[6] configuration, our results will not be dependent on the validity of an equivalent circuit model.

II. Analytical Model

The basic cell of the structure to be analyzed is shown in Figure 1. The field within the drift tube radius r=a is taken to consist of two terms: the first term is independent of z reflecting symmetric behavior in the gap; the second term has z-dependence $\sin(\frac{\pi z}{g})$ reflecting antisymmetric behavior in the gap. Thus, the axial electric field in the gap is

$$E_z = E_o \frac{J_o(kr)}{J_o(ka)} + E_1 \frac{I_o(\sigma r)}{I_o(\sigma a)} \sin\left(\frac{\pi z}{g}\right) \tag{1}$$

where

$$k = \frac{\omega}{c} \text{ and } \sigma^2 = \left(\frac{\pi}{g}\right)^2 - k^2$$

with the azimuthal magnetic field and radial electric field given in terms of E_o and E_1 by Maxwell's equations. In the present formulation, the fields explicitly satisfy the Floquet theorem. Hence, the axial electric field in the region between the drift tube radius r=a and the wave guide radius r=b can be expressed in the form

$$E_z = e^{i\phi\frac{z}{L}} \sum_{-\infty}^{\infty} f_n e^{i2n\pi\frac{z}{L}} \begin{Bmatrix} F_o(K_n r)/F_o(K_n a) \\ G_o(\mu_n r)/G_o(\mu_n a) \end{Bmatrix} \tag{2}$$

where

$$F_o(K_n r) = Y_o(K_n r) - J_o(K_n r)\frac{Y_o(K_n b)}{J_o(K_n b)} \quad \text{for } k^2 - \left(\frac{n\pi}{L}\right)^2 = K_n^2 \geq 0 \tag{3}$$

$$G_o(\mu_n r) = K_o(\mu_n r) - I_o(\mu_n r)\frac{K_o(\mu_n b)}{I_o(\mu_n b)} \quad \text{for } \left(\frac{n\pi}{L}\right)^2 - k^2 = \mu_n^2 \geq 0 \tag{4}$$

The azimuthal magnetic field and radial electric field are given in terms of f_n by Maxwell's equations.

The coefficients f_n are expressed as a linear combination of E_o and E_1 by requiring continuity of E_z at r=a over the full cell length. If we then require that the average magnetic field and the first harmonic of the magnetic field in the interior and exterior regions match at r=a within the gap, we obtain a 2x2 system of homogeneous linear equations for E_o and E_1. A non-vanishing solution requires the vanishing of the determinant

$$\begin{vmatrix} \frac{J_1(ka)}{J_o(ka)} - \frac{g}{L}\sum_{-\infty}^{\infty}\frac{k}{K_n}\frac{F_1(K_n a)}{F_o(K_n a)}T_n^2 & -\frac{g}{L}\sum_{-\infty}^{\infty}\frac{k}{K_n}\frac{F_1(K_n a)}{F_o(K_n a)}T_n S_n \\ -\frac{g}{L}\sum_{-\infty}^{\infty}\frac{k}{K_n}\frac{F_1(K_n a)}{F_o(K_n a)}T_n S_n & \frac{k}{2\sigma}\frac{I_1(\sigma a)}{I_o(\sigma a)} - \frac{g}{L}\sum_{-\infty}^{\infty}\frac{k}{K_n}\frac{F_1(K_n a)}{F_o(K_n a)}S_n^2 \end{vmatrix} = 0 \qquad (5)$$

where

$$T_n = \frac{\sin[(\phi + 2n\pi)\frac{g}{2L}]}{(\phi + 2n\pi)\frac{g}{2L}} \quad \text{and} \quad S_n = \frac{2}{g}\,\frac{(\frac{2n\pi + \phi}{L})\cos[(\phi + 2n\pi)\frac{g}{2L}]}{[(\frac{\pi}{g})^2 - (\frac{\phi + 2n\pi}{L})^2]} \qquad (6)$$

As in the previous work, the above result is identical to that obtained for a multi-cell structure by minimizing

$$q^2 = \frac{\int dV(\nabla \times \vec{E})^2}{\int dV\ \vec{E}^2}$$

subject to a variation of E_o and E_1 in each gap and the frequency k. In this case, the integral must be taken over the volume of the multi-cell cavity.

Equation (5) is just what one would obtain from the previous formulation,[1] except that now the phase advance per cell, ϕ, is included explicitly, and we have only a 2x2 determinant independent of the number of cells to be considered. (In the previous formulation for N cells, it was necessary to solve a 2N x 2N determinant.)

The resonant frequencies are those which satisfy equation (5). Since ϕ is included explicitly in the calculation of the resonant frequency, dispersion curves are readily obtained. It is not necessary to find relative field strengths in the gaps in order to determine ϕ, although the relative values of E_o and E_1 determine the symmetry properties of the field.

III. Numerical Results and Discussion

A computer program has been written which solves Equation (5). An initial frequency is guessed for a particular ϕ, and then the program searches for the frequency in that vicinity which makes the determinant vanish. In order to satisfy accuracy requirements in the previous formulation, it was necessary to take high values of n before the asymptotic values could be used. If N=1 (one cell) was the case, then n_{max} = 10 was satisfactory, while for N=7, n_{max} = 500 was necessary. The present calculation is independent of N and $|n_{max}|$ = 10 suffices in our calculation of dispersion curves. So, while the present method does not work for non-uniform structures, we have gained a simple, accurate method for studying the behavior of several pass bands. In particular, we explored band closing at the π-mode by varying the drift tube radius to achieve tuning of the equivalent coupling cell.

It should be pointed out that the achievement of band closing by proper adjustment of the drift tube radius leads to geometries which are not of practical interest. Moreover, the existence of stems will perturb those modes with radial electric field. However, we should be able to obtain some insight into the behavior of dispersion curves when one achieves band closing in multi-stem or tuned post drift tube configurations.

A second part of our study is concerned with the behavior of higher bands.

A. Band Closing

We used the above program to search for a possible closing of pass bands at the π-mode by varying the drift tube radius and observing how the frequencies of the π-mode of the first two pass bands vary. For a structure with b = 10.16 cm, g = 6.096 cm, and L = 15.24 cm we found that the first two pass bands came together (with a non-zero slope) for a drift tube radius a = 8.905 cm and a wavelength λ = 24.83 cm. Figure 2 shows the first two pass bands for the closed case, as well as for both a neighboring undercoupled and overcoupled case.

B. Higher Bands

We also used the program to study the pass band behavior for the more realistic geometry a = 1.9 cm, b = 10.16 cm, g = 6.096 cm, and L = 15.24 cm. The first three pass bands are shown in Figure 3. The first two pass bands look as one would expect. However, the third pass band exhibits a surprising zero slope near $\pi/2$ which is not consistent with an alternating structure equivalent circuit model in which only adjacent cells are coupled.

One possible explanation for this behavior is the limitation of only two terms in the gap in our calculation, which is clearly incorrect at very short wave lengths. Even if the calculation should prove to be reliable, it is possible to construct an equivalent circuit model with next nearest neighbor coupling which yields a dispersion curve that has a vanishing slope for a phase advance per repeat length between 0 and π.

Figure 4 shows the electric field distribution for modes at the ends of the pass bands for this case.

IV. Conclusions

We have developed a method for obtaining dispersion curves in a periodic stemless drift tube structure. The method has been used to explore the behavior of such a "real" structure (as opposed to an equivalent circuit model) for a geometry corresponding to the closing of a gap in the dispersion curves. We confirm the fact that at such a closing the dispersion curves cross at finite angle, thus implying a desirable insensitivity to errors.

We have further determined the nature of the field distributions for several of the modes at the ends of pass bands. A point of interest is that the π-mode in the lowest TM band sometimes corresponds to a field distribution in which E_z is anti-symmetric within a gap.

References

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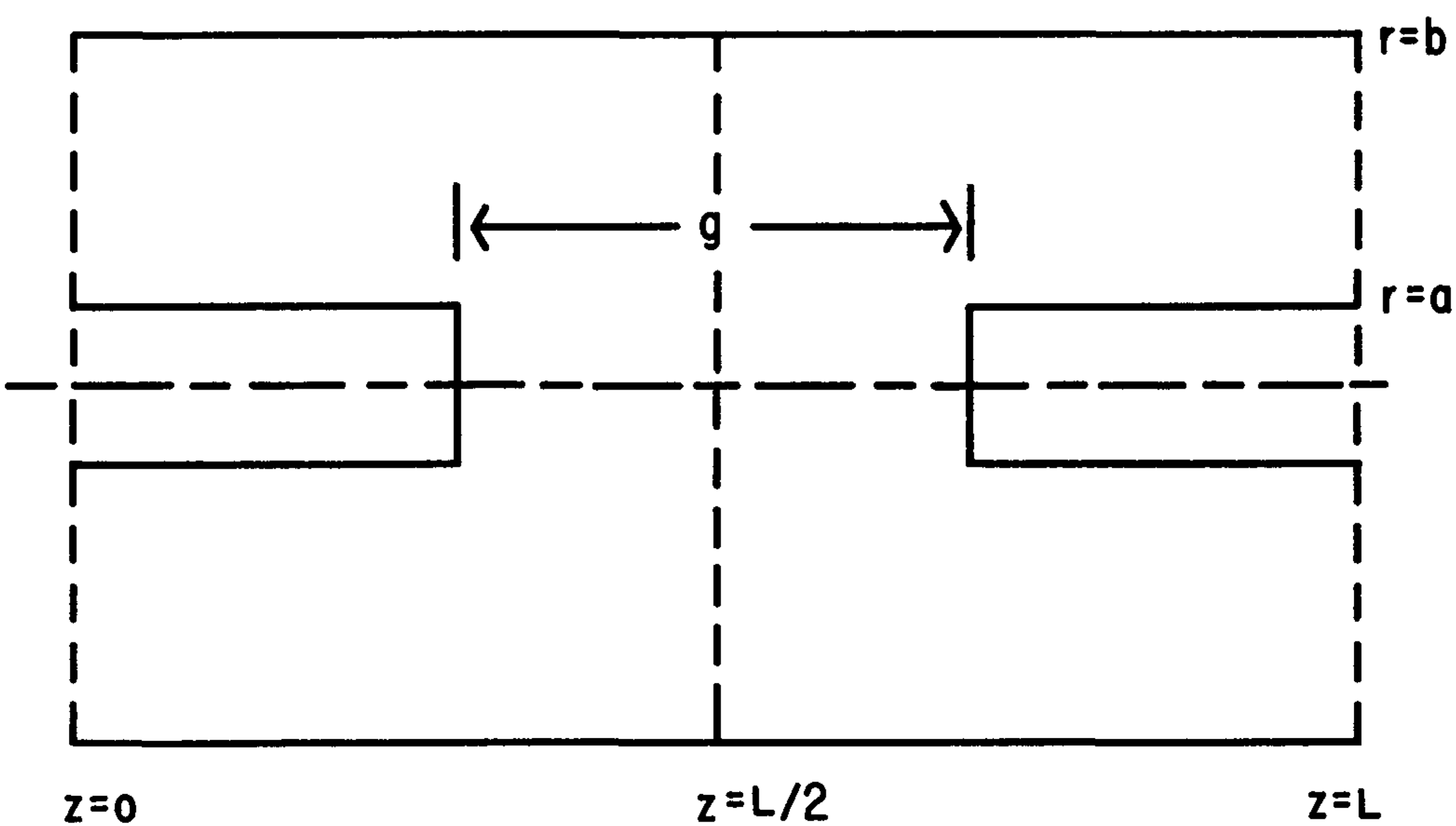

Fig. 1. Drift-tube cell geometry.

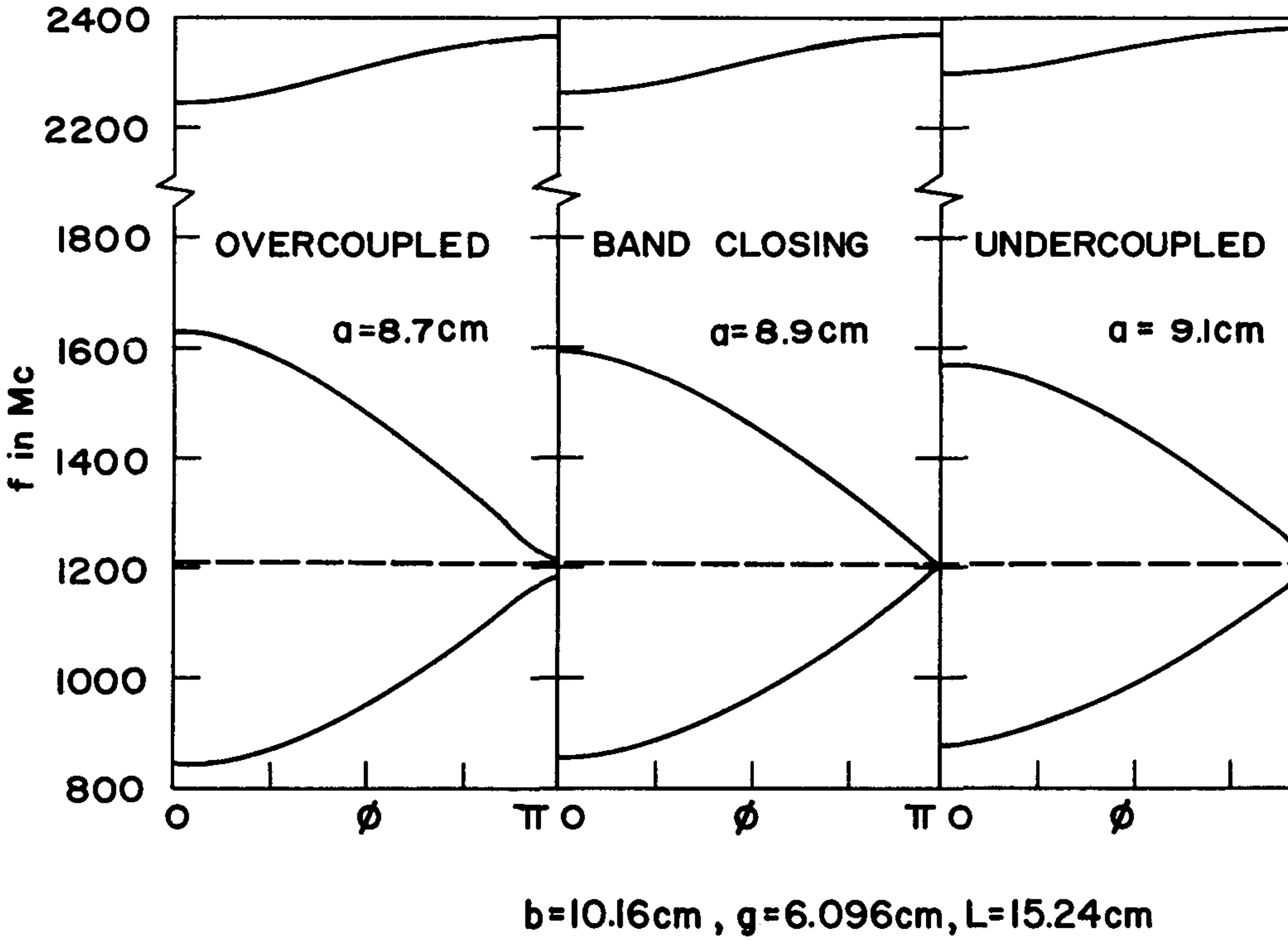

Fig. 2. Dispersion curves in the vicinity of band closing.

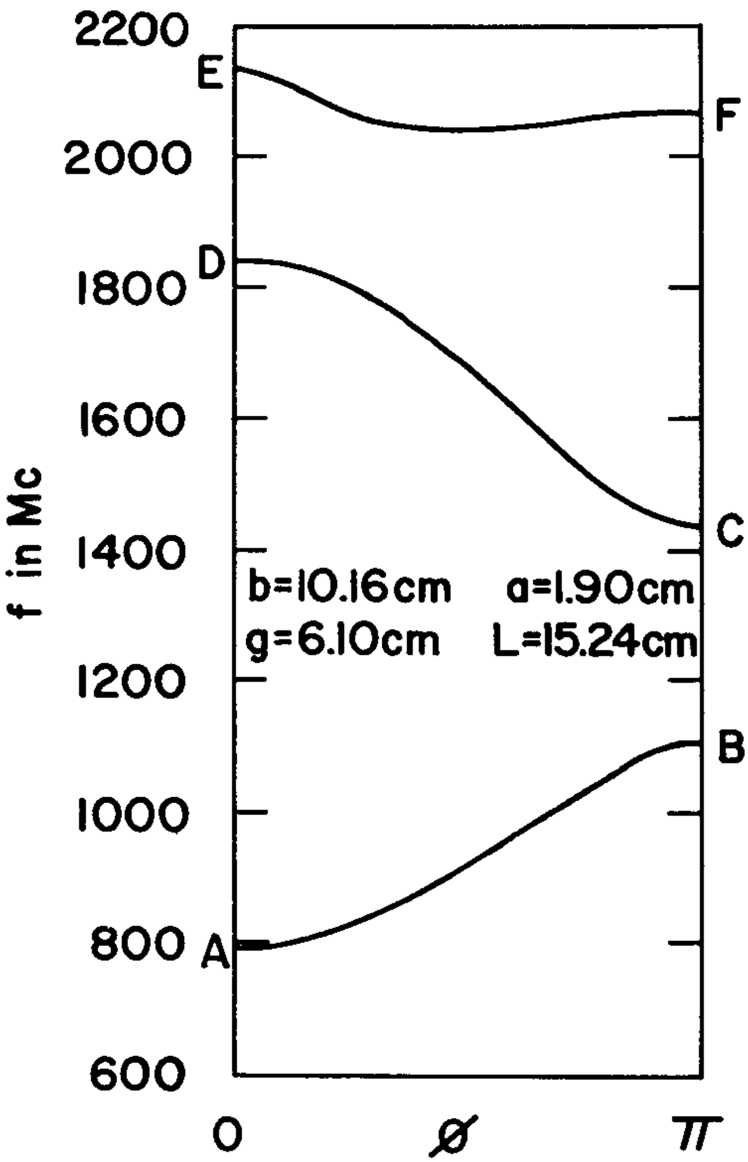

Fig. 3. Dispersion curves for conventional geometry.

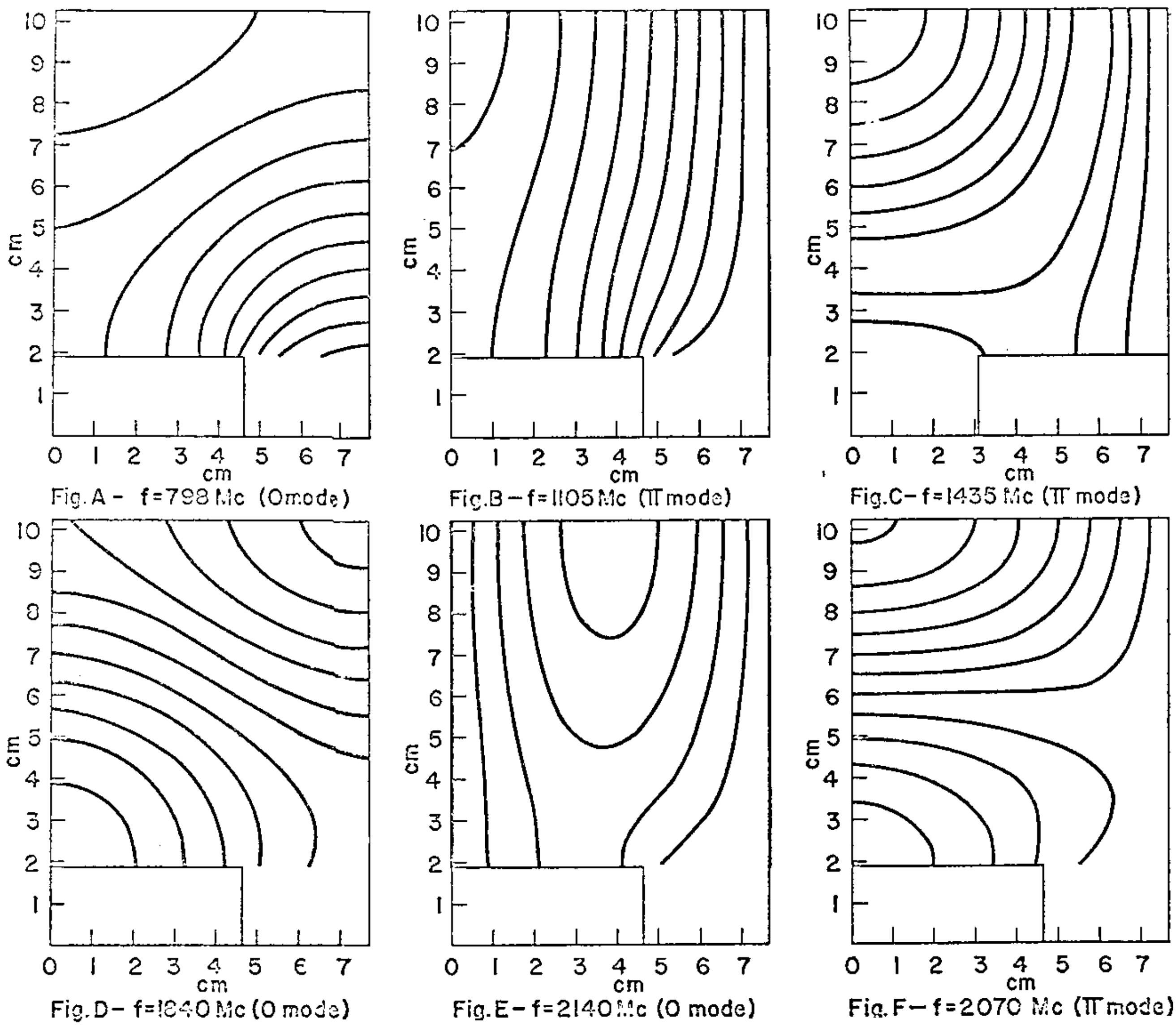

Fig. 4. Field plots for modes corresponding to Fig. 3.